QUALITATIVE REPRESENTATION AND REASONING ABOUT SHAPES

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Abstract. In this paper we present an approach to the qualitative representation of shape and its use. We use a qualitative coding scheme founded on landmarks in the shape. The scheme encodes a qualitative representation of angles, relative side lengths and curvatures at landmarks. We then show how such a representation can be used as a basis for reasoning about shapes using extracted shape features. We conclude with a preliminary analysis of 12 sketches of the architect Louis Kahn and show how they may be categorised based on these shape features.

1. Introduction

Designers use sketches in a variety of ways. One of the most significant ways is not simply as external memory but as a design medium on which they can reflect and with which they can have a "conversation" (Schön, 1983; Schön and Wiggins, 1992). The difference between sketches as external memory and sketches as a design medium is profound. As external memory the implication is that a sketch represents something unique which is to be held. As a design medium a sketch represents something to be interpreted and understood in different ways and to be made more concrete later as further decisions are taken. In general, the use of sketches as a design medium occurs at the conceptual stage of designing while the use of sketches as external memory occurs at the detail stage of designing.

Clearly, the computational representations at these two stages need to be different to allow their fundamentally different purposes to be carried out. Sketches become drawings at the detail design stage and are a means of specifying individual objects with the goal of transferring information about objects so they can be directly recognised and fabricated or constructed. Current CAD systems have largely been developed for this purpose. However, they do not serve adequately to represent the information in sketches at the conceptual design stage. Why not? One way to conceive of the difference between sketches at these two stages is to draw a distinction between what is being represented. At the detail design stage what is being represented is an individual – in object-oriented terms it is an instance. What is being represented at the conceptual design stage is not as well formed and does not yet have the characteristics of an instance: it is a class of potential objects from which individuals may be spawned in due course.

This paper takes the idea of sketches at the conceptual design stage as classes and develops computational representations for them such that they can be interpreted and analysed in various ways. To do this a qualitative representation scheme for shapes will be introduced; a scheme from which features may be inferred. In Section 2 sketches as shapes and their roles are described further prior to introducing a qualitative schema for shape representation. Section 3 describes how such a representation can be used to analyse shapes. The paper closes with a discussion of the issues this approach raises

2. Sketches and Shapes

2.1 SHAPES IN SKETCHES

Sketches in designing are normally composed of aggregations of lines that form a contour of some shape on two-dimensional space. Even though sketches and shapes are similar in their usage, there are some important differences. Sketch is a more general term referring to any aggregation of lines with some design significance. Shape, however, is more specific term, which is a set of closed and connected lines either with rectilinear or curvilinear line segments where vertices and/or nodes are located in defined positions on a two-dimensional plane. Shape exists at either sketch level or detailed design level with different denotations. A shape at the detailed level denotes a unique design solution to a specific design problem with all its numeric data. A shape at the sketch level, however, denotes a group of related categories of shape classes with no necessary numeric data. The latter rather focuses on the pictorial salience and shape patterns that are depictions of following elements:

- qualitative aspects of contour description in the form of patterns;
- qualitative and symbolic descriptions of shape attributes for the contour of a shape; and
- identity of shape attributes and an indication of their range values.

Thus, the description of sketches in any symbolic scheme may be treated as the problem of describing distinctive shape characteristics at the categorical level. Since these shape characteristics of sketches can be conceptualised as features (Jared, 1984), the representation of sketches becomes the issue of recognising, capturing, and representing these shape features as discrete symbols.

2.2 DESIGN FEATURES IN QUALITATIVE REPRESENTATIONS

We show the encoding of shape characteristics in terms of three discrete stages of representation. We use the ideas of chain coding, landmarks and qualitative symbolic representations as our starting point (Freeman, 1961; Egenhofer and Al-Taha, 1992; Jungert, 1993).

(i) physicality → symbol (P → S)

This is the phase where our qualitative representation scheme operates. Our approach is to represent characteristic physicality of a shape through three basic shape attributes, called Q-codes, and encode them into qualitative sign values:

vertex angle at a landmark (A)
relative length of edges at a landmark (L)
curvature of a boundary segment (K).

A qualitative encoding scheme have been devised for these shape attributes using the Q-codes shown in Table 1 (Gero and Park, 1997a; Gero and Park, 1997b).

TABLE 1. Definition of Q-codes.

	A-code	L-code/K-code
Numeric value range	$0 \leq \theta \leq 2\pi$	$-\infty \leq l,k < \infty$
Landmark set	$\{0, \pi\}$	$\{-\infty, 0, \infty\}$
Interval set	$\{[0,0],(0,\pi),[\pi,\pi],(\pi,0)\}$	$\{(-\infty,0),[0,0],(0,\infty)\}$
Q-code set	$\{A_{nil},A_-,A_0,A_+\}$	$\{L_-,L_0,L_+\}$, $\{K_-,K_0,K_+\}$

Symbol sequences are defined as follows:

- Q-code, α: α = {β_i½β   {A,K,L}, *i*   {-,0,+}}
- Q-sentence, σ: σ = {α_1 α_2 ... α_m} where $m = length(\sigma)$
- Q-word, ω: {ω_i α_j, *i*=1, *j* [1,*m*] or ω = α_j ... α_k, ½k - j½=*i*, *i*=[2,*m*]}

(ii) symbol → regularity (S → R)

As a result of the Q-code representation, the physicality of a shape is described as a sequence of symbols that is assumed to denote those pictorial characteristics of a shape. Some of those pictorial characteristics are easier to identify from the distinct structural regularities in encodings, while others are rather complicated because they appear in more complex and specific patterns of encodings.

We have used a linguistic analogy to investigate the structure of these symbol encodings. A basic element that represents an elementary physicality of the basic shape attributes is termed a Q-word. A symbol sequence describing a closed and connected shape contour is termed a sentence (Q-sentence). Any repeating structural pattern in terms of symmetry, iteration and alternation (Martinoli et. al., 1988) is termed a phrase (Q-phrase). Thus, analogically, the symbol sequence can be conceptualised as a hierarchy of Q-codes → Q-words → Q-phrases → Q-sentences → Q-paragraphs and so on.

The transformation from symbol sequence (unstructured) to regularities (structured) brings three interpretation possibilities. They may be interpreted as:

(i) a repetitious symbol structure;
(ii) a syntactic pattern mapping to well-known sketch patterns with specific design meaning; and
(iii) a symbol pattern having syntactic importance, which is not necessarily related to well-known shape patterns.

Firstly, the repetition in symbol structure is a distinctive character, which is easily recognised as syntactic regularity. Repetitions are normally categorised into three basic types in symbol processing, namely, iteration, alternation, and symmetry (Martinoli et. al., 1988).

- *Iteration*: repetition of symbols or a pattern of symbols in a regular interval (example: aaa..., ababab..., abcabcabc...).
- *Alternation*: repetition of symbols or a pattern of symbols in an irregular interval (example: abcabdeabfgab..., abcabdabe...).
- *Symmetry*: repetition of symbols or a pattern of symbols in a reflective symmetrical manner (example: abcdcba, abcddcba).

If the syntactic repetition is recognised, then the sequence of symbols is thought to have a regularity that is related to specific shape patterns.

Secondly, a pattern of symbol sequences can be identified as denoting specific categories of shape classes that are well-known or familiar in contour. These are shape patterns with specific labels that are mostly the names that denote those shape categories, examples are shown in Figure 1. The symbol description of those patterns in Figure 1 produces specific symbol sequence such that that syntax can be treated as specific shape knowledge.

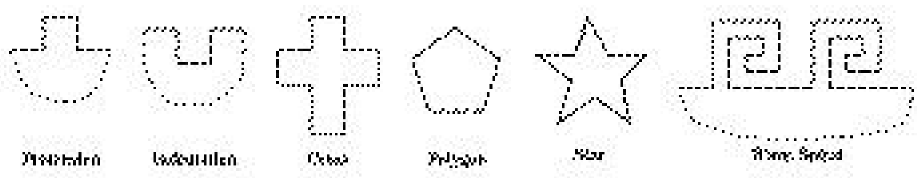


Figure 1. *Well-known shape patterns.*

Thirdly, other than these two previous types of symbol regularities, shape pattern regularities of symbol sequences can only be identified by investigating the patterns and occurrences of words (Q-words) even though they do not match any pre-existing label.

(iii) regularity → feature (R → F)

When syntactic regularities are identified from the symbol sequences, then shape features are analysed either by matching with an existing feature knowledge base or by creating a new mapping to specific design semantics. In this qualitative representation approach, features are handled either by identifying words within a sentence or by analysing the symbol pattern of the sentence. Words are analysed in terms of their syntactic patterns and their occurrences.

In addition to finding shape features, we can use those features as the basis for reasoning about shapes. We can, for example, determine categorical information about groups of shapes.

Figure 2 shows examples of the linguistic analogy at the Q-level, using illustrative shapes and their encodings.

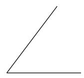
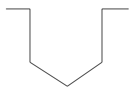
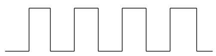
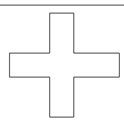
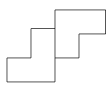
CHUNKS	ILLUSTRATIONS	ENCODING EXAMPLES
Q-CODE		A-
Q-WORD		(A- A+ A+ A+ A-) A- 3*(A+) A-
Q-PHRASE		(A- A+ A+ A-) (A- A+ A+ A-) (A- A+ A+ A-) (A- A+ A+ A-) 4 * (S[A- A+])
Q-SENTENCE		(A- A+ A+ A- A+ A+ A- A+ A+ A- A+ A+)
Q-PARAGRAPH		(A- A+ A- A+ A- A- A- A+ A- A+ A- A-)

Figure 2: *Examples of the linguistic analogy.*

3. Reasoning About Shapes Using A Qualitative Representation

3.1 SHAPE FEATURE IDENTIFICATION

The following experiment analyses sketches that are part of the development of a layout and form design by the architect Louis Kahn in terms of design similarity. Design similarity is considered as the design value that is computable by comparing the encodings of shape features of various categories. The comparisons of sketches have been processed at the symbolic description level in which shape classes are distinguished from each other through the identification of common shape features.

Encoding of Sketches

Sketches in Figure 3 are transformed into Q-code encodings as sentences. Here, the contours of outdoor space bounded by rectangular building are encoded as the shape taken from the background. All the black shapes are considered to be attached to the rectangular boundary, and the contour of outdoor shape is considered to be closed and connected. The twelve sketches from A to L then produce 12 different sentences which become the objects to be analysed.

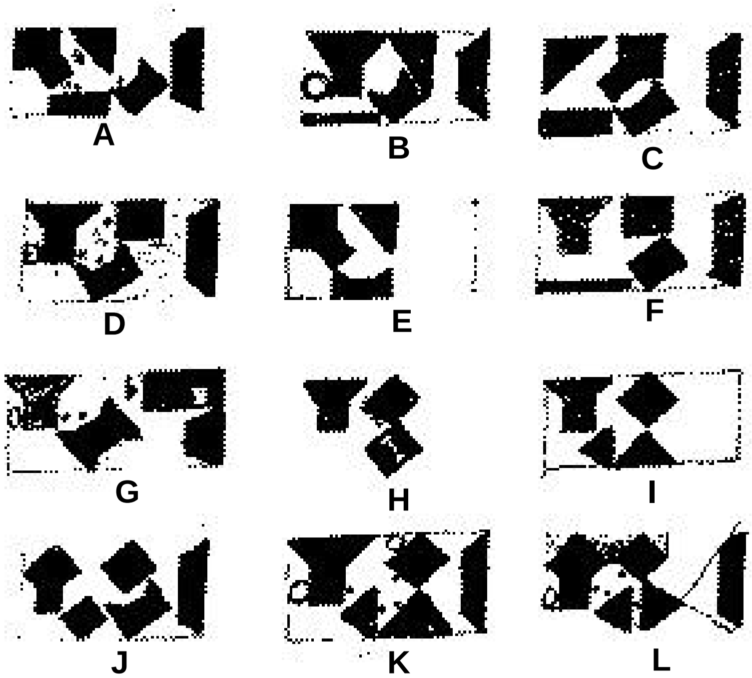


Figure 3. *Kahn's sketches (Ronner and Jhaveri 1987).*

Occurrences of Words

The symbol structures of each sentence have been analysed for patterns of words and their occurrences. The results of this analysis can be presented as bar graphs such as the one shown in Figure 4. Similar graphs can be produced for each of the sketches.

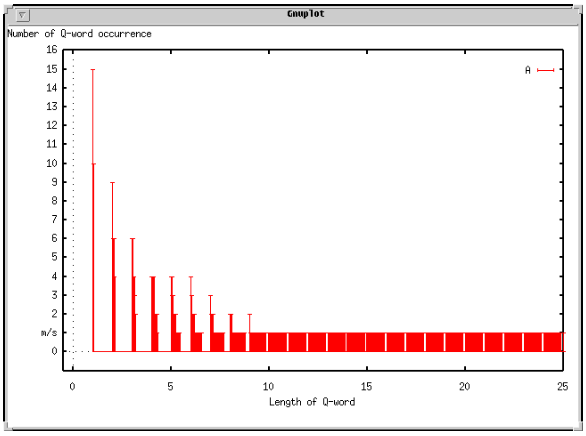


Figure 4. *A bar graph of word occurrences as word length changes (Sketch A).*

As expected, short words occur more frequently than long words. It looks as if such a distribution obeys Zipf's law (Zipf, 1949) but this has not yet been tested. Generally, any word that occurs more than once in a sentence is considered to have design significance. Its significance increases with the number of occurrences, except for word length of unity.

Regularity in Syntax Structure

Five shape feature categories have been considered. These are: indentation, protrusion, alternation, iteration and symmetry. Two types of shape features are recognised as indentation and protrusion categories if the system could identify the syntax pattern as (A_- n*(A_+) A_-) and (A_+ n*(A_-) A_+) respectively.

3.2 SHAPE FEATURES

Once the regularities of syntax patterns have been identified, each word is categorised into one of the above five shape feature categories. Based on identified words, commonalities of feature characteristics are determined by comparing matchings and mismatchings of each shape's features to the description of their category that is defined by the aggregation of all its common shape features. Comparisons can now be made either of a shape to the category or of one shape to another.

3.3 SIMILARITY MEASURES BASED ON CATEGORY

We modified the similarity measure equation proposed by Estes (Estes, 1994) to:

$$Sim(\mathrm{A,B}) = t^{r} s^{N-k}$$

where:

- A, B: shape or shape category
- *k:* number of matching shape features types
- *N:* total number of shape features types in the category
- *r:* occurrence of matching shape feature
- s: numeric value for mismatch, $0 < s < 1$
- t: numeric value for match, $t \geq 1$.

The following five tables show the similarity measures of the 12 sketches in Figure 3 in the five categories described in Section 3.1.

Indentation Category

Table 2 shows the similarities in the indentation category represented as (A_- n*(A_+) A_-) for values of n from 1 to 3.

TABLE 2. Similarity of indentation category.

	A	B	C	D	E	F	G	H	I	J	K	L
(A_- A_+ A_-)	t^2	t^3	t^3	t	t^3	t	t^2	s	t	t^2	t	t
(A_- 2*A_+ A_-)	t^4	t^3	t^3	t^4	t	t^3	t^3	t	t^2	t^3	t^3	t^4
(A_- 3*A_+ A_-)	s	s	s	s	s	t	s	t^2	t	t^2	t	t
	t^6s	t^6s	t^6s	t^5s	t^4s	t^5	t^5s	t^3s	t^4	t^7	t^5	t^6

Protrusion Category

Table 3 shows the similarities in the protrusion category represented as (A_+ n*(A_-) A_+) for values of n from 1 to 5.

TABLE 3. Similarity of protrusion category.

	A	B	C	D	E	F	G	H	I	J	K	L
(A_+ A_- A_+)	s	s	t	s	s	s	t	s	t	t	t	t
(A_+ 2*A_- A_+)	t^4	t^5	t^4	t^3	t	t^3	t	s	s	t^5	t^2	t^4
(A_+ 3*A_- A_+)	t	t	s	t	t	t^2	t	t	t	s	t	s
(A_+ 4*A_- A_+)	t	s	s	t	t^2	s	t^2	t^2	t^2	t	t	t
(A_+ 5*A_- A_+)	s	s	t	s	s	s	s	s	s	s	s	s
	t^6s^2	t^6s^3	t^6s^2	t^5s^2	t^4s^2	t^5s^3	t^5s^1	t^3s^3	t^4s^2	t^7s^2	t^5s^1	t^6s^2

Alternation, iteration and symmetry categories are considered only for the words up to length 6. It is assumed that the shapes from these sketches are not sufficiently complex to require an analysis of words longer than this. All word lengths are measured in Q-codes. For this analysis the only Q-code used was the A-code.

Alternation Category

Table 4 shows the similarities in the alternation category of these 12 sketches.

TABLE 4. Similarity of alternation category.

word lngth	A	B	C	D	E	F	G	H	I	J	K	L
1	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0
2	t^4s^0	t^4s^0	t^4s^0	t^4s^0	t^3s^1	t^4s^0	t^4s^0	t^4s^0	t^4s^0	t^4s^0	t^4s^0	t^4s^0
3	t^6s^2	t^5s^3	t^6s^2	t^5s^3	t^4s^4	t^5s^3	t^6s^2	t^6s^2	t^5s^3	t^8s^0	t^5s^3	t^7s^1
4	t^8s^3	t^6s^5	t^7s^4	t^6s^5	t^5s^6	t^7s^4	t^7s^4	t^7s^4	t^6s^5	t^8s^3	t^6s^5	t^4s^7
5	t^7s^{10}	t^7s^{10}	t^7s^{10}	t^5s^{12}	t^5s^{12}	t^6s^{11}	t^6s^{11}	t^7s^{10}	t^5s^{12}	t^8s^9	t^5s^{12}	t^4s^{13}
6	t^6s^{16}	t^6s^{16}	t^4s^{18}	t^5s^{17}	t^4s^{18}	t^5s^{17}	t^5s^{17}	t^7s^{15}	t^3s^{19}	t^8s^{14}	t^2s^{20}	t^4s^{18}
Sum	$t^{33}s^{31}$	$t^{30}s^{34}$	$t^{30}s^{34}$	$t^{26}s^{38}$	$t^{23}s^{31}$	$t^{29}s^{35}$	$t^{30}s^{34}$	$t^{33}s^{31}$	$t^{25}s^{39}$	$t^{38}s^{26}$	$t^{24}s^{30}$	$t^{24}s^{40}$

Iteration Category

Table 5 shows the similarities in the iteration category of these 12 sketches.

TABLE 5. Similarity of iteration category.

word lngth	A	B	C	D	E	F	G	H	I	J	K	L
1	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0
2	t^1s^2	t^0s^3	t^2s^1	t^1s^2	t^1s^2	t^0s^3	t^2s^1	t^1s^2	t^2s^1	t^3s^0	t^2s^1	t^3s^0
3	t^2s^2	t^3s^1	t^3s^1	t^1s^3	t^3s^1	t^1s^3	t^1s^3	t^0s^4	t^0s^4	t^3s^1	t^0s^4	t^1s^3
4	t^4s^2	t^4s^2	t^4s^2	t^4s^2	t^1s^5	t^4s^2	t^0s^6	t^0s^6	t^0s^6	t^4s^2	t^2s^4	t^4s^2
5	t^0s^8	t^0s^8	t^0s^8	t^3s^5	t^2s^6	t^5s^3	t^0s^8	t^1s^7	t^1s^7	t^1s^7	t^1s^7	t^1s^7
6	t^2s^0	t^0s^2	t^0s^2	t^0s^2	t^0s^2	t^0s^2	t^0s^2	t^0s^2	t^1s^1	t^0s^2	t^1s^1	t^0s^2
Sum	$t^{11}s^{14}$	t^9s^{16}	$t^{10}s^{15}$	$t^{11}s^{14}$	t^9s^{16}	$t^{13}s^{12}$	t^5s^{20}	t^4s^{11}	t^5s^{20}	$t^{13}s^{12}$	t^8s^{17}	$t^{11}s^{14}$

4.3.5. Symmetry Category

Table 6 shows the similarities in the symmetry category of these 12 sketches.

TABLE 6. Similarity of symmetry category.

word lngth	A	B	C	D	E	F	G	H	I	J	K	L
1	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0	t^2s^0
2	t^3s^1	t^3s^1	t^3s^1	t^3s^1	t^3s^1	t^4s^0	t^3s^1	t^3s^1	t^4s^0	t^4s^0	t^4s^0	t^4s^0
3	t^3s^4	t^4s^3	t^4s^3	t^3s^4	t^3s^4	t^4s^3	t^2s^5	t^3s^4	t^3s^4	t^5s^2	t^4s^3	t^5s^2
4	t^4s^5	t^3s^6	t^4s^5	t^4s^5	t^3s^6	t^5s^4	t^2s^7	t^3s^6	t^3s^6	t^4s^5	t^4s^5	t^4s^5
5	t^3s^7	t^2s^8	t^1s^9	t^3s^7	t^2s^8	t^5s^4	t^1s^9	t^2s^8	t^2s^8	t^4s^6	t^1s^9	t^4s^6
6	t^3s^3	t^2s^4	t^1s^5	t^3s^3	t^1s^5	t^1s^5	t^0s^6	t^2s^4	t^0s^6	t^1s^5	t^0s^6	t^2s^4
Sum	$t^{18}s^{20}$	$t^{16}s^{22}$	$t^{15}s^{23}$	$t^{18}s^{20}$	$t^{14}s^{24}$	$t^{21}s^{17}$	$t^{10}s^{28}$	$t^{15}s^{23}$	$t^{14}s^{24}$	$t^{20}s^{18}$	$t^{15}s^{23}$	$t^{21}s^{17}$

4. Results

The similarity measures, of the 12 sketches, to the five shape feature categories produces an order of categorical similarity which is shown in terms of the percentage similarity to the most typical member of the category. The values are adjusted to produce the average similarity measure to be 50% to the most typical member (prototype) of the category:

- indentation category: (100 J); (76 L); (57 K); (52 A B C); (43 I); (39 D F G); (30 E); (22 H);
- protrusion category: (100 G K); (98 J); (60 A C L); (37 D); (23 E I); (22 B); (14 F); (5 H);
- alternation category: (100 J); (64 A H); (49 B C G); (46 K); (45 F); (42 E); (35 D); (32 I); (29 L);
- iteration category: (100 F J); (61 A D L); (47 C); (40 H); (37 B E); (29 K); (14 G I); and
- symmetry category: (100 F L); (83 J); (57 A D); (39 B); (33 C H K); (27 E I); (13 G).

From these results we can claim the following:

- for each sketch what shape features are more significant;
- which sketches are more similar to the shape classes defined either by each shape feature category or some combination of shape feature categories; and
- we are able to compare one sketch to two or more other sketches to assess the similarity of design ideas in terms of shape features. Comparisons can be made either by a single shape feature category or by a combination of two or more shape feature categories.

5. Discussion

Such qualitative shape representation and its subsequent analysis provide the beginning of a wider range of possibilities of reasoning about shape. It becomes tractable to develop shape categories using an extension of the shape similarity concepts described earlier in this paper. Categorical similarity can be measured by summing the similarity measures between one exemplar and all the other members of the category. The similarity to category measure becomes:

$$SimilarityToCategory(\mathrm{A}) = \Sigma\ Similarity(\mathrm{A}, \mathrm{X})$$

where A is a shape in the category and X is all the members of the category.

A wide range of similarity measurements and predictions can be made (Estes, 1994), including relative typicality of a shape for each exemplar *(Ex)* as follows:

$$RelativeTypicality(Ex)=SimilarityToCategory(Ex)/\Sigma SimilarityToCategory(X)$$

We can demonstrate categorisation with the following examples. The exemplars in the categories share commonalities that are constructed with a square (SQR) and three protrusion (P) features. In the membership, reflective symmetry is included while rotational symmetry is excluded.

SQR+3P category

This shape category includes those shape exemplars with a square and three protrusions on edges as members. There are five members in this category as shown in Figure 5.

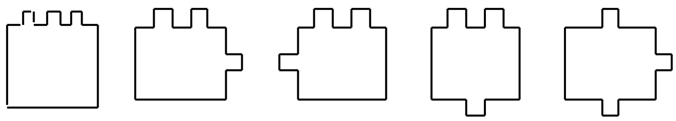


Figure 5. *Shape category SQR+3P.*

SQR+2P+1C

This shape category includes those shape exemplars with a square, two edge protrusions and one-corner protrusion (C) features as members. The membership is given for the following exemplars in the Figure 6.

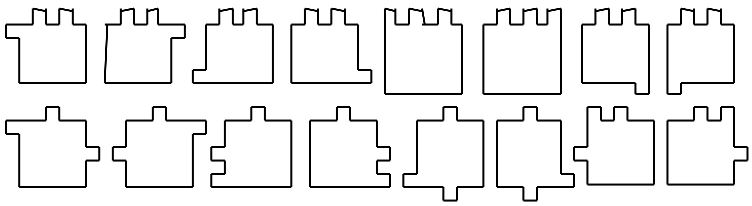


Figure 6. *Shape category SQR+2P+1C.*

SQR+1P+2C
The shape features that characterise this category include a square, one edge protrusion and two corner protrusions. The membership is given for the following exemplars in the Figure 7.

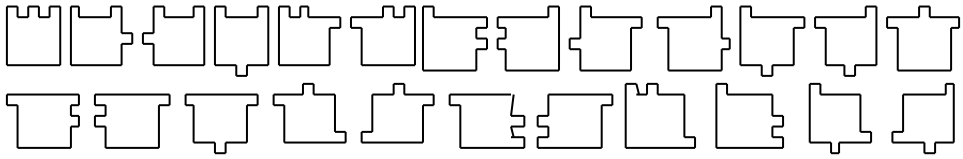


Figure 7. *Shape category SQR+1P+2C.*

SQR+3C
This shape category is characterised by the shape features of a square and three corner protrusions. The membership includes the following exemplars in the Figure 8.

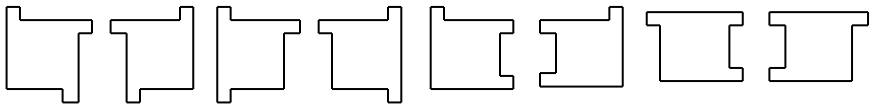


Figure 8. *Shape category SQR+3C.*

These four categories defined above share common features. They contain a square (quadrilateral in its class) and three occurrences of protrusion variations. In that sense these four categories can be thought of as sub-categories of a parent category which is defined to include these commonality. This can be represented as ***SQR+3PV*** indicating that it contains features that represent a square and three protrusion variations.

Thus, we can see how the qualitative representation of shape allow for a wide range of reasoning to be carried out over individuals and groups of shapes.

Qualitative representations of shapes have been developed for a variety of other applications (Frank et al, 1992; Frank and Campari, 1993). The focus in designing includes the need to re-represent shapes to allow alternate interpretations of those shapes – what Schön (1983) has called “reflection”. This provides a foundation for models of designing that include the “situatedness” of designing (Gero, 1998). Thus, the ability to interpret a shape as belonging to a number of different categories derived from its qualitative representation is important. The approach adopted in this paper builds on existing ideas from qualitative representations of shape and space and focuses on the reasoning possible in developing and interpreting features derivable from structural regularities in those representations. Those structural regularities can be mapped onto semantic labels of interest during designing.

Acknowledgments

This work is supported by a grant from the Australian Research Council, Grant No. A89802104 and a Overseas Postgraduate Research Award. Computing resources are provided in part by the Key Centre of Design Computing and Cognition.

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This paper is a copy of: Park, S-H. and Gero, J. S. (1999) Qualitative representation and reasoning about shapes, *in* Gero, J. S. and Tversky, B.

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