Modulation and Mathematics in Generative Building Design

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The rise of design computation as a paradigm in design process has certainly brought increased attention to the mathematical basis of form giving in architectural design. Numerous types of generative systems ranging from shape grammars to parametric design have been developed in the past, where computers are used as a tool to generate building form automatically. This paper focuses on the role of number sequences and mathematical methods of controlling and introducing systematic change (modulation) to generative systems. In a graduate level class, VBA programming through a CAD system was used to develop software that can parametrically generate 2-d and 3-d forms. As a conclusion, the paper argues that forms generated as such are not monotonous, and that being particularly attentive to the mathematical basis of variation and change in parametric generators allows one to explore change in a more systematic fashion.

Keywords*: Generative System, mathematics, modulation, CAD.*

Introduction

In the past, several prominent architects and design thinkers have focused on the issue of the relationship between mathematics and design, among which Le Corbusier, Palladio and many of the classical revivalist architects can be counted (Ching, 1996; March, 1999). It is common knowledge that ancient Greek and Roman architectures have been based on strong proportioning systems. These efforts obviously reflect a time when digital technology was not available as a widely used tool and designers did not have the benefit of using such tools. With the increase in computer usage, a greater interest in the relationship of mathematics to art and architecture emerged. For example, the organizers of the fifth interdisciplinary conference on Arts, Mathematics and Architecture, ISAMA/CTI, 2004 (www.isama.org/: June, 2004) sponsored by computer science as well as art communities, lists geometric art, mathematical visualization, tessellations and tilings, fractals, CAD systems as well as architectural form giving as part of their field of interest (https://facweb.cs.depaul.edu/sluecking/isama-cti.htm: June, 2004). This paper presents an effort to incorporate mathematical principles into generative systems in a class held in Spring 2003. AutoCAD's programming language Visual Basic for Applications was used as a development platform. Among the concepts explored were proportion,

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Form giving typically requires the manipulation of geometric entities through the process of locating forms in Cartesian space, sizing and resizing, reorienting, and deciding how forms will be located relative to each other as well as relative to the whole. As designer performs these tasks manually through hand drawn sketches or through cardboard models, scale, measurement and exact dimensions and sizes of things are not yet a concern, although later at design development phase, forms must be accurately sized and quantified. What mathematics can provide when incorporated into the early phase of the design process is that quantification and sizing can now become part of the conceptual framework for architectural design. For example, when Le Corbusier worked with „Modular" as a system of measurement based on the proportions of the human body (Ching, 1996), he was incorporating size, quantity and scale into his designs at the very early stages of design, whereas in most other cases, sizing and measurement become almost an afterthought, often governed by the requirements imposed by structural and functional concerns. The potential of mathematics in design exploration is more varied and interesting in the early phases of the design process than it is at later stages. This is where computing comes into play. While manual methods of design did not support the incorporation of mathematical principles into design exploration, computers can be designer's strongest ally when exploring the design implications of mathematical principles in early design phase.

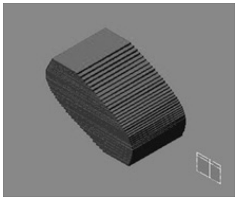
Within this context, concepts such as proportion, repetition, modulation, systematic change and variation as well as mathematical models of physical phenomena can be incorporated into generative systems for design. While repetition and proportion represent more traditional aspects of mathematical principles mostly implemented through parametric design systems where form parameters are constrained through an underlying controlling order (Podovan, 1999; Tavernor, 2002), modulation and systematic change have not been as widely investigated.

Systematic change in a generative system

As mentioned above, the examples summarized in this article will be from a graduate class taught in the Spring of 2003. Among the assignments given to the class were facade, plan parti and 3D massing generators for architectural design. Not only the repetitive, modular, parametric aspects which might at times be seen as monotonous and boring, but also modulation (systematic changing, altering) were explored as a design principle. Methods such as parametric design and shape instancing were found to be more amenable to modular design and modulating, (i.e. altering, changing, transforming) based on different mathematical principles in developing such generative software systems. Systematic modulation can include a surprise element that would normally not exist with other types of parametric design. For example, in this study, following Fibonacci series as the elements of a façade generated surprising results. Again using a similar mathematical principle in generating 3-D volumes created solid-void relationships that are hard to imagine without the aid of the computer (Fig.1). The examples seen in Fig. 2 also explore solid modeling operations of subtraction and union. In these examples, the values for the parameters were inputted by the user, which gave the opportunity to test different combinations of proportion and ratio as well as specific combinations of these such as golden section. Clearly, without the help of computing,

(1)

(2)



(3)

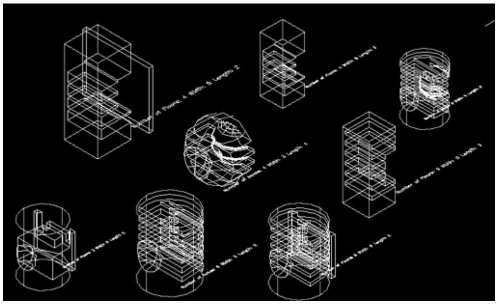


Figure 1
Subtraction and addition as part of the solid parametric generator.

Figure 2
Wire frame view of the solid model generator.

Figure 3
Sculptural variation in form giving.

such parametric explorations would be very difficult, if not impossible. While color was used to simply represent and differentiate the variety of geometries and the operations used in generating these forms, the unexpected effect of the residual color of the objects that were subtracted was a good surprise. This led to discussions regarding the potential of incorporating variations in materials into design to reveal the underlying algorithm and principle in form generation, thus an algorithmic aspect leading to a design insight.

One of the difficulties in such generative systems is matching the parameters/proportions used with the specific generated form after the form is displayed on the screen. In the 3-d solid model generator seen in Fig's 1 and 2, the parametric values for each design were automatically generated as a text object and printed next to each design. This was intended to assist the designer in understanding the generative implications of the numbers assigned to each parameter. Fig. 2 has examples in wire frame view where text can be seen.   It also allowed the testing of different ratios and proportions. The number of floors, the nature of the initial form, the proportion of the cut into the form were some of the parameters that can be manipulated in this particular example. Some of the most interesting forms were generated when extreme values with big differences were assigned to the form parameters. For example, assigning a very low number to the number of floors led to the solution seen at the very upper left corner of the forms seen in Fig. 1 as well as to the form at the very lower left corner of the wire frame view seen in Fig. 2. These were a lot more sculptural in nature, as opposed to the architectural character of the other solutions.

Modulation

Systematic variation through the manipulation of form parameters by using mathematical functions was another exercise that was undertaken in this class. Since trigonometric functions typically pro-

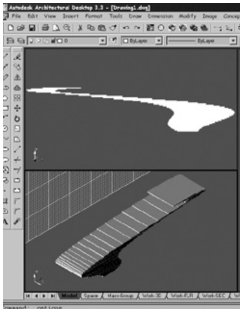
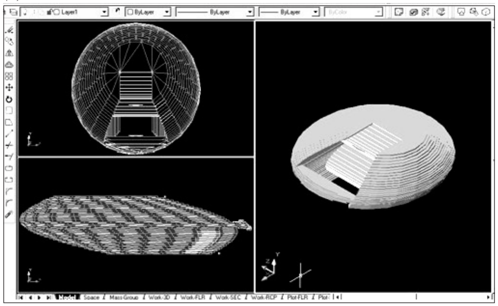


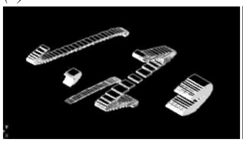
Figure 4
Trigonometric variation in form giving.

vide harmonic variation, this project led to the exploration of trigonometric functions of sine and cosine as the basis of form giving. Solid objects were generated and manipulated through VBA code to generate a very wide range of forms, some of which implied architectural solutions while others were more sculptural which can be the basis of furniture design. For example, the example seen in Fig. 3 initially started as an effort to generate different designs for chair design by the author of this article. Initially more conservative values were assigned that led to more sculptural forms. On the other hand, as the parameters of the trigonometric functions were manipulated with more extreme values, more architectural looking forms as seen in Fig. 4 and 5 were

(5)



(6)



(7)

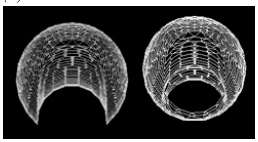


Figure 5
Forms with architectural character generator with trigonometric variation

Figure 6
Vertical modulation emphasized with horizontal modulation represented with more conservative values as well as more extreme value changes.

Figure 7
Parametric variation in the horizontal, for forms with architectural character.

generated. The initial algorithm was also an effort to mimic the production process where layers of sheet material will be cut and assembled to generate the final product as seen in Fig 3. This also had the potential to use CAM equipment to produce the physical form represented by the generated design. The layering of sheet material also had the potential to eventually assign different values to the thickness of the sheets, informed by the real world properties of materials. This aspect is yet to be explored and reserved for future work.

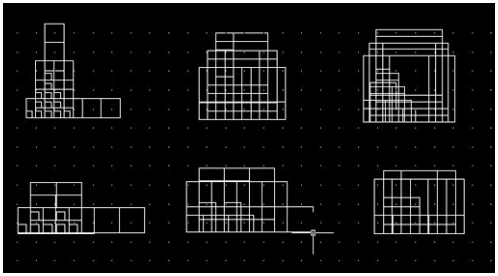
On the other hand, more architectural looking solutions, obviously led to the reading of each layer as a floor slab, while the whole form was read more as a whole building or larger parts of a building, such as ramps or stairs that are sculptural looking. The most interesting and unexpected aspect was the repetition of the harmonic rhythm in the vertical which did not exist in the initial designs that were generated (Fig. 5). Large differences between the values assigned to the horizontal-shift parameters were most instrumental in exaggerating the impact of the underlying trigonometric function (Fig. 6).

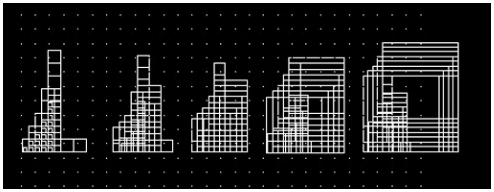
Preconceiving the generation of forms seen in figures 5, 6 and 7 through the same algorithm was almost impossible. In this particular case, each time a parameter was manipulated, software users were almost glued to the computer screen to see what was emerging. Even after testing numerous parametric variations, there was still quite a bit of surprise element as the forms emerged.

Please see appendix A for the partial VBA code used in generating the forms seen in figures 3 through 7.

Facade modulation

The shapes seen in figures 8, 9 and 10 were generated as a guide to articulating facades with a variety of proportions. A variety of number sequences including the Fibonacci series of 1,1,2,3,5,8,13,… (sum of the previous two numbers generates the next number) were used to generate the vector lines that can be used as construction lines during façade design. In Fig. 8, the top row shows shapes that





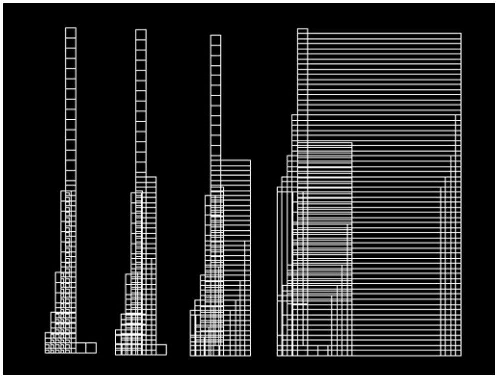


Figure 8
Façade modulation.

Figure 9
Fibonacci series for façade design.

Figure 10
Exponential variation of the modulation.

were generated as a result of using 1,3,5,3,1 as the number of vertical bays with a modulation factor of 1, 3 and 5 from left to right respectively. In the same figure, the lower row is where 1,2,1,2,1 was used as the number of vertical bays with a modulation factor of 1,2,3 from left to right respectively.

On the other hand, in Figure 9, Fibonacci series was used to generate the shapes. The numbers used were 1,2,3,5,8 from left to right while a modulation factor of again numbers from Fibonacci series 1,2,3,5,8 were used for the five different figures from left to right. The variety of overlaps and proportions created a rich variety of orderly forms that can be used as construction lines in the design of a façade.

On the other hand, Fig. 10 represents exponential variation, where 2,4,8,16,32 were used to generate the number of floors, while 2,4,8,16 were used to modulate and generate the forms seen in Fig. 10 from left to right respectively.

Summary

As a summary, generative systems can be helpful as one explores variation and systematic change though the use of underlying mathematical principles of proportion and number sequences. These can assist in generating 3-D solid objects as well as in creating construction lines that can be helpful in façade design. In either case, the next step would be to inform the process of modulation through the assignment of number sequences that mimic manufacturing and construction processes. This is yet an untapped area that needs to be further explored.

On another note, the final proportion (such as a slender façade verses a more square one) as well as the number and nature of the subdivisions imply a variety of building typologies such as high rise, low rise, etc., whereas more sculptural verses more elemental types imply materiality such as masonry, wood frame, steel frame, etc. Furthermore, in the examples above, the nature of the subdivisions indicates systematic variation in window treatment and window sizes, avoiding monotony which can typically be a problem when façade elements are simply repeated. The façade exercise was also intended to demonstrate that modular did not always mean monotonous design. All of the exercises were helpful in talking about emergence in design and the surprise element in systematic change and variation.

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Appendix A

Partial VBA Code for solid model generator:

```
For nCnt = 0 To 40
Set objBox = ThisDrawing.ModelSpace.AddBox(bPt, width, width + Abs(width * Sin(Pi / 40 * nCnt)), Height)
,Set objBox = ThisDrawing.ModelSpace.AddCylinder(bPt, width + Abs(width * Sin(Pi / 40 * nCnt)), Height)
objBox.Color = 125
objBox.Update
bPt(1) = bPt(1) - (width / 2.1 + Abs(width / 1.5 * Sin(Pi / 20 * nCnt) * 1.4))
,bPt(1) = bPt(1) - width / 2
Set objSubs = ThisDrawing.ModelSpace.AddBox(bPt, width + Abs(width * Sin(Pi / 40 * nCnt)), width + Abs(width * Sin(Pi / 40 * nCnt)), Height * 1.1)
,Set objSubs = ThisDrawing.ModelSpace.AddCylinder(bPt, width / 1.3, Height)
,Set objSubs = ThisDrawing.ModelSpace.AddTorus(bPt, width / 1.5, width / 2)
objBox.Boolean acSubtraction, objSubs
objBox.Update
 bPt(1) = bPt(1) + width / 2 + Abs(width / 1.5 * Sin(Pi / 20 * nCnt) * 1.4)
,bPt(1) = bPt(1) + width / 2
bPt(2) = bPt(2) + Height
Next
```