

The generation of Chinese ice-ray lattice designs on 3D surfaces

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We revisit Stiny's ice-ray grammar and adapt its shape rules, including additional rules for augmenting a polygon by inscribing a triangular shape, to apply to twisted surfaces. We describe the constructive process of rule application for twisted polygons, shortly describe the implementation of the grammar, and illustrate its use in the design of a façade, the shape of which is defined as a twisted polygon.

Keywords: Design generation; shape grammars; ice-ray lattice designs, twisted surfaces.

Introduction

The motivation for writing this paper comes from the design of a building façade using Chinese ice-ray lattice structures (Figure 1). Ice-ray lattices are a particular form of traditional Chinese lattices constructed between 1000 BC and 1900 AD, as catalogued by Daniel Sheets Dye (1949), which do not have a regular structure, but rather mimic the ice-lines formed in the ice-formation process. The Federation Square building in Melbourne by Lab Architecture Studio served as an example in the design of the façade, although the pattern used in the Federation Square building is far more regular. The façade of the building in Figure 1 is composed of twisted polygons. While Chinese ice-ray lattice structures can be projected onto twisted polygons, we consider a non-

projected solution based on solid geometry.

Stiny (1977) defines a parametric shape grammar for the generation of Chinese ice-ray lattice designs. Stiny's exemplar grammar has four constructive rules allowing for a convex polygon to form two new convex polygons, with approximately equal areas, by placing a single line between two of the original polygon's edges. Specifically, the rules state that any triangle, convex quadrilateral or convex pentagon, with area greater than some given constant, can be augmented once by placing a line between two of its edges to form, respectively, a triangle and a convex quadrilateral, a triangle and a convex hexagon, two convex quadrilaterals, or a convex quadrilateral and a convex pentagon. Stiny also suggests some additional constructive rules for allowing polygons to be augmented by inscribing a triangular or other polygonal shape, generating other, slightly more complex, ice-ray designs.

Stiny's shape grammar is designed to construct traditional Chinese ice-ray lattice designs, but can be adapted and applied to conceive more contemporary designs. Lab Architecture Studio also considers

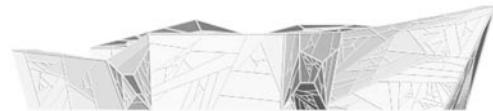


Figure 1
An elevation of the building.

Figure 2
Four constructive rules that split a convex polygon into two new convex polygons by placing a single line between two of the original polygon's edges.

Figure 3
Three additional rules that inscribe a triangular shape between three edges of a convex polygon.

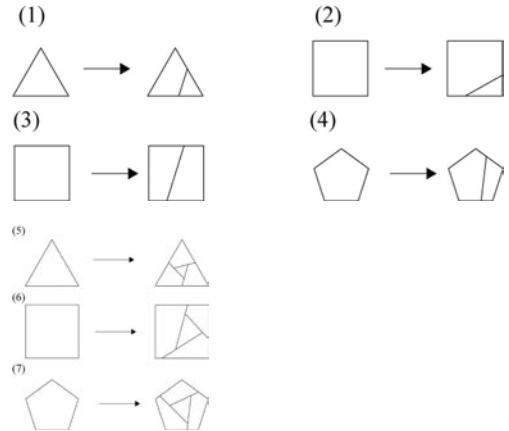
a fractal pattern, similar to a Chinese ice-ray pattern, in the design of the façades of the SOHO Shangdu building, to be completed in Beijing in 2007. These façades are also not flat but are folded along some of the lines of the fractal pattern.

In this paper, we adapt the shape rules in Stiny's ice-ray grammar, including the additional rules for augmenting a polygon by inscribing a triangular shape, to apply to non-flat surfaces, such as twisted surfaces. The four original shape rules each specify the placing of a single line between two edges. In a twisted polygon, such a line can be placed in a similar way leading to two new twisted polygons. The additional shape rules for inscribing a triangular shape specifies the placing of three lines extending from a triangle to each touch one of the edges of the original polygon. Thus, the inscribed shape is connected to the original twisted polygon by three points, forming a (flat) triangle.

Shape rules

Figure 2 shows the original shape rules in Stiny's grammar. These are parametric shape rules, they apply to any triangle (rule 1), convex quadrilateral (rules 2 and 3) or convex pentagon (rule 4), respectively. Note that Stiny's shape rules are augmented with labeled points in order to guide the generation and to prevent rule application to the same shape more than once. As a result, Stiny also includes one extra rule – a termination rule – to erase a labeled point. These are omitted here (see below for an explanation in light of the implementation).

Figure 3 shows three additional shape rules that inscribe triangular shapes into a triangle (rule 5), convex quadrilateral (rule 6) or convex pentagon (rule 7), respectively. Stiny also suggests allowing for convex polygons to be augmented by inscribing pentagonal (or hexagonal) shapes. This suggestion is not retained as it is not generally applicable in the context of a twisted surface, whereas the seven shape rules that are presented here are. In case of rules 1 through 4, a single line is placed between two points on two of the original

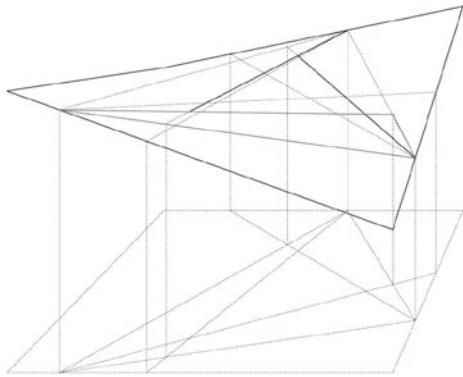


polygon's edges. Such a line can always be constructed. In case of rules 5 through 7, a triangular shape is constructed that connects in the plane of the triangle to three points on three of the original polygon's edges. Since three points always (uniquely) define a plane, such a triangular shape can also be constructed (see Figure 4 for the case of a twisted quadrilateral).

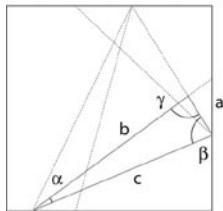
Rule application

The implementation described below is not of an actual shape grammar, that is, it does not adhere to the definition of a shape as *any* "finite arrangement of straight lines of limited but nonzero length" (Stiny, 1977), nor does it rely on shape recognition algorithms. The implementation instead uses the notion of a polygonal face as defined by the CAD software application it is embedded in. Furthermore, the generation is guided by the order in which the faces are created. In this way, no auxiliary information, such as labeled points, is required in the parametric rules.

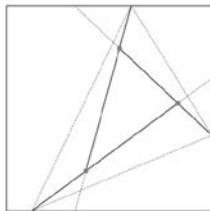
Stiny's shape rules are constrained in two ways. First, they only apply to polygons with area greater than some given constant. Secondly, in the case of rules 1 through 4, the resulting polygons have approximately equal areas. Additional constraints in terms of the lengths of the polygons' edges are necessary in or-



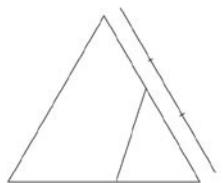
$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$



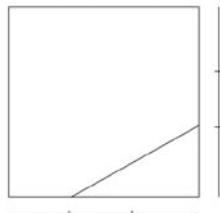
(1)



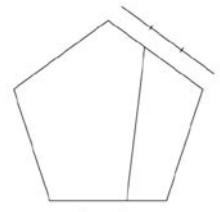
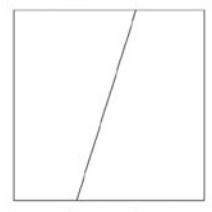
(2)



(3)



(4)



der to ensure that an already short edge is not further subdivided. In the current implementation, all con-

straints are expressed in terms of the lengths of the original polygons' edges. Figure 5 illustrates the constraints on the endpoints of the single line placed in rules 1 through 4. Each endpoint is picked randomly within a section of the polygon's edge that is defined by two parameter values specified by the user (e.g., 0.35 and 0.65, where the edge's endpoints have values 0 and 1). In the current implementation, a single random value applies for both endpoints.

Figure 6 illustrates the constraints on the endpoints of the inscribed triangular shape in rules 5 through 7. Each triangular shape is constructed from six points, three of which define the connecting points to the edges of the original polygon, and thereby the plane that embeds the triangular shape, and the other three points define the respective planes, each perpendicular to the plane of the triangle, that embed the edges of the triangular shape. Here too, each point is picked randomly within a section of the polygon's edge. In the case of a triangle (rule 5), these three points define the connecting points, the other three points are picked from the part of the respective edge between the connecting point and the original edge's vertex with parameter value 1. In the case of a quadrilateral (rule 6), the same construction is applied, with the exception of the first edge, where the first point defines one of the auxiliary points and the second, connecting point lies between the first point and the original edge's vertex with parameter value 0. In the case of a pentagon (rule 7), further variations on this construction technique are considered in order to define the three connecting points and the three auxiliary points.

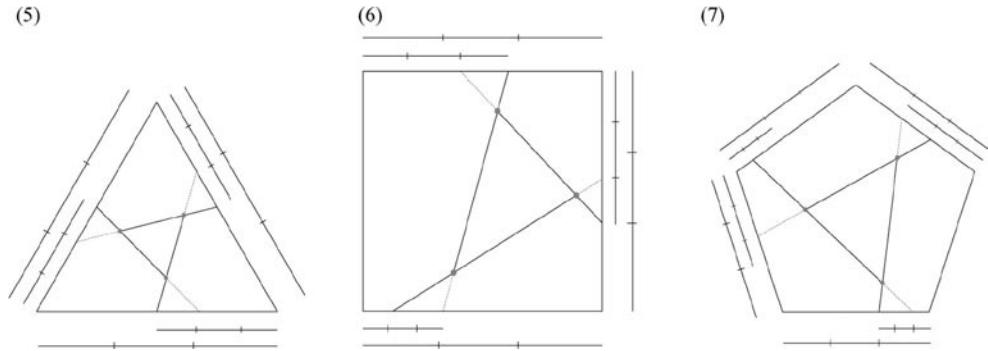
Grammar implementation

The ice-ray grammar for twisted surfaces is implemented in MEL (Maya Embedded Language), the scripting language of the Autodesk® Maya® software. The generation process is guided by the order in which the polygonal faces are created. Rule selection is naturally dependent on the number of sides to the polygon and is further defined randomly. Also, the selection of edges containing the endpoints of the

Figure 4
The construction of a triangular shape inscribed in a twisted convex quadrilateral, starting from three points on three of the twisted quadrilateral's edges.

Figure 5
The construction of the single line placed between two edges of the original convex polygon.

Figure 6
The construction of the triangular shape inscribed between three edges of the original convex polygon.



single line or triangular shape is randomly defined among the number of possible permutations. Rule application results in one or more new polygonal faces, but each edge that is created is also separately represented as a linear curve. Each face and curve is assigned a level designator in the process. Upon completion of the generation process, each linear curve is extruded according to a profile that reflects upon the level designator of the curve. As such, the generation specifies not only the final form of the façade but also its building technical structure, which can then be exported and analyzed for stability using a finite element software application. Figure 7 shows an exemplar generation.

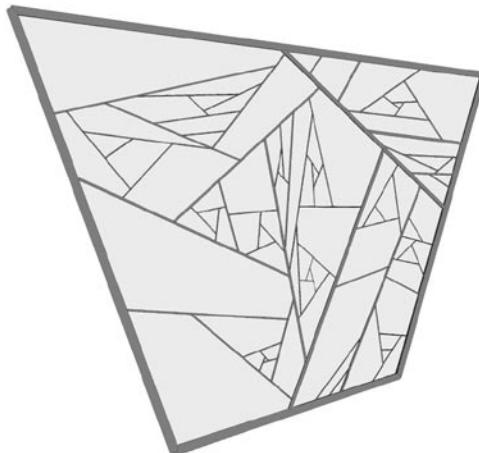


Figure 7
The design of a façade using the ice-ray grammar. The initial shape of the façade is a twisted convex quadrilateral.

Conclusion

Stiny's ice-ray grammar has been implemented repeatedly, either as a specific implementation¹ or as an exemplar application of a (more general purpose) shape grammar interpreter (e.g., McCormack and Cagan, 2002). In this paper, we describe another specific implementation of this grammar, however applied to 3D, twisted surfaces. Both the four original shape rules and the additional shape rules for inscribing a triangular shape apply equally to twisted polygons. Rather than projecting the resulting lattice structures onto the twisted polygon, we describe the application of the shape rules to twisted polygons using solid geometry.

References

- Dye, D.S.: 1949, *A Grammar of Chinese Lattice*, Harvard University, Cambridge, Mass.
- McCormack, J. P. and Cagan, J.: 2002, Supporting designers' hierarchies through parametric shape recognition, *Environment and Planning B: Planning and Design* 29(6), pp. 913–931.
- Stiny, G.: 1977, Ice-ray: a note on the generation of Chinese lattice designs, *Environment and Planning B: Planning and Design* 4, pp. 89–98.

¹ <http://web.mit.edu/haldane/www/icerays/icerays2/index.html>: April 2001