Sharing hidden power

Communicating latency in digital models

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As digital spatial models take on the complex relationships inherent in a lattice of dependencies and variables, how easy is it to fully comprehend and communicate the underlying structure and logical subtext of the architectural model: the metadesign? The design of a building, the relationships between a host of different attributes and performances was ever a complex system. Now the models, the representations, are in the early stages of taking on more of that complexity and reflexivity. How do we share and communicate these modelling environments or work on them together?
This paper explores the issue through examples from one particular associative geometry model constructed as research to underpin the collaborative design development of the narthex of the Passion Façade on the west transept of Gaudi's Sagrada Família church, part of the building which is now in the early stages of construction.

Keywords: *Design communication; CAD; CAD CAM; mathematical models; architecture.*

Introduction

Geometry is the study of space and architecture is its creation by construction or subdivision. (Blackwell, 1984) To communicate space we ultimately depend on some level of symbolic representation. This may be natural language, physical models, projections (exploiting our own perception and conceptual reconstruction of space via optics) (Evans, 1995), immersive virtual reality where we are less conscious of the site of projection, or other pragmatic or poetic conceits. There is always some level of abstraction at work.

Mathematical models use mathematical language to describe the behaviour of a system. The level of abstraction is high to simplify the problem and ultimately, through deductive reasoning, to prove what is consistent (and refute what is not). (Lakatos, 1976, Stevens, 1990) Electronic computation has supported the harnessing of a little of this logic to construct spatial models that are in effect behavioural systems. A single exploratory design model can open up a field of spatial possibilities (albeit a field highly determined by the modelling process); constraint modelling and optimisation can reveal ranges of design solutions that meet spatial or other design performance criteria.

In digital modelling we can create any number of unseen relationships between the parts of the model. We may imagine that these will be apparent. Within an associative geometry model it is possible to create a lattice of relationships in which the spatial opportunities are various and latent. This is complex in comparison to a simple dumb geometrical diagram showing one static solution. Communicating the underlying structure on which the latent possibilities and corresponding constraints depend is an interesting challenge.

Animation is a gift in demonstrating both the geometrical consequences and the manipulations of parameter values or relations in order to bring about different spatial outcomes. It can show aims, opportunities but this still may not convey the underlying logic once the network diagram of what is in train is really substantial. Experience with modelling using associative geometry indicates that it is important to be able to communicate at this meta-level if communication is to be a constituent act of creative endeavour rather than a simple exchange of information. It is also an important question to understand in applying this modelling approach to complex projects involving large teams often including many disciplines.

This paper examines examples of associative geometry models used to collaborate at a distance on the design development of the narthex, or upper colonnade, of the Passion Façade at the west transept of Antoni Gaudí's Sagrada Família church (Figure 1). It notes difficulties encountered, some of the underlying reasons for these and how they might be overcome. In summary it explores the challenges of communicating architectural models, the operation of which may be largely obscured even in animate visualisation. The archaeological, historical and analytical basis of the decisions for the particular constraints established are outside the scope of this paper.

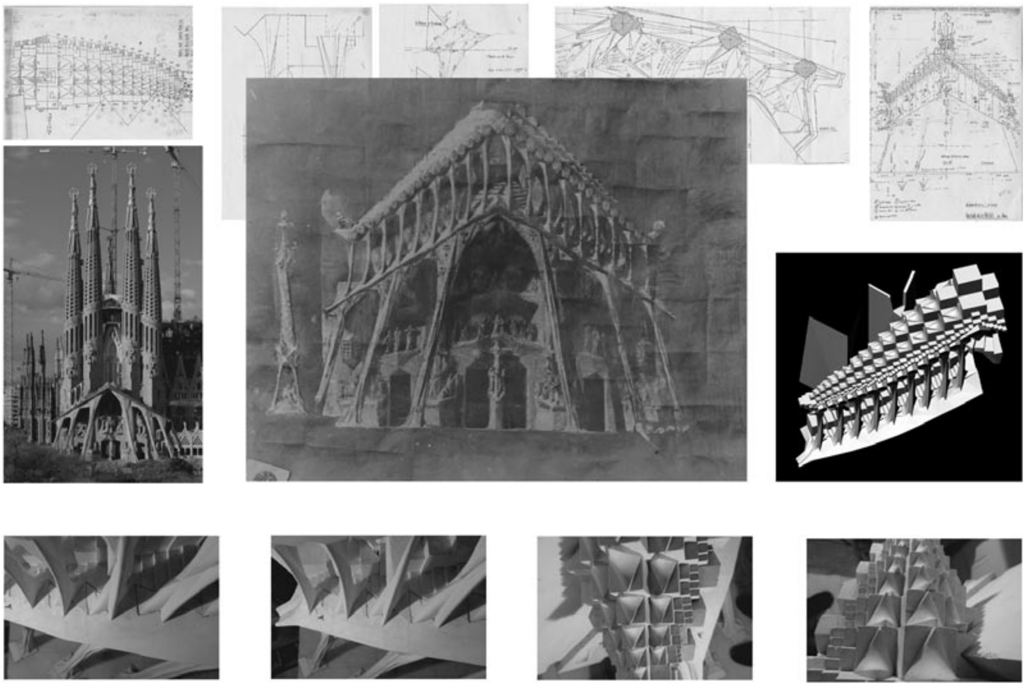

Figure 1
Centre left:ortho corrected photograph of the Passion Façade before narthex construction commenced; centre: original 1917 photograph of Gaudi drawing;centre right: digital reproduction of a 1980s plaster model of the narthex; top row: Cardonner's sketches and notes for the plaster model; lowest row: details of the plaster model.

Examples from the colonnade for the narthex of the Passion Façade of the Sagrada Familia church

The frieze of hexagonal prisms and the column spacing.

In the associative geometry model, a large degree of flexibility exists for the spacing, orientation and inclination of the columns of the colonnade but the way in which these can be manipulated is constrained by the relationships between the columns and the hexagonal prisms above them.

Each column is pinned by a centre point on its central axis which is also on a 3D curve combining a straight line in the frontal plane and conic in plan. These points can 'slide along' this curve. The axis line is defined by this column centre and a second point that lies on the lower edge of the hexagonal prism above. This point lies on this edge, aligning the centre of the column with the centre of the prism but can effectively slide along this lower edge of the prism (Figure 2).

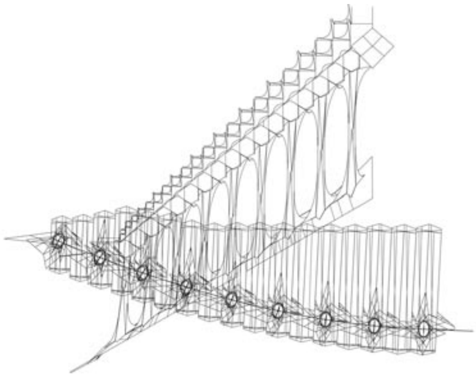

Figure 3
Wireframe plan of the columns and hexagonal prism frieze overlaid in front of an elevation of one half of the narthex colonnade (not to similar scale). The plan shows that the major axes of the ellipses that define the column "throats" are arrayed on, and normal to a parabola in plan. The hexagonal prisms arrayed over the columns are parallel to one another but rotated 3 degrees in plan relative to the axis of the church transept.

Each column has an elliptical basis to its cross section and the major and minor axes of these ellipses are normal and tangent to a plan view parabola respectively, thus they are not orthogonal in the frontal plane but increasingly 'radial' as you move from the centre of the colonnade (Figure 3).

There is a strong base coordinate system that

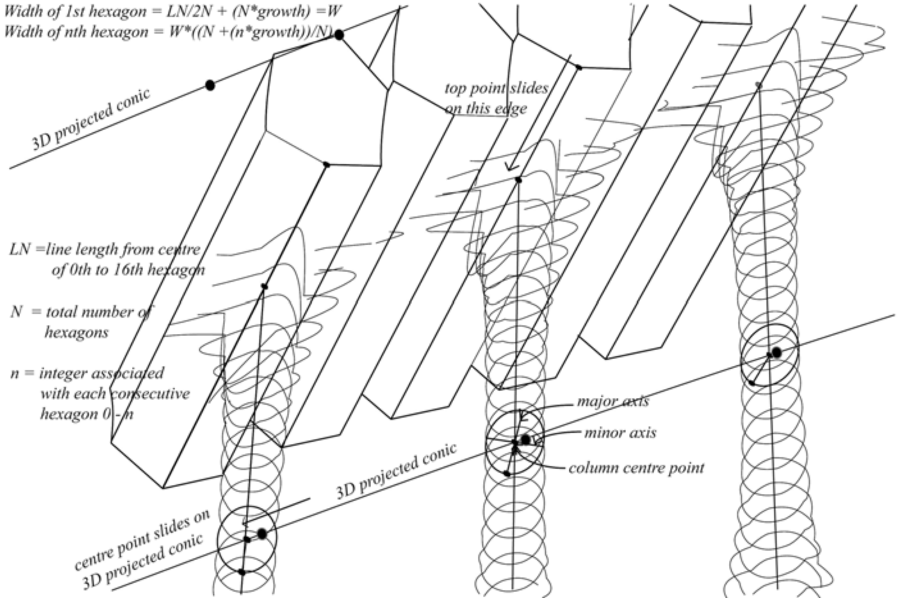

Figure 2
Distribution and growth of hexagonal prisms and relationship of columns to prisms

is more than operational – it is the plan orientation of the whole church and strongly embedded in the thinking and conceptualising of the design and construction teams. The narthex is part of the west transept and specifically the west facing Passion Façade which is, as a whole, orientated parallel to the YZ plane of the main coordinate system. This is noted merely to emphasise that while the component geometrical systems in the model are not easily rationalised in this Cartesian system, the Cartesian system is far from arbitrary or without meaning in the architecture.

Each of the organisational strategies is simple in its own right. However, all three interact to have a constraining influence on the positioning and orientation of the columns and the manner in which they can be moved, or otherwise transformed and re orientated.

Thus, requests such as: 'Lean the columns in a little more' or 'try progressively to increase the inward lean of the columns from the highest to the lowest' appear conceptually straight forward but are in reality procedurally complex due to the inherent constraints generated by other drivers of the model structure.

Considering these proposals in more detail: 'lean the columns in a little more' clearly means incline their central axes by an angle relative to the vertical (the starting neutral state of these axes in constructing the model) so that the upper part of the column leans in to the building and away from the frontal plane. But does it mean that this rotation of the axis should be in the plane through the major axis of the ellipse, in other words, normal to the conic curve on which all the columns are arrayed? This seems likely intuitively and much more likely than an orthogonal lean in the base coordinate system given the radial array of the columns along the parabolic path. However, there is a problem: the column axis is not just constrained to a centre point on the project conic curve. It is also constrained to intersect the lowest edge of the hexagonal prism so it can only slide long this edge taking it out of the plane through the ellipse major axis. The results are unpredictable. There is no opportunity to simply say: top column vertical and each subsequent one x degrees further off vertical. The model can be manipulated but its current structure means that this occurs almost haptically, making small incremental changes in the positions of the upper points on the hexagon edges and corresponding shifts in the position of the centre points on the projected conic curve.

The combined effect of the shifting orientation of the column axes in plan determined by the parabola and the parallel rotation of the hexagonal prisms produces an effect on the column inclination that can be calculated for any particular combination of parameter values but is beyond intuitive prediction.

Does the model have to be this way and as rigidly structured? Why not construct the columns with a central axis that has rotational degrees of freedom aligned with the ellipse axes? This would imply relaxing the relationship between the tops of the columns and the hexagonal components of the frieze, possible but outside the current brief.

This example is a good illustration of the facility of software to allow a user to construct a logical model in a thoroughly Euclidean manner. There is no mathematical notation. Concealed algorithms have been invoked to create shapes, find intersections, normals, etcetera, all in ways closely analogous to the most ancient construction using physical models and compasses. The opportunity to interate with new parameter values, or using the same idiom, new proportions within the individual relationships without reconstruction of the model is the only uniquely computational contribution to the process.

Nevertheless the constructed relationships are not self apparent in the three dimensional visualisation of the geometry model. They can only be discovered through interrogating the graph of relations. Thus they are not readily available in human-human interaction or collaborative design interaction and the implications of 'lean the columns in more' are beyond intuitive reach.

The stepping cubes of the 'Cresteria'.

A second simple but more 'mathematically' generated part of the model provides a possible counter example.

The upper part of the cornice is composed of stepping cubes of stone, simple planar geometry in comparison to the combinations of helicoids, paraboloids and hyperboloids that compose much of the buildings surfaces and structure. The steps like the hexagonal prism frieze that support them 'grow' towards the centre of the colonnade and the precise number of steps and their rate of growth is not categorically discernable from the primary evidence: the 1917 photograph of the Gaudí drawing. While the hexagonal frieze is seen to relate to the (changing) column spacing below it, the relationship between the units of the frieze and the steps above is less clear. This situation is reversed in a long worked-on model completed in the 1980s, over half a century after Gaudí's death. This information is given to set the scene for the type of flexibility, variable iterations and support for deliberations that might be expected of a contemporary digital design model.

The stepping cornice can be seen to be composed of repeating units of three to five steps laterally across the composition from front to back, or roughly normal to the front plane. Each of these successive units is deeper and higher in roughly equal proportion to the one that preceded it as you move up the sloping cornice from the outside to the centre and within the unit the steps ascend from the front to the centre and descend again towards the back. The exact alignment of the steps from one unit to the next varies as you ascend. There were many ways to structure this but a decision was taken early to, once more, provide a framework of

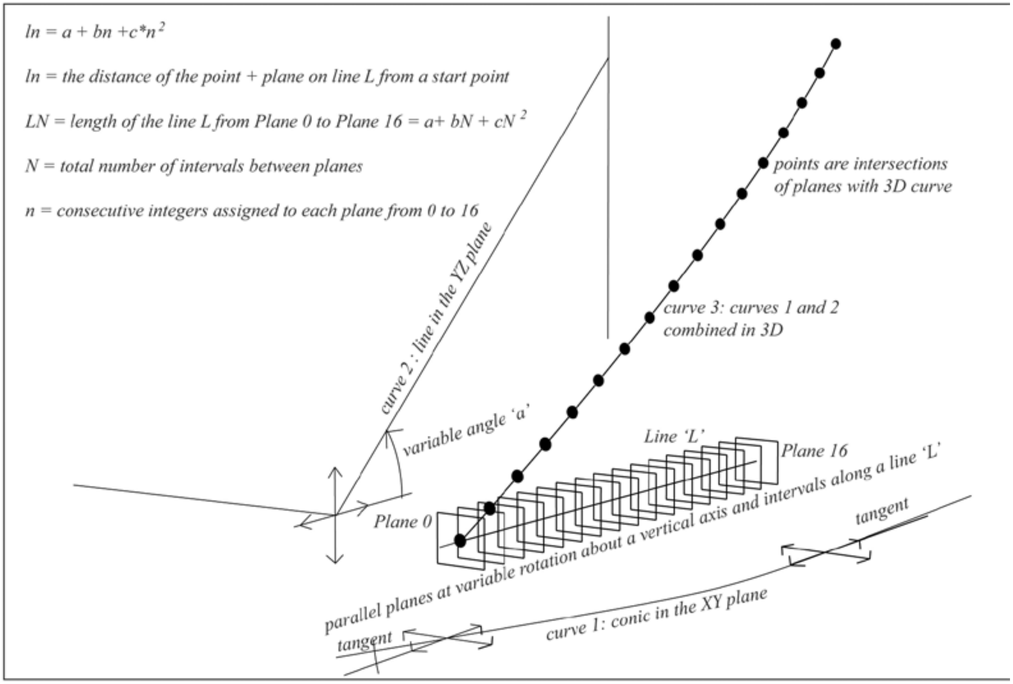

Figure 4
The diagram shows how a 3D curve is defined by combining a parabola in plan and a straight line in front elevation. Parallel planes are distributed (in a parametrically variable way) along a line setting up the growth series in the steps through the definition of their distribution.

three dimensional curves based initially on parabolas (quadratics) in plan and straight lines in elevation/vertical section. Onto this framework a system of variable (parametric) growth would be introduced (points of intersection on the curves) that could vary in type and rate of change, in other words, it could conform to various functions as well as being parametrically variable for each. Figure 4 is a simplified diagram showing one of the 3D curves and its intersection with parallel planes at variable intervals. In this process a parameter would also be introduced for variable instances or number of stepping units within a given overall length (this length also parametrically variable) for the whole assembly. This would maintain the maximum flexibility for fine tuning the best fit for the stepping assembly to the graphical primary evidence in a number of different ways once the model was completed.

Here, once more, geometrical construction is employed to set up the primary relationships. The curves that provide the foundation can move anywhere in space, change their overall length through a single linked parameter, should site information or some other input change, there are angle parameters for each of the lines governing the pitch of any particular ascent, and the plan shape of the curves can change through the tangent angles and shape parameter of the parabola. The stepping units are understood to be parallel to one another but the angle of the plane governing their collective interfaces to the ordinal planes can vary parametrically. The base of the whole assembly conforms to a plane parallel to the top of the tilted frieze of hexagonal prisms. But there is also a further level of sophistication in defining the variable growth of the stepping units.

Perhaps the least flexible aspect of the model is the granularity of the units. Each of these was introduced as an independent instance of a single parametric model re-parented on the local points to

The principal curves through the stepping 'cresteria' were, like those for the hexagonal prisms and column throats also found to be parabolic in plan and straight lines viewed in the front plane.
The growth in the depth of the steps was not the same linear pattern as with the hexagons, however. The position of the front of each step was found to be statistically a quadratic function of the set number. Form:

$$a + b*n + c*n^2$$

'a' turned out to be 0.1
b and N are variables that can have their values changed in a spreadsheet
'c' is calculated automatically: c=(((L-a)/N-b)N
this seemed in accord with the parabolic plan

`Angle relation to hexagons` (deg)	90
`lowest plane offset` (mm)	438
`Highest plane offset` (mm)	-16000
N	20
b (mm)	360
`height of top points` (mm)	1050
`Inclination of top points` (deg)	-3.1
`inclination of third points` (deg)	-2.5
`height third points` (mm)	350
`Height 4th points` (mm)	200
`Inclination 4th points` (deg)	-2
`runoff angle(YZ)` (deg)	2
`fintype2_01 Front point ratio`	1
`fintype2_03 upstand ratio`	0.25
``height of 2nd points`` (mm)	1000
``Inclination of 2nd points`` (deg)	-3

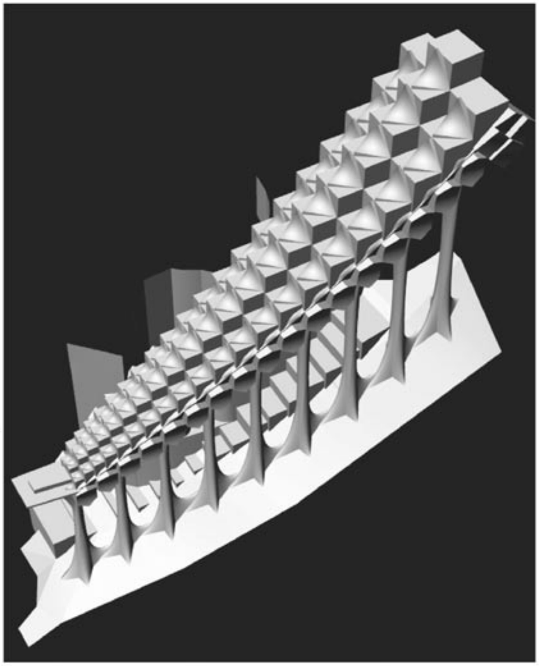

Figure 5
The growth series in the 'Cresteria': table or diagram showing the basic user parameters for growth and instances of steps and equations or relations that link the parameters to activate these user parameters.

determine its location, proportions and dimensions. Once these are introduced into the overall model they are no longer linked to their prototype so any global change to the unit involves extensive local editing or dispensing with the superceded and reinstantiating all the instances of the new version.

Returning to the choice of the quadratic for not only the plan section shape of all the component parts of the colonnade but also for the initial formula for the growth of the stepping cornice of cubes, or 'cresteria', this was based on careful measurement (and analysis using curve fitting software) of, firstly, and most authoritatively, the original photograph, secondly, the later plaster model, for which we also had access to Cardoner's original notes and, thirdly, the site information from the 'as built' surroundings. The straight pitch lines through the all the ascending lines of steps suggested that the integral would be a second order function and the curve analysis software confirmed that next to a polynomial order n, the quadratic was the best fit, and a statistically good fit, common curve for the distance between adjacent steps plotted against step number. This had a seductive consistency about it. However replacing the quadratic by a completely different function presented little challenge to the modeller as long as that function and the values in it were passably consistent with the step distribution. Anything too outlandish can result in self intersecting surfaces or impossible command structure (for instance intersections defined for entities that no longer intersect) within the actual surface units.

The final version of this part of the model has so far been robust in allowing for the iterative variations requested in the team. Early versions had shortcomings. In particular the stepping unit interfaces were defined vertically but the horizontal relationships were not maintained under parametric variation. However the only design group variation that was not well accommodated in subsequent versions was the decision to move from a five step module based on the 1980s plaster model to a simpler understanding of the original as a three step module. In this case the limitations of having chosen the discrete unitary model as discussed above came into play. What if the configuration of the steps across the cornice had conformed to a meta structure more similar to that adopted for the steps longitudinally? Could these also have included the growth factor and variable instancing?

Discussion

Both these case studies which have been 'isolated' from the associative geometry model of the narthex of the Sagrada Família church employ the age old approach to constructing geometry to create the associations. There are some subtle differences, however, between the meta structure of the two parts of the model. These differences seem to impact on the facility with which the model can respond flexibly to unpredictable inputs and requests for iteration in a shared design process. Clearly there are many different types of relationship at play here and it would be unwise to draw sweeping conclusions from the experience of working with two parts of the model with contrasting objectives. The first example is confounded by three deterministic geometrical decisions with mutual impact: first, the analysis of the prisms in the frieze as parallel sided and parallel to one another, second, the columns found to be arrayed and aligned on a curve in plan projection (nominally a parabola, symmetrical about the central plane of the colonnade) and thirdly for the column central axes to align with the central plane of alternate hexagonal prisms, the growth in the prisms thus governing the change in column spacing. The interplay between the rectangular array of the frieze and the parabolic array of the columns directed the way in which the column inclination and alignment could be varied.

While conceding that the first example is more complex and contestable than the second, the most significant difference between the two case studies seems to be the fundamentality of the '*user controls*' in the second example compared to the first.

What if the first example had been constructed from a similar starting point of user controls in the schema? Thus the centre points on the columns might still be arrayed on a three dimensional curve, their internal ellipse axes arrayed on the two dimensional plan component of this curve, defined not by the software's conic algorithm but by a parametric quadratic equation, easily superseded in subsequent versions or iterations. The inclination of individual columns might be controlled by two angle parameters in relation to the (neutral vertical) position of the central axes in each of the directions of the major and minor axes of their elliptical plan. In this way it might or might not coincide precisely with the centre of the corresponding hexagonal prism in the frieze, were these still in a rectangular array, but could be arranged to do so algebraically or it might drive a corresponding parabolic or other curved array in the frieze. In this way more possible relations would be explored and compared to the original evidence.

The proposition here is clear; that the human interaction should lead the modelling process. Relations should correspond to likely expectations of the models variability and control. The conundrum is also clear. It is the very variability of the model that uncovers potential ranges of possibilities that leads to design explorations, or these very human desires.

The examples are taken from a highly constrained, proscribed situation in working towards a detailed design for construction from limited documentation of what is regarded as a definitive design proposal: Gaudí's own restored design models and surviving evidence of drawings.

This work sponsors future research into ways of generalising procedures based on the experience of these and similar case studies and better ways of understanding, predicting and abstracting model 'use' information for designing models.

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