Interactive Parametric Design and the Role of Light in Byzantine Churches

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Byzantine church design depended heavily on natural light which was used for evocative purposes. The orientation of the main axis of the church, the form of the apse and the location and size of its windows are affected by the need for sunlight to shine at the altar on a desired time and for a certain duration. Until now the process of accurately taking account of all the above parameters has been rather difficult and tedious. This paper illustrates the use of digital tools both for the analysis of the geometry of existing apse designs and the parametric generation of new ones. A sophisticated computer program was used to calculate sun angles during the Byzantine period incorporating calendar changes. In addition, Bentley System's Generative Components software was used to construct a parametric model that allows the user to define the geometric conditions and parameters of the apse. The software would then iterate through multiple solutions to satisfy the desired conditions as well as allow the user to change the conditions at will. The tools were used to discover the geometry of the apse of the Byzantine church of Hagia Sophia in Constantinople and to analyze the apse of the Post-Byzantine church of the Xeropotamou monastery on Mount Athos in Greece. The paper concludes with a discussion of the role of parametric tools for architectural analysis and the generation of possible design solutions.

Keywords: *Parametric Design; Generative Components; Light; Byzantine Churches.*

The role of light in Byzantine churches

According to a recent theory, Byzantine church design made use of natural light for the generation of an atmosphere supportive of the liturgical acts (Potamianos, 1996). One of these light effects was the admission of a light shaft on the altar through one of the apse windows on a significant time of day of important celebration dates. According to Christian dogma, the significant time was the Byzantine third hour during which the Holy Ghost descended to sanctify the offerings presented to God by the priest in front of the altar[1]. This central liturgical rite necessitated the incidence of a light shaft on the offerings resembling the descent of the Holy Spirit. In order for this to occur, the church orientation was determined by the Byzantine timekeeping method. This method was based on the assumption that both day and night were subdivided into twelve hours each. But since the day varied in duration according to the season the hours varied likewise becoming longer in the summer and shorter in the winter. The third hour always corresponded to the middle of the morning, but, compared to our timekeeping method, it appears to vary according to the season (Grumel, 1958).

The orientation of the church axis was determined by the dates on which this light shaft was thought it had to occur. For a small church, used a few times a year, the orientation was usually determined by the dedication date and thus there would be a single opening in the apse. Larger churches had their axis oriented toward the equinox probably for three reasons: 1) around the equinox important feasts of Christianity were placed, 2) the side apse windows could accommodate major feasts at summertime and wintertime correspondingly, and 3) the equinoctial orientation was independent of calendar errors. The system of ritual lighting involved the dome design (Potamianos, 2004), the proportions of the church in respect to the way its interior spaces ought to be viewed (Potamianos, Jabi, Turner, 1995), and a number of additional issues (Potamianos, 1996). This paper is limited to the generation of a parametric model which attempts to systematize observations specific to the geometry of the apse in plan as related to the ritual admission of light on the altar.

According to our observations, in designing the church apse, a circle was established first which determined both the interior side of the round apse wall and the location of the altar. The circle center seems to have been placed on the junction of the church axis with the interior side of the wall of the main building. The location of the altar was determined by this circle but the exact position of the priest was at times determined by a polygon circumscribed to it. A second larger circle was drawn, to account for the thickness of the wall, around which another similar polygon was circumscribed. Regular polygons were usually employed. The number of their sides, however, varied considerably from one church to another. The type of polygon used is in general thought to be a matter of stylistic preference while we maintain that to a certain extent it was determined by the intention to direct visible light shafts toward the altar in the manner outlined above. We posit that in order to make a decision on the type of polygon to be used two or more important dates had to be set and the relative direction of sunlight be established for each.

To test this hypothesis, we used the Byzantine church of Hagia Sophia of Constantinople as an example from which to derive the basic design rules. Hagia Sophia was the imperial church in which most major celebrations were held[2]. In Justinian times the emperor participated in thirteen major celebrations per year[3]. The building that stands today was built by Justinian, between 532 and 537, and designed by Isidore of Miletus and Anthemius of Tralles, a physicist and a mathematician[4]. Its dedication to Hagia Sophia, did not refer to any saint by that name but to Christ as the Wisdom (Sophia) or Word of God made flesh, which is confirmed by the fact that the patronal feast was celebrated at Christmas (Mainstone, 2001, p. 133; Downey, 1960, p. 16), on December 25th[5]. Checking the orientation of the church we found that its axis is precisely aligned with the third hour of the equinox at an azimuth angle of 123.4º [6]. To verify that the angles observed in the realized building correspond to the alignments of sun azimuth angles with the altar on the particular dates and hours of the year of its design (532) a specialized computer program was used[7].

A major finding of our analysis is that, for the apse design, the aforementioned basic circle appears to have been used along with a circumscribed regular heptagon (7-sided polygon) about it with one of its chords set parallel to the exterior wall and the opposite apex located precisely in front of the altar (Figure 1). An odd-numbered polygon is not a usual choice for an altar. However, we observed that

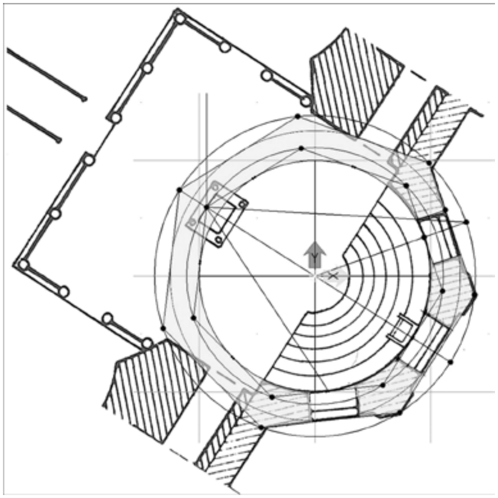

Figure 1
Hagia Sophia apse ground plan with superimposed sun azimuth angles and heptagon as generated by the parametric model.

a heptagon was also used for the apse design of St. Sergius and Bacchus church (527-536), generally considered as a prototype study for Hagia Sophia (Mainstone, 2001, p. 157). The heptagon has an angle at center, corresponding to the chord, of 360º/7=51.4º. The heptagon apex at the altar aligns on December 25th of 532, with the sun azimuth of the third hour (146.9º) shining through the axis of the southernmost window. The method of calculation, using the aforementioned computer program is as follows: On 25/12/532 the sun rose at 7:23:40 a.m. (7.3944 a.m). Until 12 noon 4.6056 (12-7.3944) hours intervened which divided by half (4.6056/2=2.3028) give the third hour for which an azimuth angle of -33.14'l from South or 180º-33.14º=146.86º from North is given by the program. This aligns exactly with the line drawn from the point before the altar to the middle of the southernmost window of the apse (taken at the interior face of the wall).

Subsequently, the angle between these dates (equinox and Christmas) is established (146.9º-123.4º=23.5º). This angle was, in this case a nearly inscribed angle, that is, emanating from the point at the circumference where the altar stood and opening up toward the part where the windows were placed. It was nearly but not quite inscribed because it actually started from the apex of the circumscribed heptagon. When this angle was transferred precisely at the circumference it was changed into 24.6º. The corresponding angle at center would be twice as large (i.e. 24.6º X 2 = 49.2º). Comparing the relationship of this angle to the corresponding angles at center of the two closest polygons we get 360º/8=45º and 49.2º-45º=4.2º for an octagon while we get 360º/7=51.4º and 51.4º-49.2º=2.2º for a heptagon. Thereby the heptagon is the most suitable polygon[8].

However, the difference is not significant enough to justify the selection of the heptagon over the octagon. Apart from the heptagon's advantage in respect to the determination of the position of the priest by an apex there might be additional reasons which might be stemming from the symbolic significance of the heptagon[9]. Additionally, a regular heptagon is considered to be impossible to construct by *legal use* of a compass and a straightedge. Instead, Greek geometers knew about a construction method invented by Archimedes that they termed a *Neusis*-construction (Holme, 2002, pp. 90-93). It is not unreasonable to conclude that a mathematician such as Anthemius of Tralles would illustrate his prowess by building a challenging geometrical construction.

The construction of the parametric model

In order to construct the parametric model, we used Bentley System's *Generative Components* software (Figure 2). Briefly, Generative Components is a powerful object-oriented and parametric system for the construction of geometric entities that can be interlinked through parameters. The definition of a parametric model creates, conceptually, a tree structure that dictates how parameters are inherited among entities. The parametric model for Hagia Sophia followed the above description of the geometric relationships. Yet, it allowed the user to input the nondependent values such as the radius of the apse, the wall thickness, the azimuth angle of the church axis, and the azimuth angle of the incident sun ray.

A script iterated through the possible number of polygons and each was presented to the user. An embedded script also analyzed the polygon to suggest an 'ideal' shape given an evaluation function. The resulting drawing was then placed on the plan of the church apse for verification.

The church of the Xeropotamou monastery

We will test now the parametric model on the Post Byzantine church of the Xeropotamou monastery on Mount Athos in Greece which employs a rather curious polygon for the apse plan. The monastery was founded in 956 and dedicated to Saint Nicephorus (Polybiou, 1999, p. 19). From 1270 on it was dedicated to the 40 martyrs. The church that stands today was built in 1762[10]. A precise survey of it by the Center for the Holy Mountain Heritage Preservation shows the church axis at an azimuth angle of 122º. On the third hour of the 40 martyrs' feast (March 9th of 1762) the program gives us an azimuth angle of 122.5º. On the third hour of Saint Nicephorus feast (February 9th of 956) the program gives us an azimuth angle of 136.5º which aligns with the altar through, the now filled in but still distinguishable, southernmost window of the apse tangentially with its left jamb (Note 11). Thus the inscribed angle 136.5º-122º=14.5º corresponds to angle at center of 14.5ºX2=29º. The regular dodecagon (12-sided polygon) has an angle at center of 360º/12=30º while the 13-sided polygon an angle at center of 360º/13=27.6º. The dodecagon is closer to the angle between the two important feasts and thus chosen for the apse design (Figure 3).

Concluding Remarks

We found strong evidence that light played a major role in the design of Byzantine churches. Furthermore, we found the design of the church apse in particular and the placement of its windows is most probably derived from a rigorous understanding of geometry, sun angles and the behavior of light. Current parametric software provides powerful tools for complex geometrical analysis of historical buildings as well as the ability to think *constructively* as well as *algorithmically* about architectural design. After an initial phase of construction of basic geometric entities and relationships, an embedded algorithm was able to reason about the state of the model, vary the parameters according to an evaluation function and automatically generate a new solution. The user would then be able to seamlessly continue modifying the model. This seamless transition between the algorithm and the construction is perhaps what we find the most promising aspect of parametric software as an analytical tool as well as a true design aid. Since the aspects of the design relating to light are not limited to a two dimensional plan we intend to build, for future work, a fully three-dimensional parametric model and use it not only to analyze a larger number of historical churches but as a compositional aid to new ones.

Endnotes

1. According to the New Testament the Holy Ghost descended on the third hour of Pentecost in the form of light to enlighten the Apostles and enable them to teach the Christian faith. Acts of the Apostles b, 1-15. Correspondingly on the third hour, during the Eucharist celebration, the Holy Ghost is believed to descend upon the offerings and transform them into Christ's Body and Blood which can similarly enlighten those who receive them (Downey 1960, pp. 114, 127).
2. Such as Christmas (Dec. 25th), Epiphany (Jan. 6th), Good Friday, Easter, Monday after Easter, Pentecost, Exaltation of the Cross (Sept. 14th) etc. Major feasts were also the Annunciation (March 25th), Circumcision (Jan. 1st), Transfiguration (Aug. 6th), the anniversary of the foundation of the city (May 11th), and Constantine's and Helen's feast (May 21st) (Downey, 1960, pp. 16, 115).
3. Book of Ceremonies cited in Mainstone (2001, p.

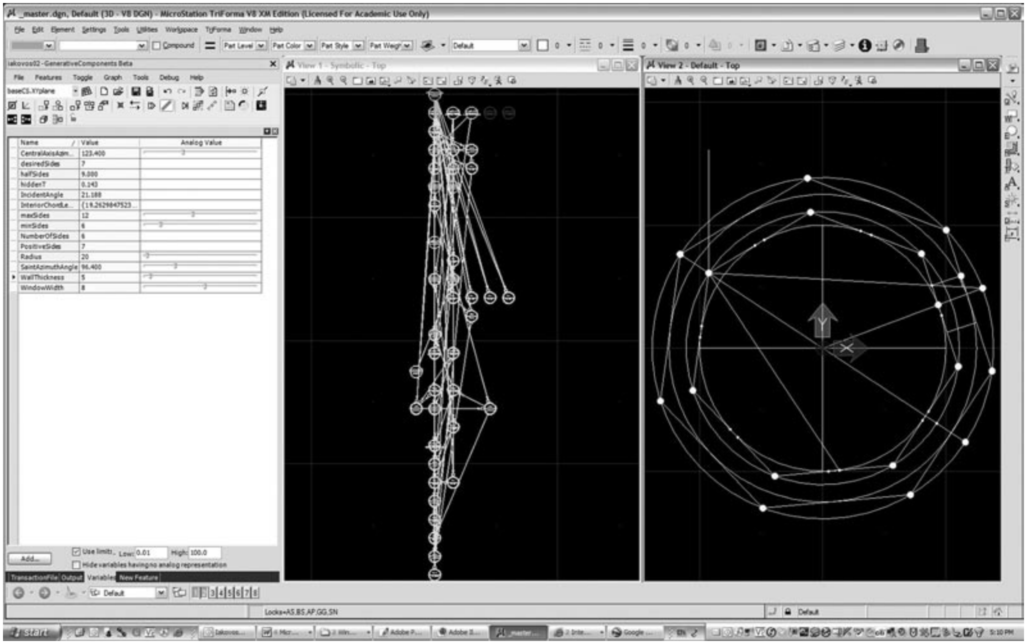

Figure 2
The parametric model showing from left to right: 1) Graph variables, 2) Symbolic view of the hierarchy, and 3) Resulting geometric construction.

231) states that the emperor participated in the liturgy on seventeen occasions a year. However, four of them were feasts introduced after the time of Justinian.

4. The first church was built by Constantius in 360 and was burned in 404. The second church was built by Theodosius II in 415 and was burned in January 532. (Mainstone, 2001, ch. 5).
5. Proof to Christmas celebration on this date is Paul Silentiarius' comment on the forthcoming next day Christmas celebration in his speech delivered on the church dedication after its second rebuilding by Justinian on December 24th of 562 (Mainstone, 2001, p. 219).
6. This azimuth angle is published by Van Nice (1965), whose work is the most precise survey available of the church to date.
7. Program code by Jeffrey Sax © 1991-1992 distributed by Willman-Bell, Inc. no. 33854 as elaborated on by Associate Professor Richard Sears of the University of Michigan.
8. Additional dates were served as well but due to space shortage this cannot be dealt with here.
9. The heptagon carries multiple symbolic connotations among which is its association with Virgin Mary, since 7 is thought of as the virgin number. It also corresponds to the then thought of as the 7 planets of the planetary system which included the 5 planets, the sun and the moon. Furthermore, it stands for the fusion of divine and human since number three, symbolizing divine nature, combines with four, symbolizing human nature.
10. It is significant that demolition work began on March 9th of 1762, i.e. on the 40 martyrs' feast day (Polybiou, 1999, p. 74).
11. The year of the initial dedication to St. Nicephorus has been used for the calculation because it must have been kept as a direction transferred from the older demolished church and realized

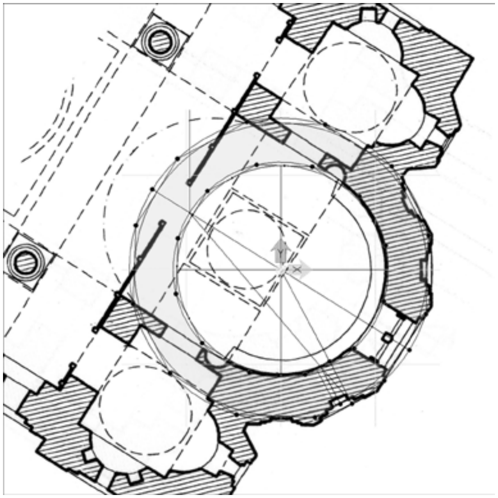

Figure 3
Xeropotamou monastery church ground plan with superimposed sun azimuth angles and dodecagon as generated by the parametric model.

as the direction of the southernmost window. It is also remarkable that the direction commemorating St. Nicephorus is also found on an extant church model, unique of its kind, constructed in 1762 by Constantine, a royal engineer of the Ottoman state (Polybiou, 1999, p. 50, 73).

References

Downey, G.: 1960, Constantinople in the Age of Justinian, University of Oklahoma Press, Norman, OK.

Grumel, V.: 1958, Traité d'Études Byzantines, I: La Chronologie, Bibliothèque Byzantine, Presses Universitaires de France, Paris, pp. 163-165.

Holme, A.: 2002, Geometry: Our Cultural Heritage, Springer-Verlag, New York.

Mainstone, R. J.: 2001, Hagia Sophia: Architecture, Structure and Liturgy of Justinian's Great Church, Thames and Hudson, New York.

Polybiou, M.: 1999, To Katholiko tes Mones Xeropotamou, Ypourgeio Politismou, Athens, p. 19.

Potamianos, I., 1996, Light into Architecture: The Evocative Use of Natural Light as Related to Liturgy in Byzantine Churches, Dissertation, University of Michigan, Ann Arbor.

Potamianos, I., Jabi, W., Turner J.: 1995, Exploring the Proportions of Middle Byzantine Churches: A Parametric Approach, Sixth International Conference on Computer-Aided Architectural Design Futures, pp. 483-493.

Potamianos, I.: 2004, The Mathematics of the Ideal Dome, in Weber R., and M. Amann (eds), Aesthetics and Architectural Composition - Proceedings of the Dresden International Symposium of Architecture, pp. 66-71.

Van Nice, R.L. 1965, Saint Sophia in Istanbul; An Architectural Survey, Dumbarton Oaks Center for Byzantine Studies, Washington, D. C.