Restitution and Interpretation of Spatial Representations

A New Approach for Teaching Representation

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The use of computers has changed the practice of spatial representations. The users are no longer drawers but modelers who need to be able to check the coherence of models. The teaching of representations has to adapt with this evolution, especially in Schools of Architecture. A pedagogical way is to give interpretation tools of spatial representations through projective properties (incidence or affine properties) and consequence of intrinsic constraints (parallelism, orthogonality, and symmetry). The application of this knowledge is essential for the rebuilding of existing 3D objects or for a design process, with the restitution of 3D models from sketches. These approaches are illustrated in a pedagogical way, using dynamic geometry, in the restitution of the polyhedron of the engraving "Melencolia I" of A. Dürer, and in a dynamic sketch of a skylight inspired of the Vitra museum of F. Ghery.

Keywords: *restitution; perspective; teaching; geometric algebra; sketch.*

Introduction

Computations of 2D images from 3D scenes are a common reality, even for complex scenes and for animations. The increase of computing capacity is used to generate very sophisticated images, but computers still have difficulties to give an interpretation of the space indicated by drawers. At the same time, the users, invaded with images, are faced with the increasing need to check coherence of models.

In this context, the teaching of classical perspective appears to be a cultural heritage of the history of occidental representations, but is no longer operational. Meanwhile, the inverse problem of 3D restitution of a 2D representation, is no longer a specific problem concerning only specialists (restitution from photography of a destroyed object for instance), but it has a growing significance:

- For computer, the goal is to interpret sketches and documents
- For architects, the aim is to build a mental reconstruction of 3D objects in order to check and evaluate spatial representations.

We propose that automatic 3D restitution methods, developed in the last decade, could be involved in a new approach of the teaching of representation techniques.

In this article, we will first expose specific problematics of 3D restitution in automatic process and the way we experiment solutions. In a second part we will explore the ways it can be used in a pedagogical context.

3D restitution problematic

It is possible to rebuild a 3D object from a single 2D perspective representation and a list of spatial properties (parallelism, orthogonality…). For instance, only one (or none) right-angled parallelepiped exists (except considering an homothetic transformation) as a restitution of a perspective representation. Actually, the drawing is never absolutely correct, and the reconstruction always amounts to correct it to be coherent with the stated spatial properties. The aim is to correct drawing close enough to the original.

In a more general way, in the GINA project[1], we experiment a 3D reconstruction system, built around a projective geometry core, using the Grassmann Cayley algebra and we have chosen to separate this 2D correction from 3D restitution of the correct 2D projection. Some intrinsic projective properties of polyhedrons can escape from the operator. So we experimented the way to check the coherence of polyhedrons representations, using the "labeling method".

Pedagogical applications

The teaching of geometry, particularly, space geometry, has almost disappeared. This leads us to set up a geometric culture in order for students to be able to check the coherence of projections.

The dynamic geometry software, used in our teachings, allow to explore the behavior of geometric constructions in real time, in different representation spaces.

According to the kind of projection, the properties used for restitution can be divided in:

- Incidence properties of projective geometry,
- Parallelism properties, and partially, orthogonality properties, in affine geometry
- Metric properties in Euclidean geometry.

In a central perspective, the knowledge of vanishing points allows to express parallelism considering incidence. Mutually, the interpretation of an incidence as a 3D parallelism is not obvious and has to be checked or assumed.

Two situations can be considered with regard to the level of precision of drawing and the knowledge of the represented 3D object:

1. The projection is supposed to be precise (photography, paintings, drawings or engraves of masters). The coherence of the representation is rigorous and some hypotheses on objects can be evaluated (using auxiliary plane method, parallelism conditions, and in some cases, orthogonality conditions). That's what we will show in a study of the famous engraving "Melencolia I" of Albrecht Dórer, where several hypotheses on the represented polyhedron will be discussed. In addition with this study the use of a 3D restitution software has an important impact: the student has to determine what are necessary and sufficient conditions to restitute an object.

An other aspect is the use of duality between faces and vertices. This approach allows a systematic construction of intersections and can be used to check constructions. It is analogue to the Cremona method used in mechanics and can be used in orthogonal projections (axonometry) and extended in central projection (perspective).

2. The projection is a perspective sketch where accuracy is not a goal, the aim is to express spatial intentions. In this case, the computer has to give an adapted help to the designer for drawing in perspective. The system has to be able to propose corrections to be validated or not. It is possible to propose help to perspective drawings by the use of special projective snaps. For instance it is possible to propose an orthogonal trihedron compatible with an initial orthogonal trihedron taken as a reference. An example of such an application will be shown in the restitution of a skylight (inspired by the Vitra museum of F. Ghery).

Case of restitution with a precise perspective : the polyhedron of "Melencolia I" of A. Dürer

The famous engraving of Albrecht Dürer, "Melencolia I", realized in 1514, shows a complex scene in

[1] www.ariam.archi.fr/gina

perspective where is represented a massive object usually seen as a stone polyhedron (description of Panofsky, 1943). This masterpiece and this particular object inspired a lot of artists (cf. exhibition in Paris, Mélancolie, génie et folie de l'occident (Clair, 2005). The remarkable sharpness of the engraving and the perfect knowledge of the laws of perspective exhibited by Dürer (1525) allow to be confident with the precision of the representation. It gives an emblematic example of a quiet simple but non-trivial object known by a perspective. In order to limit our study we consider only few elements of the engraving: the polyhedron, the boundary of the engraving and the

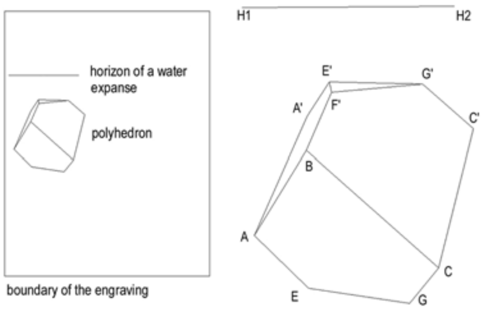


Figure 1
Schema of some elements of "Melencolia I" and names of vertices

horizon of a water expanse (Figure1).

Coherence of the 2D representation : incidence properties, hypotheses of plane faces.

The first question about this object is to check that the sides of the polyhedron are plane. A consequence of plane sides in the common case of a truncated 3-faces pyramid, is that the 3 lines of the truncated faces have to converge (to the missing vertex). If we consider the pyramid (A'BC'E'F'G'), the lines in projection (A'E'), (BF') and (C'G') must cross in a unique point B' (in 3D as in projection). In figure 2 (left) the intersections give three points (B'1, B'2, B'3). The difference may be due to errors in the interpretation of the position of vertices in the engraving : edges are not always rigorous lines, and vertices are not easy to determine, especially for small angles between lines ((AA') and (A'E') for instance). A correction of the

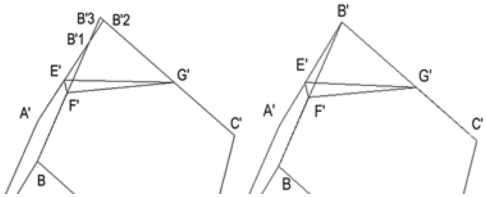


Figure 2.
Intersection of three planes, in the perspective of the polyhedron. Incoherent perspective with three points of intersection instead of a unique one (on the left) and proposal of a coherent perspective (on the right)

drawing must be done to continue study of the objet assumed to be a polyhedron. For instance B' is assumed to be B'3 (Figure 2, right), because the points A' and E' seem to be less reliable than other points. The "labeling method" could also be used.

With a coherent perspective of a polyhedron, it becomes possible to determine hidden lines, using auxiliary planes (Ciblac, 2004), thanks to the general propriety after Steiner (Pirard, 1967): "if in a polyhedron, a side has n edges, and if the projection of n-1 adjacent edges from n-1 vertices, are known, it is possible to build the n^{th} adjacent edge projection." The construction of the hidden lines is given for the hexahedron (ABCDA'B'C'D'), where D is the intersection of (AE) and (GC) (Figure 3, right). It is compared with a preparatory sketch of A. Dürer (Figure 3, left) where hidden edges are represented. The point D' is not visible in the engraving but determined only by incidence considerations (no assumption on parallelism nor orthogonality).

Parallelism and symmetry

Some assumptions on parallelism can be made on the polyhedron:

- The hexahedron is a parallelepiped
- The two triangles EFG and E'F'G' are parallel, and horizontal.

The addition of these two assumptions gives an interpretation of the polyhedron as a truncated parallelepiped by two horizontal planes.

Considering the hypothesis of a parallelepiped, we can propose a compatible perspective of this object using vanishing points of parallel directions. Some vertices have to be slightly moved to satisfy

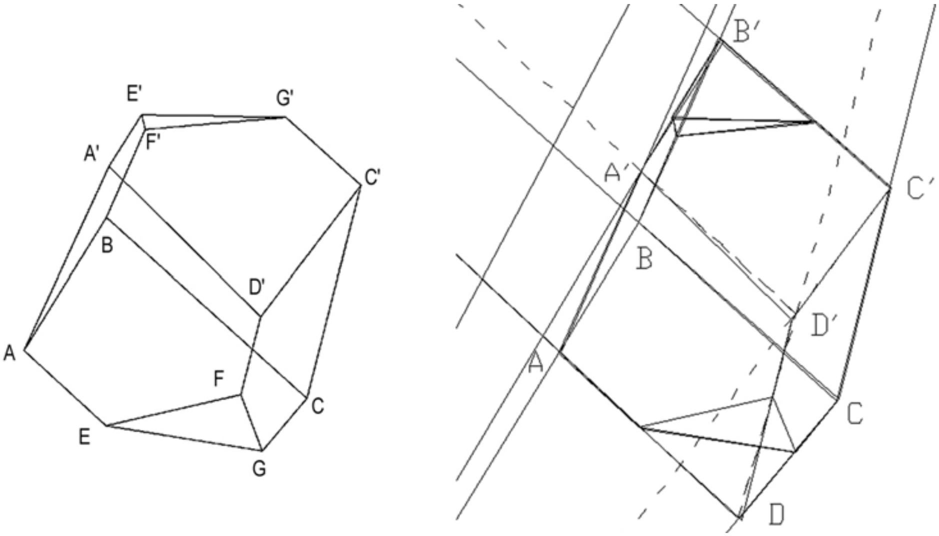


Figure 3
Polyhedron after a sketch of A. Dürer where hidden edges are represented. The sketch is inversed to correspond to the engraving (on the left). On the right, comparison of constructed hidden edges on the engraving (dashed lines) and the sketch

this hypothesis. A proposal of a parallelepiped close to the polyhedron of the engraving is given. In figure 4 (left), the positions of the vanishing points of the directions of edges (pf1, pf2 and pf3) are given. The orthocenter h of the vanishing points is outside of the triangle (pf1,pf2,pf3). That means that the three directions are not orthogonal. A comparison with the points of the engraving (Figure 4, right) shows some slight offsets. The point D' is directly the intersection of the lines drawn from the vanishing points and the corresponding points A', C' and D.

Since (ABCDA'B'C'D') is a parallelepiped, the two triangles (A'BC') and (AD'C) are parallel and symmetrical according to the center of the parallelepiped.

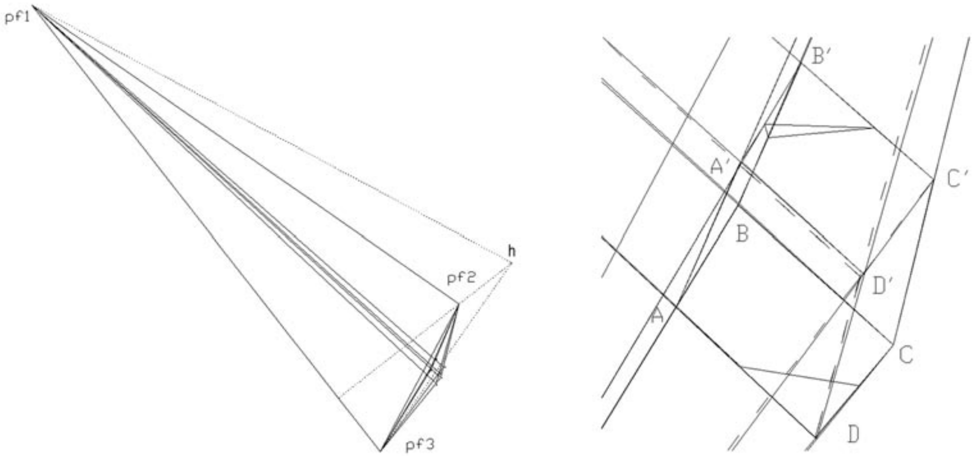


Figure 4
Vanishing points of the directions of edges of a parallelepiped closed to the polyhedron (left). Comparison of the projection of the parallelepiped an the drawing of the engraving (right).

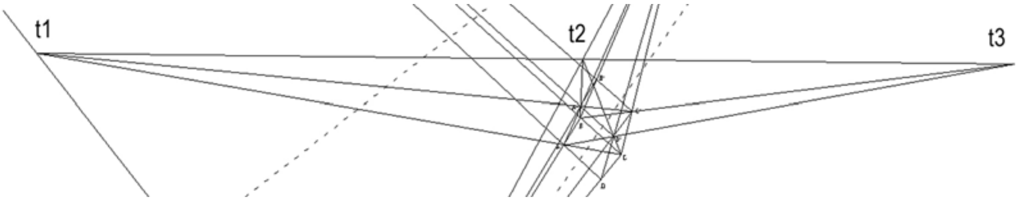


Figure 5
Vanishing points p1, p2 and p3 of the sides directions of the triangles (A'BC') and (AD'C)

Hence, (AC)//(A'C'), (BA')//(CD') and (BC')//(AD'). The corresponding vanishing points of these three directions are respectively t1, t2 and t3 (Figure 5).

A first assumption of symmetry is to consider the upper and lower planes (EFG) and (E'F'G'), parallel to the planes (A'BC') and (AD'C). It involves :

- (EG), (E'G'), (AC) and (A'C') are parallel,
- (FG), (E'F'), (BA') and (CD') are parallel, and
- (EF), (F'G'), (BC') and (AD') are parallel.

With this hypothesis, the point F, that is visible in the sketch but not in the engraving, is constructed from E and G. The hidden vertices are compared with a sketch of A. Dürer where hidden edges are represented (Figure 6).

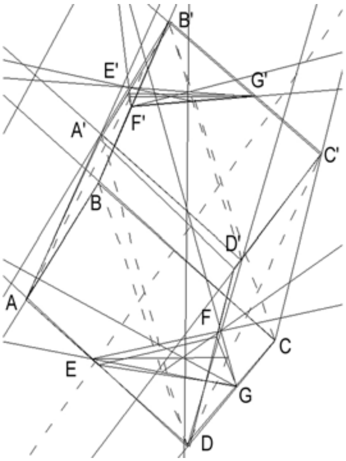


Figure 6
Rebuilt polyhedron on a parallelepiped with parallel triangular sides, compared with the sketch of Dürer

Use of duality for correction

The application of duality between faces and vertices on coherence of axonometric projections allows to establish flatness of sides or their belonging to parallel planes or to a beam of

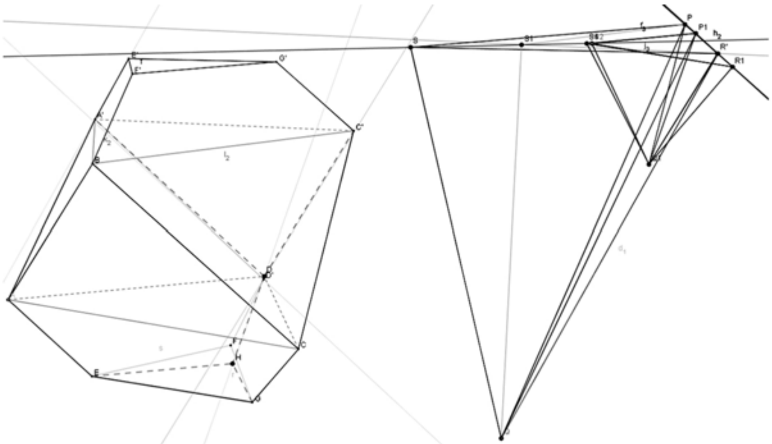
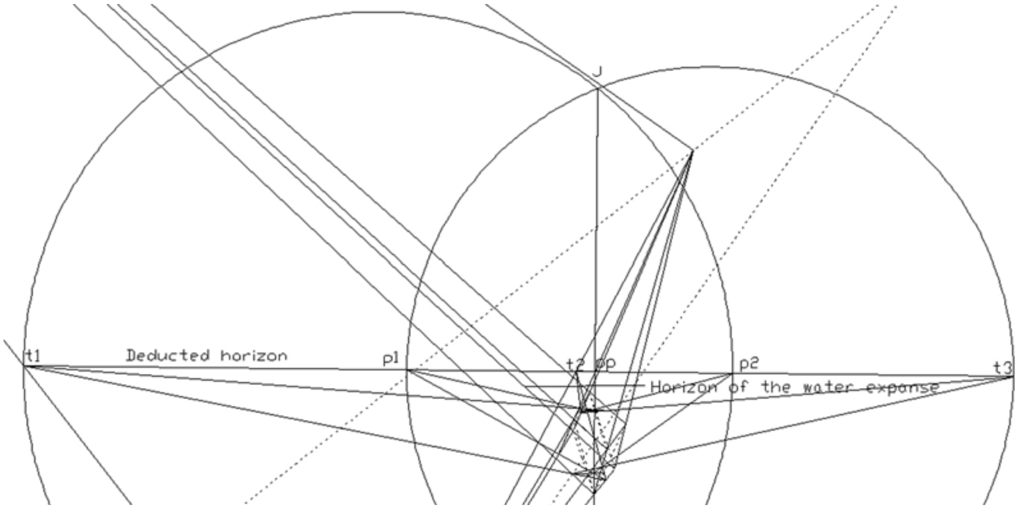


Figure 7
Corrected polyhedron and sketch (left) and dual figure (right)

Figure 8
Vanishing points of two couples of orthogonal directions (t1,p2) and (t3p1) and deduction of the principal point pp, and the principal distance equal to (pp,J)



planes. We applied it in central perspective (Figure 7) to dynamically find coherent hidden lines with the hypothesis of parallelism of triangular sections.

Orthogonality: determination of principal point and principal distance.

A second assumption of symmetry is to consider the sides (EFG) and (E'F'G') as equilateral triangles. Considering this hypothesis, medians are orthogonal to the edges of the triangles. Medians are easily built in the perspective of a parallelepiped, so that three couples of orthogonal directions and their vanishing points are determined. The principal point pp and the principal distance (pp,J) can be deducted, if the projection plane is supposed perpendicular to the triangular sides (Figure 8). If they are also supposed in horizontal planes, they give a deducted horizon, parallel to the horizon of the water expanse, but a little above it.

Reconstruction using Photomodeler

The knowledge of the principal point and principal distance gives the position of the center of projection (or eye). With the perspective and the assumptions of parallelism it is possible to rebuild the polyhedron by construction of lines in 3D with a modeler. It can also be rebuilt using specific software like Photomodeler2 (from EOS systems Inc., Vancouver, British Columbia) with other hypotheses. Photomodeler is a software that is able to rebuild a 3D object by identifying visible points on different photographs. A centered image, corresponding to a known focal distance (35mm, for instance) and a precise size (24 mm x 36 mm, for instance), is given (Figure 9, left). The use of only one perspective cannot give any result with Photomodeler because it rebuilds objects from at least 2 views. If we assume a symmetry of an angle of 120° around the (B'D) axis, the engraving can be considered as three different views of a unique objet in an initial position and after rotations of 120° and 240° from the initial position (Figure 9, right). No correction of the position of vertices nor additional hypotheses are necessary to use the software, because the knowledge of the principal point and the principal distance, and the hypothesis of symmetry are implicitly given by the views. With three views, hidden vertices can be omitted because

[2] www.photomodeler.com

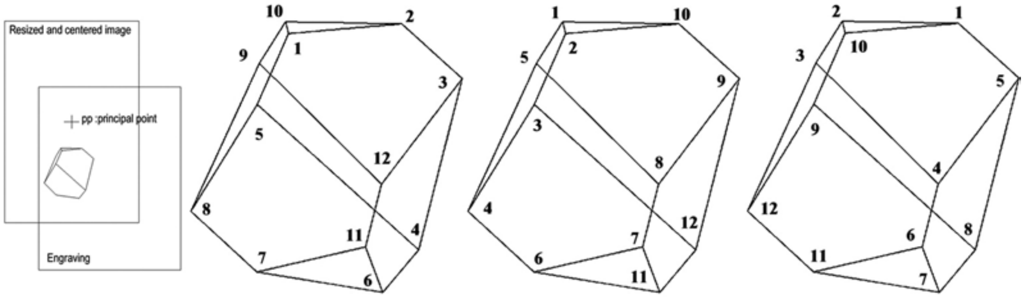


Figure 9
Centered and resized image of the engraving for Photomodeler for a 35mm focal distance (left). Position of vertices in the initial position (on the left), after a rotation of 120° (center) and after a rotation of 240° (on the right) from the initial position

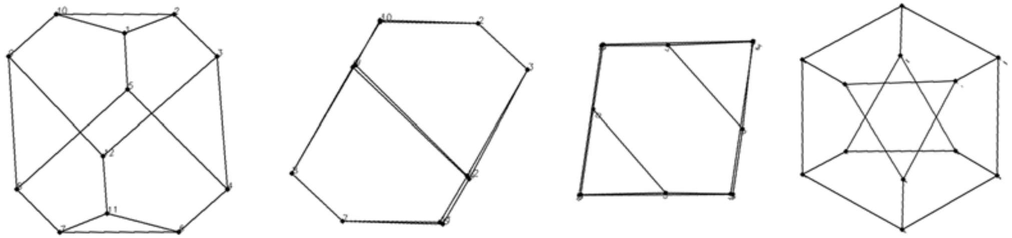


Figure 10
Axonometric views of the rebuilt polyhedron with Photomodeler

points have to be given at least twice. The restitution using the sketch of Dürer (Figure 10) is very close to a symmetrical polyhedron built with to equal equilateral triangles and six equal pentagons constructed on a rhomb.

Case of restitution when sketching : inclined skylight

We propose here a dynamic perspective representation of a skylight inspired by the Vitra museum of F. Ghery. The base of the building is constructed in right-angled parallelepiped. The skylight placed on it is supposed to be a freely inclined right-angled parallelepiped. It is possible to make a perspective construction including the constraint that two right-angle parallelepipeds are represented, with variable relative position of the elements. Because it is possible to propose an orthogonal trihedron compatible with an initial orthogonal trihedron taken as a reference. The 3D restitution is immediate since the vanishing points of tri-rectangle trihedron are known. Two different positions of the skylight are given from the dynamic construction (Figure 11).

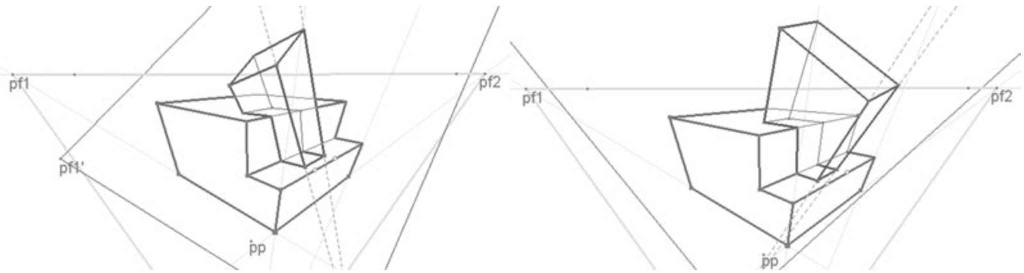


Figure 11
Two positions of a right-angled parallelepiped in a dynamic perspective

Conclusion

This applied proposition aims to heighten awareness on the importance of an adapted teaching of representations, founded on an indispensable geometric knowledge. In addition of these applications some theoretical tools could be introduced in the learning path. Today, projective geometry is used in Nurbs and the Grassmann Cayley algebra allows to realize projective cores for rebuilding software.

These considerations about restitution tools and teaching approaches show the evolution of the needs for comprehension and modeling of space. Software allowing a direct restitution still have to be proposed to architects and could be a new way for conception. The teaching of representations has to be an initiation to these problematics and we hope it will have pedagogical repercussions.

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