AN EXPERIMENT IN COMPUTATIONAL COMPOSITION

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ABSTRACT

A compositional study based on a visual interpretation of information theory is introduced. An algorithm is presented that relates variety in spatial parameters to visual information, along with a genetically inspired mechanism for refining a design through cycles of incremental cumulative changes. Two- and three- dimensional examples are shown.

Descent

From disorder (a chaos)
order grows
- grows fruitful
The chaos feeds it, chaos
feeds the tree.

William Carlos Williams

1. INTRODUCTION

Recent research efforts on computational approaches to design have emphasized the structural aspects of parametric composition, or have concentrated on grammatical aspects of design [Mitchell 1988]. While these approaches are promising, the problem of what St .Thomas Aquinas called 'concreation' , the concurrent determination of both the compositional structure *and* the sensual form, have not been adequately addressed. With the advent of parametric design, and with grammatical or other knowledge-based approaches, the issue of how to use computation to determine proportion becomes very important. The present paper attempts to revive the issue of proportion in architectural composition, and suggests that the multitude of approaches used throughout history may have been approximations to a much more fundamental concern, the provision of adequate visual information to demand, and sustain, visual interest.

The second motivation prompting this work is of a historical nature. It is the beginning of an overt attempt to relate the use of computers in architecture to the history of formal approaches to art and design, from classical times to the present, but in particular to the explorations of the Constructivists, the artists of De Stijl, and their followers. That is the tradition that computational composition must recognize and claim as its precursor, the tradition to learn, explore and expand.

2. BACKGROUND

2.1. Some questions.

Malevich paints a series of rectangles on a canvas, some larger, some smaller, some at this angle, some at another, placed at odd locations. We call this a composition. What guided Malevich to size and place those shapes in that particular way?

In cities such as Paris, in places where a strong, unified architectural context exists, the presence of a modern building among the mass of classical buildings feels like an absence: a hole in the fabric. What is missing?

It is often said that an artist *knows when to stop*. This presumably means that the artist can recognize when something has reached an acceptable state of completion, has a visual goal in mind. Malevich has a visual goal in mind (among other goals), and a way of achieving it. What is this goal?

A person looking at the city, on the other hand, characterizes the modern grids as *blank*, perceiving a contrast between older buildings and newer ones. In the former, one perceives a presence of a quality, in the latter an absence. Like Malevich, the viewer has a way of distinguishing one from the other, by some criterion. But what is that criterion?

The difference is not one of the number of parts between one composition and another. After all, of the infinite ways that eight rectangles could be arranged on a canvas, only few would be seen as compositions, and fewer still as those of Malevich.

It can be argued that Malevich constructed his compositions in very precise ways, and that we, as viewers, can recognize that the compositions are thus not random, perceiving the presence of a geometric construction even if the construction itself evades us. But then, why is the construction interesting to us?

Two answers seem possible: first, since constructed objects show a high degree of interrelationship, we can conclude that 'coherence' is something that we have the intrinsic capacity to discern and which we value, since it helps us distinguish between randomness and order. Second, we can speculate that we respond to constructions because of what constructions produce, that is, that the constructions are means for achieving some other goal. But, once again, what is that goal?

The work described in this paper addresses both of these possibilities; to the extent that it describes an implemented algorithm for computational composition it presents a generative 'construction' of forms to which we respond; and to the extent that the algorithm relates visual interest to information content, it attempts to answer the questions posed above: The absence we perceive in modern cities, on a visual level, is an absence of visual information; the goal of the visual artist, on a visual level, is to provide sufficient visual information to maintain our interest over multiple exposures to the work.

The primary message of composition is the indication of the presence of intelligence, indicated by a complex pattern of interrelationships, providing information. The secondary messages, qualifications of the first, and of necessity also communicated through pattern, are the responses of intelligence to the world, through emotion. As Beethoven's late quartets demonstrate, intelligent, interrelated patterns inform our emotions.

2.2. The autonomy of proportion.

The use of computational approaches to design is as long as the history of architecture. Computation in a geometric, numeric or symbolic sense was surely an important determinant of the earliest architectural forms. Classical architecture, from the design of a simple element, to the arrangement of elements in an order, to the placement of a building on a site, always seeks to achieve very particular formal goals [Tzonis 1986] . In the middle ages masons' marks, their *signa lapidaria*, were designed on particular *master keys* or *grids* [Eco 1986]. Gothic cathedrals, striving to pursue *the aesthetics of the organism*, the aesthetics of form *and* proportion, were governed by similar geometric relationships [Eco 1985]. In the renaissance harmonic, musical ideas became dominant [Wittkower 1971]. The advent of the scientific revolution made architects seek scientific ways to explain proportional systems [Pérez-Gómez 1985]. Even with the coming of the modern movement, with its break from the past, proportion continued to be pursued by those factions of the movement that maintained the primacy of aesthetic concerns.

The persistence of the concern for determining proportions indicates that while there is no explicit agreement on a particular system of proportion that is superior to all others, there is *implicit* agreement that the expressive power of architecture is in great part due to the masterful modulation of proportions within and between elements. Two observations must be made: first, that the issue of proportion arises independently of the particular explanation given at any time; and second, that until this century, the pursuit of proportion has followed its own course, construction being its servant, and other pragmatic concerns being subordinated to the formal intention.

The autonomy of proportion is most clear in traditions where the compositional form is most clearly defined, since when the number and types of elements that are required is well specified, and their general arrangement clearly understood, the excellent is distinguished from the ordinary primarily by virtue of its proportional relations. This implies that the problem of the proper determination of proportions is most pressing in precisely those situations that yield to systematic description: grammars, parameterization and hierarchic substitution, - that is to say, in those approaches that are at the forefront of today's computer aided design.

2.3. Constructive art and computer art.

The beginning of the twentieth century brought with it a new sensibility in the arts. The great historical forms were challenged, only to allow the implicit investigation of the autonomy of form and proportion to become explicit. Around 1910, after Cubism's initial challenge to representation, several artists began exploring non-objective compositions in systematic ways. Kandisky, Malevich, Van Doesburg, Mondrian, Gabo, Klee, and others, pursued particular aspects of composition without reference to an external reality. Van Doesburg distinguished between concrete art and abstraction, claiming that concrete art was not related to any external reality but was a record of a pure *mental flow*; Kandisky spoke of painting *mental images*; Malevich wrote that Suprematism aimed at demonstrating *the supremacy of pure sensation,* and that its *essential content is the totality of non-objective natural excitations without any goal or purpose*; and Mondrian, influenced by the philosopher Dr. Jan Schoenmaakers, claimed to be aiming at *the representation of relations,* a *plastic mathematics* [Rotzler 1989]. All used constructive techniques to control formal relations, and all pursued the connections between visual art architecture, and music.

Naum Gabo wrote: *[...the Constructive idea...] has revealed a universal law that the elements of a visual art such as lines, colours, shapes, possess their own forces of expression independent of any association with the external aspects of the world; that their life and action are self-conditioned psychological phenomena rooted in human nature; that those elements are not chosen by convention by any utilitarian or other reason as words and figures are, they are not merely abstract signs, but they are immediately and organically bound up with human emotions. The revelation of this fundamental law has opened up a vast field in art giving the possibility of expression to those human impulses and emotions which have been neglected.* [Rotzler 1988]

The various branches of this movement continue to develop, and since the seventies a body of *computer art* has developed that is properly the outgrowth of these early modern explorations. From the initial imitations of Mondrian and Riley by Michael Noll, to mathematical explorations by Frieder Nake, to the explorations of space and order of Manfred Mohr, the hypertransformations of Vera Molnar and the stereoscopic sculptures of Kenneth Snelson, this body of work extends the initial exploration of constructive art in the realm of computing and actively engages the strengths and limitations of technology.

With De Stijl, Suprematism, Constructivism, and at least the aesthetic aspect of the Bauhaus, and their descendants in constructive and, more recently, computer art, the arrangements of shape and their relations are seen as sufficient means of expression. The autonomy of form has been discovered. What is being expressed is something internal, 'non-objective', an 'inward spring,' a 'mental flow.' In their time such ideas may have sounded mystical. In ours they are clues to as yet uncharted cognitive phenomena. Since abstract works of art are in essence visual patterns, the assertion that they can affect us directly, without relying on reference, implies that we have access to mental mechanisms that enable us to decipher and create these patterns in ways that allow us to respond by being 'moved' through some kind of 'pattern description'. To paint 'mental images' is to paint a representation of an image created directly by the mental processes by which we decipher the world. It is to gain insight, access, and control over those processes. The least byproduct of understanding these processes will be a better architecture.

2.4. Information theory.

The autonomy of form and proportion discussed above suggests that it must be possible to discover what aspects of patterns affect us and in what ways. In this paper I begin the exploration of the idea that the arrangement of form contains visual information, and that we distinguish between different designs on the basis of this information.

Uses of information measures for art also have a fairly long history, ranging from Max Bense's [Rotzler 1988] *information aesthetics* , to Stiny's *algorithmic aesthetics* [Stiny 1978], Gibson's [Gibson 1971] *ecological approach,* and March's *generalized measures of information* [March 1975]. This experiment was conducted without reference to these works. In hindsight, it seems that the difference between those approaches and this one is in the directness with which this approach relates the measure of information to perceptual aspects of design, physical location and dimension, and the range of variation in each. Clearly, a more complete examination of precedents is needed to validate the approach taken here. On the other hand, it is worthwhile to occasionally reinvent a wheel in the pursuit of one's curiosity.

3. FUNDAMENTAL CONCEPTS

3.1. Order: intention and singularity vs. information.

The fundamental insight of this approach requires a distinction between two senses of order in design: the ***order of intention and singularity*** and the ***order of information.***

Let us consider a regular square grid. Strict regularity is not found in nature directly, and so its occurrence suggests willful organization, intention. Furthermore, only one arrangement of lines possesses the characteristic of forming equal squares of a given size; it is thus singular, and we distinguish it from the infinite number of other arrangements that do not posses this characteristic. Thus when we speak of a grid as displaying order, we are speaking of order as regularity that suggests intention and singularity.

But we can look at this grid in another way. If the dimensions that describe the cells of the grid are considered to be visual messages, then a regular grid exhibits complete entropy, a cacophony of equal (and thus redundant) voices all screaming for attention, and all sending the same message, that of the unit dimension of the

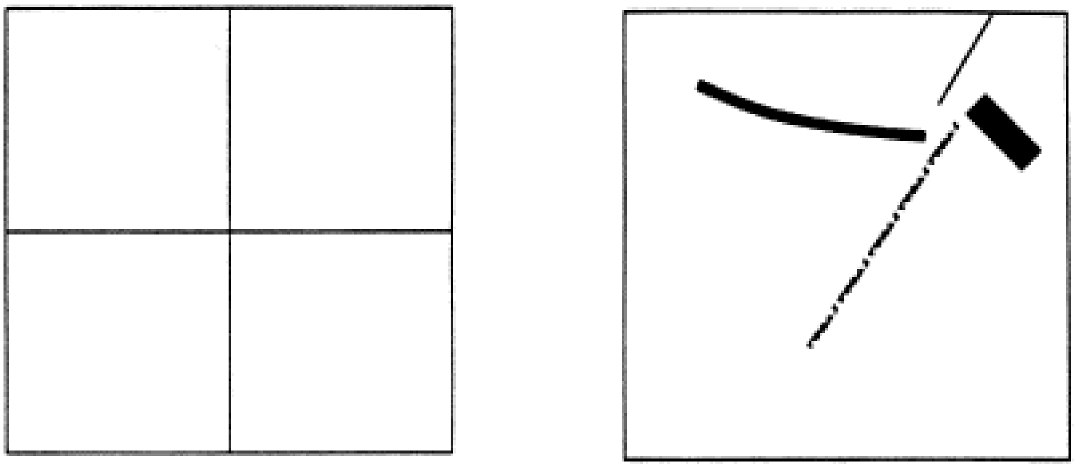


Figure 1. Two compositions.

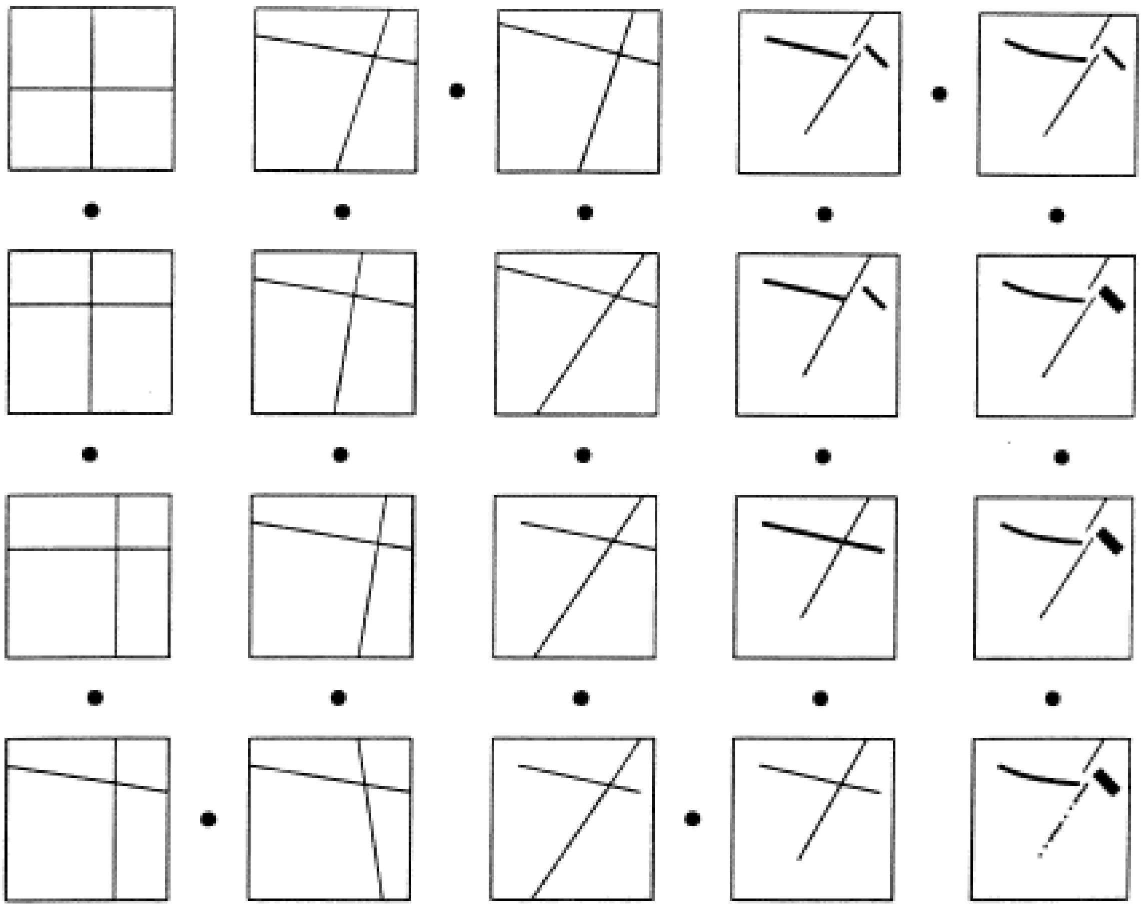


Figure 2. Derivation of the second composition from the first.

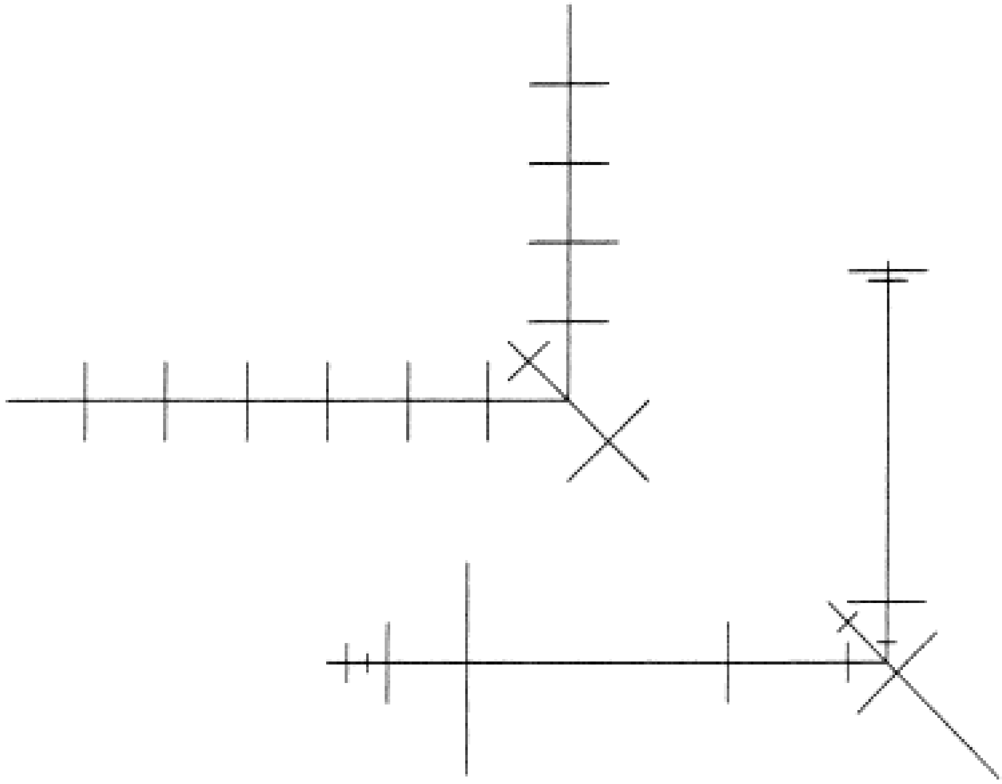


Figure 3. Two skeletal structures.

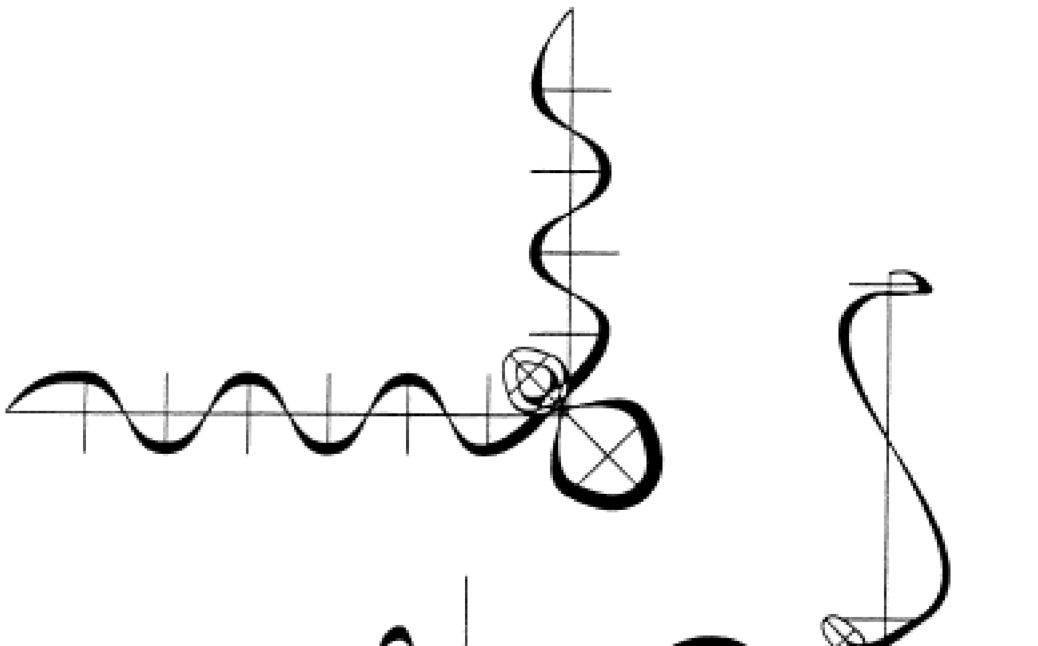


Figure 4. Two curves based on the skeletal structures.

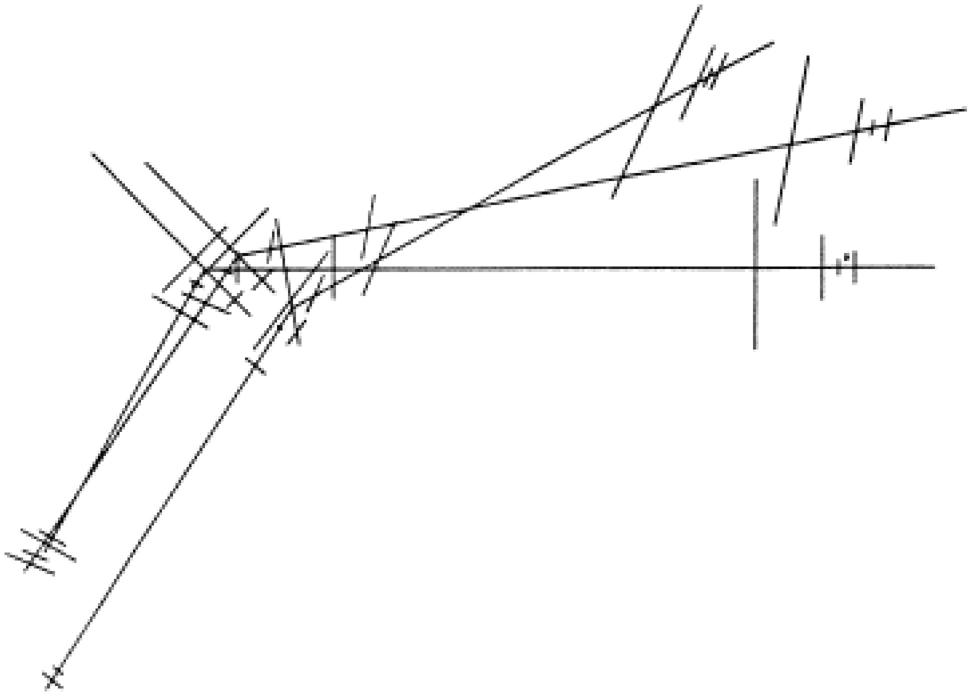


Figure 5. Transformed skeletal structures.

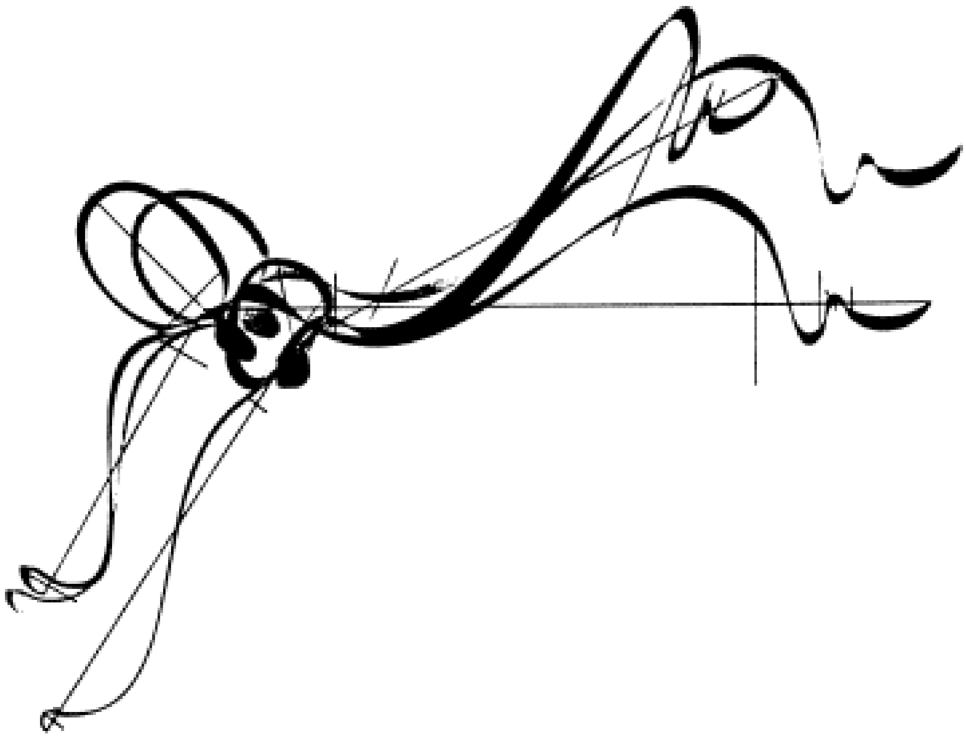


Figure 6. Transformed curves.

grid. A single dimension defines the entire grid. Having perceived this dimension we have comprehended the grid completely. We can disengage our interest.

This suggests an inversion of the conventional understanding of the regular grid: order becomes disorder. Visually, all the messages of the grid are equally likely to come through, and all are the same. The square grid is at once highly entropic and monotonous.

Thus, there are two conflicting readings of the regular grid: conceptually, it is singular and intentional, and therefore orderly; perceptually, it is chaotic and uninteresting, and therefore disorderly.

Intuitively, increased differences imply increased information content. When the dimensions of the grid are all equal the information content of the grid is minimal, and can be completely specified by a statement such as: *a square grid of unit size x.* Any alteration that introduces a difference increases the information content of the grid, and the statement needed to describe that grid, as an index of its information content, is correspondingly more complex. Consider the two images in Figure 1. One is a simple two-dimensional composition, the other a simple four-square grid. Initially, they appear unrelated. Figure 2 shows how the composition is derived from the square grid by a series of successive alterations, each of which introduces a single difference. (The differences examined in this example include dimension, angle, connection, weight, continuity, curvature, color, and so on.)

In a similar vein, Figure 3 shows two skeletal structures, each with exactly the same number of parts as the other. The spacing in the first is regular, while that of the second is varied (and thus contains more information). Two curves based on these structures are shown in Figure 4. The curve corresponding to the skeletal structure that has increased differences is more interesting, and is actually reminiscent of the famous Art Nouveau whiplash curve. In Figure 5 the differences of the second structure are repeated, varied, superimposed, and exaggerated, and the resulting design is shown in Figure 6.

Figure 7(a) shows a rectangle in a square field. In its original position, as a square in a 9-square grid, the rectangle communicates the least amount of information. Dimensional and positional differences are introduced to increase the information content of the composition. The result, after five steps, is shown in Figure 7(f).

This is the basic configuration I will use to arrive at two- and three-dimensional compositions, as is described below.

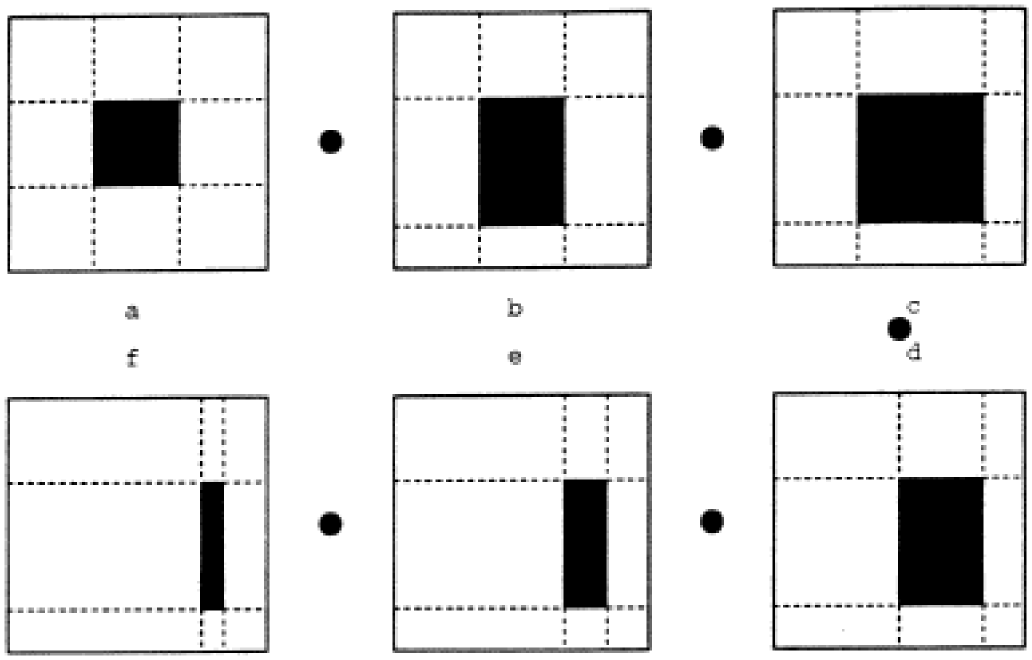


Figure 7. The initial rectangle in (a), after five steps, is transformed to (b), by increasing its information content.

3.2. Scope.

Initially, the problem investigated here is limited to the arrangement of rectangles within a square frame. The goal is to create visually interesting compositions by incrementally adjusting the locations and dimensions of these graphic elements in a way that increases visual information.

This approach is then extended to three dimensions and is used to organize parallelepipeds within the volume of a cube. The approach can be further extended to include the determination of surface attributes such as color and texture.

3.3. Representation.

Let us now consider how these ideas can be represented in a program.

A ***visual entity O*** is determined by a list of parameters (p1,p2,p3,...) that relate the entity to the visual field it occupies. For n different entities we can then define a matrix as follows:

O1	(	p1	p2	p3	…	pn	)
O2	(	p1	p2	p3	…	pn	)
O3	(	p1	p2	p3	…	pn	)
O4	(	p1	p2	p3	…	pn	)
…							
On	(	p1	p2	p3	…	pn	)

Each row in this matrix contains the parameters that define an entity, while each column contains all the similar parameters within a given composition. We define the ***internal information content*** of an entity to be the information content of an object with respect to itself, and thus to be the information content of its parameter-row. We define the ***external information content*** of an object along any one of its parameters (with respect to the other objects in the arrangement) to be the information content of the parameter column.

The parameters that describe an object can be grouped into any number of dimensions; within any dimension any number of parameters may be defined:

O1 ((p1,p2,p3),(p4,p5,p6),(p7,p8,p9,p10),...,(pn-m,...,pn-1,pn))

In the examples that follow I will be using:

O1 ((p1,p2,p3),(p4,p5,p6)) for the 2D case, and

O1 ((p1,p2,p3),(p4,p5,p6),(p7,p8,p9)) for the 3D case.

The exact use of these will be explained below.

3.4. Information content.

According to information theory, the entropy (H) of a message is given by the following expression:

$$H = p_1 Log_2(1/p_1) + p_2 Log_2(1/p_2) + p_3 Log_2(1/p_3) + ... + p_n Log_2(1/p_n)$$

where $p_1 \ldots p_n$ are the probabilities related to the occurrence of each message in a set of potential messages. The sum of $p_1 \ldots p_n$ is equal to one.

When all messages in a given set are equally likely to occur, that is, when $p_1 .. p_n$ are all equal, the value of the entropy is highest, $Log_2(n)$. When one message is certain to occur the entropy is zero. Visually, the first case corresponds to a regular grid, where the dominant visual message is that of the unit cell's dimension. The second case, on the other hand, introduces hierarchy, with one dimension becoming clearly dominant, and the other dimensions becoming subordinate in a progression of diminishing sizes.

We wish to define a measure whose value will increase as differences increase, and have a least value when differences are minimized. Therefore we define the visual information content of an object $O(p_1,p_2,p_3,\ldots p_n)$ to be :

$$VI_{internal} = c - H(p_1,p_2,p_3,\ldots p_n)$$

that is, a constant c minus the entropy of its parameters, c>H. This means that the value of the visual information, VI, will be highest when the parameters determining the object O are most different.

Similarly, a $VI_{external}$ is obtained by evaluating the entropy of all the parameters in a particular parameter column and subtracting it from c.

According to the information analogy we are pursuing the parameters $p_1 \ldots p_n$ are probabilities whose sum is equal to one. In our case $p_1 \ldots p_n$ are normalized in the following manner:

$$p_{iN} = p_i/(p_1+p_2+p_3\ldots+p_n)$$

that is, they are converted into fractions of the sum of the overall parameter row or column, as appropriate. The sum of these fractions is thus equal to one.

4. IMPLEMENTATION

4.1. Implementation.

In the two-dimensional case, this implementation defines the objects to be arranged in the composition as rectangles placed within a square field. Each object divides that field into nine distinct regions. The extents of these regions are controlled by six parameters, three along the x axis and three along the y axis. Thus each object is represented as $O(p_1, p_2, p_3, \ldots p_6)$, as shown below.

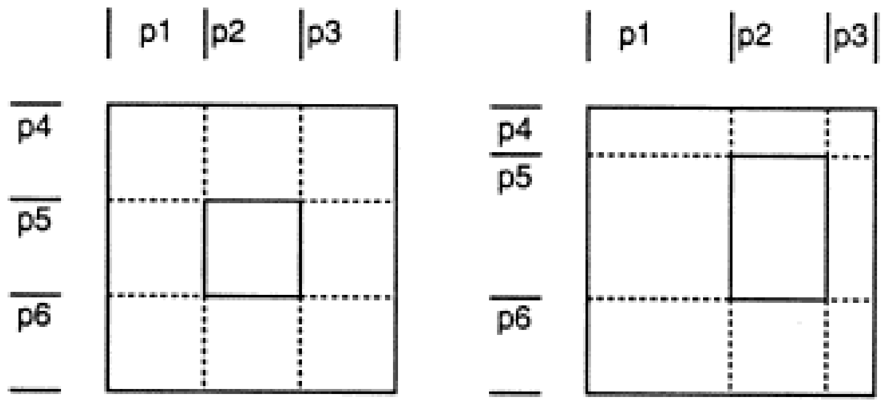


Figure 8. Representation.

The example on the left shows a parametric rectangle placed in the state of minimum information, all parameters being equal. The example on the right introduces differences to the values of the parameters, and thus has higher information content.

According to the description given above, the system can be seen as 1) trying to maximize the differences between the extents of the regions that define an object in each direction, as part of increasing its internal score, and 2) as trying to maximize the differences between the equivalent regions of different objects, as part of increasing its external score. The combined effect of these two operations is to maximize the variety of intervals between objects, and thus to increase the visual interest within an arrangement of objects.

Initially, all the objects are superimposed, i.e., they have identical parameters sets, and all the parameters are equal. For this particular implementation the field is defined as a 300 x 300 unit square, and all the parameters are set to an initial value of 100, thus defining a nine-square grid.

Mapping the parameters onto the square is an arbitrary choice. The algorithm does not associate the parameters with any particular mapping. In other words, the same parameters could have been used to describe an entirely different compositional configuration.

4.2. Genetic hill climbing: random mutation / cumulative change.

> Building is a biological process, not an aesthetic process.
>
> ***Hannes Meyer***

Rather than enumerating a very large space of possibilities explicitly, I have chosen an approach based on a genetic analogy. At any given time a fixed number of objects exists within the system. These objects are represented by their parameters, or gene strips, and the set of these gene strips constitutes the *current gene pool.* The composition literally evolves through *random mutation* and *cumulative change.* The gene strips are mutated as follows:

1) a particular gene strip is chosen randomly;

2) a particular parameter within the gene strip is chosen randomly;

3) a mutation value is chosen randomly, within a prespecified range ;

4) a particular operation is chosen randomly (at present simply addition, subtraction, or no change).

5) one of the parameter's neighbors is chosen randomly and adjusted so as to maintain the overall value of the gene strip. (Recall that the parameters function as probabilities in a message, and the sum of the probabilities must be 1.)

The change is then applied to the gene strip, and three scores are obtained. The first two scores have been described above; they are the internal and external information scores. The third score implements a form of Pareto optimality: it only considers if the internal and external results are better, worse or equal, in terms of information content, compared to those existing prior to the mutation, and gives an overall score according to the following interaction matrix:

	=	+	-
=	=	+	-
+	+	+	-
-	-	-	-

Based on this overall score the system then decides what to do with the gene: if the overall score is negative, the gene is left unaltered and another gene is chosen, as before. If the score is unchanged, then the same gene is kept, but another mutation value is chosen and applied, in an effort to force a stronger choice. Finally, if the overall score is positive, then the mutated gene replaces the previous gene. At this point a small increase in the overall information content of the design has been accomplished.

4.3. Additional dimensions.

Figure 9 shows examples of two-dimensional compositions generated by this process. Figure 10 (photographs) shows three dimensional compositions generated by extending the parameter set from six to nine parameters and interpreting those parameters as dimensions along a third axis. Thus the visual field within which the composition is generated becomes a cube. Additional parameters can be added, along additional dimensions. The resulting composition can then be generated within the corresponding hyperspace. Parameters chosen from any three dimensions can then produce a three-dimensional projection of the n-dimensional composition.

4.4. Extensions.

Several extensions are possible at this point:

1) to treat the rectangles or parallelepipeds as bounding envelopes, and to provide grammatical rules for substitution of parametric elements within them;

2) to consider both positive and negative elements, and the operations between elements (boolean and other);

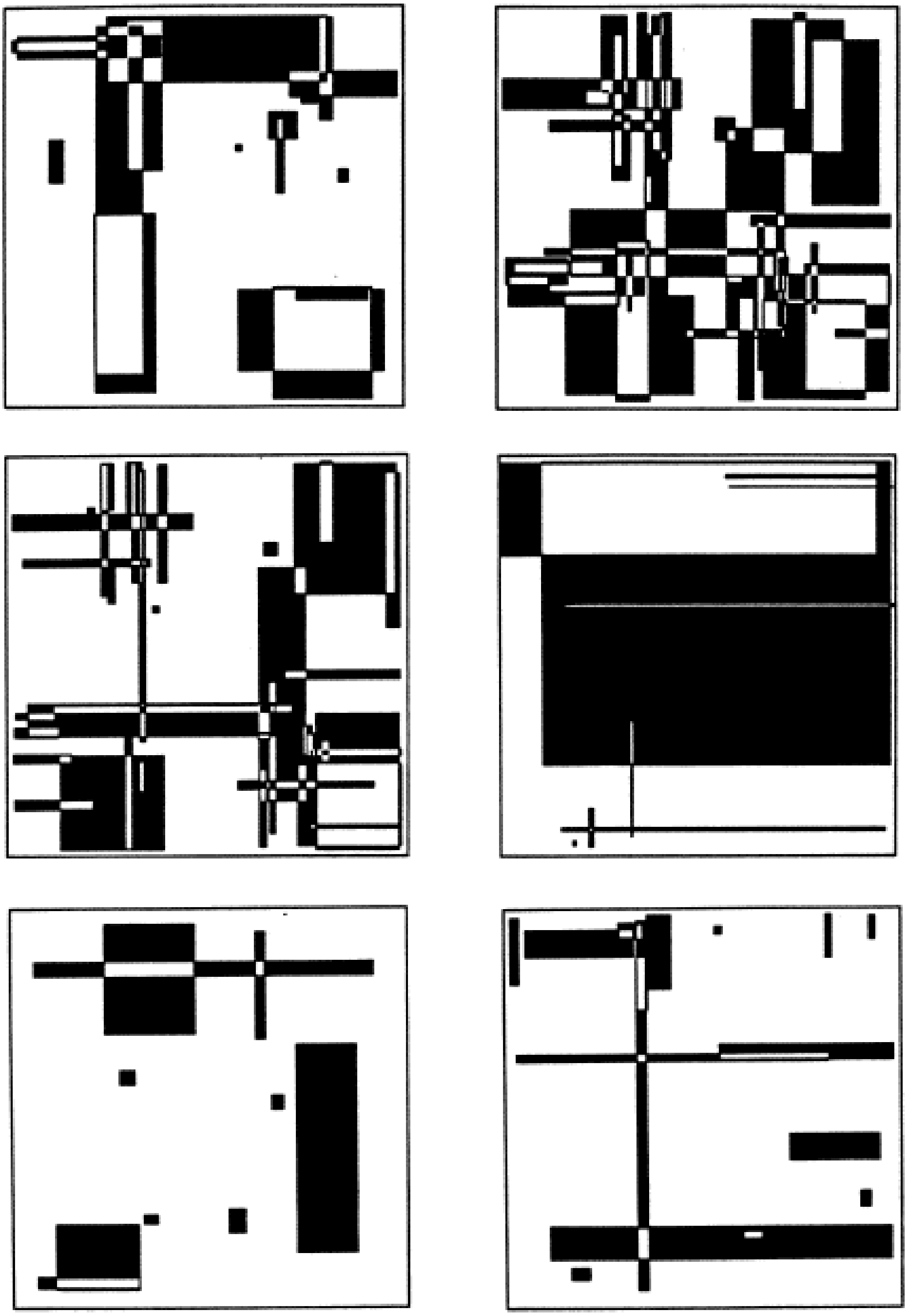
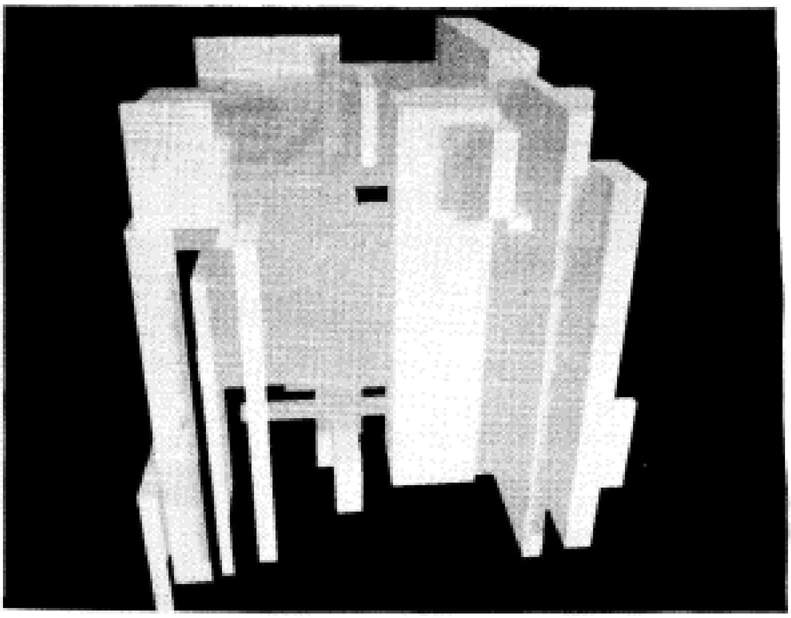


Figure 9. 2D compositions.



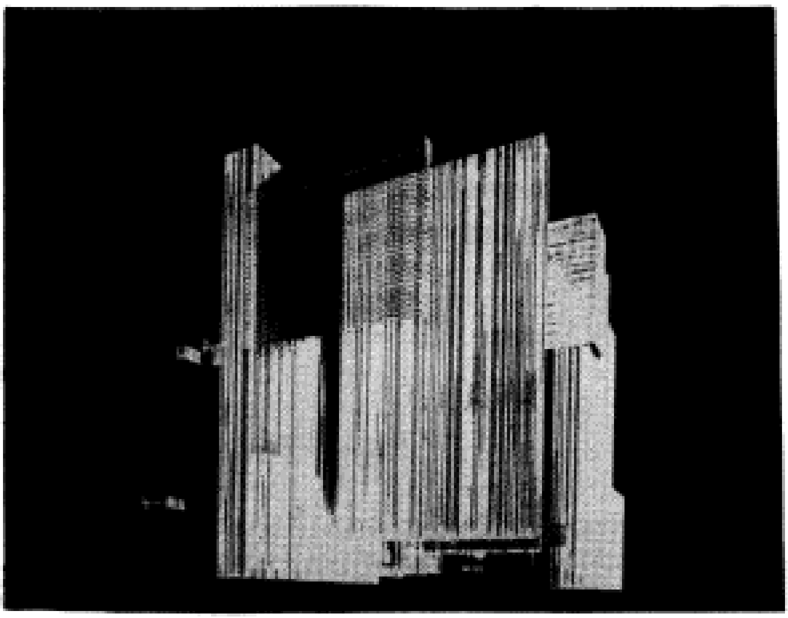


Figure 10. 3D compositions.

3) to allow the algorithm to operate recursively, so that once an element reaches a certain state it can subdivide, and its parts can be composed within it just as it was composed within its parent field;

4) to alter the initial configuration of the elements on the field;

5) to alter the shape of the field;

6) to investigate other functions relating information to visual interest.

Of course, this list is not exhaustive.

4.5. Mutation schedules.

Large mutation values give rise to bold changes in the configuration of the elements, while small values refine the configuration (literally producing compositions that appear, subjectively, more 'refined'). Intuitively, this corresponds to the initial broad strokes that set a design concept direction, followed by subsequent refinement of alternatives within a chosen scheme. It is thus possible for the behavior of the system to be controlled by using mutation schedules.

4.6. Constraints.

The generative mechanism can easily be extended to accommodate other kinds of constraints. In this implementation, for example, none of the regions was allowed to go to zero. This constraint is handled independently by the system, simply by adding another consideration to the scoring routines. The mutation and information mechanisms remain unaltered. If they generate an alternative that is not satisfactory, the scoring mechanisms will cause that gene to be ignored. In this case the operation of the system seeks to attain the maximum information within the set of constraints.

The actual form additional constraints would take varies. Any function of the parameters, either within a single gene strip, or among different ones, can be used as a constraint. Finally, the information scores themselves can be used to control the resulting composition. As we mentioned above, the information function used can itself be altered or refined in various ways.

5. ASSESMENT

5.1. Assesment.

In my own estimation, the designs produced by the system are indeed visually interesting, and bear a striking resemblance in character to the De Stijl and Constructivist aeshetic. Naturally, readers will have to reach their own conclusions. Clear differences are visible between arrangements that have evolved over 50-100 cycles, 400-500 cycles, and 1000, 4000, and 10000 cycles. The approach is robust, and can easily be extended to additional two or three dimensional parameters that control orthogonality, angle, texture, color, as we have seen.

The inversion of the normal interpretation of information theory seems to have been fruitful. It is unlikely, however, that any single measure can capture all the richness and variability of design. Additional research is needed to determine how to tailor the central formula to visual phenomena.

5.2. Application.

The examples shown in the figures above are extremely simple compositional forms. The actual application of this algorithm would be to give actual dimensions to more complex parametrically defined architectural compositions. At the one extreme, the algorithm could be used to size and place the actual space defining elements in a composition reminiscent to de Stijl; at the other it could proportion a classical composition, as well as all the elements that belonged to it. Extensions to the algorithm can allow it to operate on groups of elements within elements (as in the context of TopDown, for example).

5.3. Significance.

Knowledge about the characteristics of coherent form and the ways of producing it is inherently interesting because it elucidates important aspects of the ways in which we perceive and process information. If it can be demonstrated that we perceive coherence directly, as an aggregate measure of the global properties of an artifact, then our aesthetic sense will be shown to be a crucial and complex part of our intelligence. In the words of Aquinas: *beauty consists in due proportion, for the* *senses delight in things duly proportioned ... because the sense too is a sort of reason, as is every cognitive power.*

The implications of this approach have broader repercussions as well, especially for the actual quality of the built environment, and for architectural practice, education and research. It is clear that while our buildings may be stronger, more efficient, and less expensive than those of the past, architecturally they do not make good environments. At least in part, this is due to the economics of architectural practice. It is prohibitively costly today to spend as much time in the detailed consideration of an architectural composition as we see invested in the buildings of the past. This is partly because these tasks have traditionally been performed in extremely labor-intensive ways. Any tool that allows architects to control the design at a higher compositional level will have profound effects on the image and quality of our cities.

Computer aided design has always promised to allow architects more time for design, but most indications show that this is not true. Recent reviews have stated that even as production tools computers are not faster than draftsmen, though they do help produce more consistent and higher quality drawings. This is because present day computer systems are neutral in terms of design. They neither contain compositional knowledge, nor allow, much less encourage, compositional experimentation. The most important limitation is that they do not allow global manipulation of the relations embodied in a design, and thus actually hinder the development of coherent form, which by definition concerns both global and local formal issues. The kind of tool proposed here, on the other hand is not design-neutral. It specifically aimed at increasing architects' compositional control over a design.

Architectural education is centered on the design studio. Specific compositional knowledge can also find direct synthetic application in the studio, and analytic application in architectural history and theory courses. Tools developed with compositional ideas in mind can allow education to emphasize the intrinsic architectural concepts rather than the peripheral skills that presently make architectural education (and practice) so labor intensive. Since we have lost much of the necessary compositional knowledge through disuse, it will help restore and extend that knowledge into architectural education and practice.

Finally, new directions can open for architectural research, and for research in computer aided design, having to do with this historically neglected but nonetheless central aspect of architectural design.

5.4. Conclusion.

This short study has informally demonstrated one way in which computers can be used to explore fundamental compositional questions in a computational setting. By addressing the architectural issue of proportion directly, not by attempting to reproduce a historical example, but by inventing a compositional process, insight has been gained not only concerning the ***how*** of proportion, but also the ***why***.

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