Towards a formal logic of design rationalization

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Certain extensions to standard predicate logic are proposed and used as a framework for critical logical study of patterns of inference in design reasoning. It is shown that within this framework a modal logic of design rationalization (suggested by an empirical study reported earlier) can be formally defined in terms of quantification over a universe of discourse of ‘relevant points of view’. Five basic principles of the extended predicate logic are listed, on the basis of which the validity of ten modal patterns of inference encountered in design rationalization is tested. The basic idea of reducing modality to quantification is traced back to the philosophy of von Wright, and the approach is compared to that of related work on logic in design. © 1997 Elsevier Science Ltd.

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In a recent case study of reasoning involved in justification of design decisions, several patterns of inference were identified. (The case was about residential site planning in a Danish suburban area named Spangsbjerg; hence we shall refer to it as ‘the Spangsbjerg case’.) Some inferences followed patterns of ordinary propositional or predicate logic; others appeared to rely on a certain modal logic. In modal logics operators are introduced to modify the meaning of ordinary declarative propositions. For example, in deontic modal logic (the logic in which modes, or modalities, of obligation and permission are studied) such operators as ‘Obligatory’ and ‘Permitted’ are used, which take a declarative proposition $P$ (about an act) as their operand and change its meaning into ‘it is obligatory that $P$’ and ‘it is permitted that $P$’, respectively. In so-called alethic modal logic, modalities of possible and necessary truth are studied; they resemble those of permission and obligation.

In 1951 von Wright pointed out ‘essential similarities’ between universal quantification (‘for all . . .’) and the modalities of necessity and of obli-
gation, and also between existential quantification ('for some . . .', 'there exists . . .') and the modalities of possibility and permission (op. cit. p 2).

In the present paper we explore this insight of von Wright’s in the context of a logic of design rationalization, taking the analogy a step further, so as to define modal operators in terms of quantification. This enables us to draw upon known results of standard predicate logic. As will be shown, such reduction of modality to quantification not only suggests a formalization of an emerging ‘logic of design rationalization’ within a suitably extended predicate logic, but also gives us a touchstone on which to test the validity of such modal inferences as were unveiled by the Spangshøjbjerg case study.

Admittedly, studying design rationalization through the looking glass of formal logic leads to a fairly narrow view of the subject, leaving out many aspects—say, of a sociological, political or economic nature—that are no doubt important in understanding design reasoning and discourse. However, what is lost in breadth may be gained in depth. I believe, and intend to demonstrate in this paper, that formal logical analysis of design rationalization has something valuable to offer, notably by showing what kinds of arguments are, under certain assumptions, ‘safe’ (in the sense of resting on valid patterns of inference), and what kinds are not. Apart from the theoretical interest such a criterion of validity may have, awareness of it may conceivably be of use in practical design. It is possible, furthermore, that a formal ‘logic of design rationalization’ such as the one we shall suggest and explore may find application in computerized decision support systems.

In sections 1 and 2, respectively, elements of formal logic, and the basic idea of defining modality through quantification, are briefly presented. Particular modalities of design rationalization logic (with an associated notion of rationality) are informally introduced in section 3. These modalities are formally underpinned through extensions of predicate logic as discussed in section 4, and in section 5 the inference patterns encountered in the Spangshøjbjerg case are tested for validity by means of the logical machinery thus introduced. A review of related work is given in section 6.

1 Elements of symbolic logic and formal notation
As indicated above, the present study is one of logic, albeit with the clear purpose of contributing to design research. To keep the line of reasoning uncluttered by lengthy explanations of technicalities of logic (or short but imprecise ones), some familiarity with elementary symbolic logic will be assumed on the part of the reader. Readers to whom this presents a prob-
lem, are referred to texts on the subject; e.g. Kalish and Montague\textsuperscript{6} which covers a wide range of logic and takes a pragmatic though rigorous approach to logic, Hilbert and Ackermann\textsuperscript{7} which is more theoretical in its approach, and Quine\textsuperscript{8} which is somewhat narrower in its selection of material for detailed coverage but gentle, yet precise in its presentation, and with reasonable emphasis on logic as a practical tool (apart from presenting some original views on elementary theory of quantification and set theory). Unfortunately, and despite the considerable age and maturity of logic as a field of study, terminology and notation vary within wide bounds, as is illustrated by these books. Let us therefore briefly characterize the basic formal notation we shall adopt in this paper.

Rather than considering specific propositions, we are interested only in their forms, rendered as formulae in the manner of symbolic logic. We shall use italic capital ‘proposition letters’ ‘\(P\)’, ‘\(Q\)’, etc. as formulae for propositions whose inner structure is left unanalysed.

Furthermore, formulae representing open sentences (such as ‘\(x\) has a driver’s licence’) with one variable consist of a capital ‘predicate letter’ followed by the variable (e.g. ‘\(Dx\)’). A predicate such as D is true of all, some, or none of the objects in a given ‘universe of discourse’ and false of the others; it can be thought of as a function, taking an object (say, a person) from the universe as argument, and yielding a truth-value as result—just as an open sentence becomes a true or false proposition when its variable assumes a particular value.

Prefixing a formula with ‘\(\neg\)’ yields a new formula representing the negation of the proposition or open sentence that the original formula stood for. Furthermore a formula can be constructed from two simpler ones by connecting them with one of the connectives ‘\(\land\)’ (‘and’), ‘\(\lor\)’ (‘or’), ‘\(\rightarrow\)’ (‘if–then’), and ‘\(\leftrightarrow\)’ (‘if and only if’). Where these rules are applied to a formula already composed from shorter ones, the composite formula must be enclosed by parentheses to avoid ambiguity. For example, in ‘\(\neg (P \land \neg Q)\)’ the first negation sign is applied to the entire composite formula ‘\((P \land \neg Q)\)’, but in ‘\(\neg P \land \neg Q\)’ only to ‘\(P\)’. Furthermore, ‘\(P \land Q \lor R\)’ is not allowed, but ‘\((P \land Q) \lor R\)’ and ‘\(P \land (Q \lor R)\)’ are.

Any formula can be enclosed by parentheses, e.g. to enhance readability where it is used as part of a larger formula. A formula \textit{enclosed by parentheses} (and usually containing a predicate letter) can be preceded by an existential or universal quantifier-phrase: ‘\(\exists x\)’ and ‘\(\forall x\)’, respectively (or with other variables than ‘\(x\)’). The result is again a new formula where the parentheses mentioned delimit the scope of the quantifier. Examples: if
‘Dx’ stands for the open sentence ‘x has a driver’s licence’, ‘∃x (Dx)’ would stand for the proposition ‘someone has a driver’s licence’ (‘there exists an x such that x has a driver’s licence’), and ‘∀x (Dx)’ for ‘everybody has a driver’s licence’ (‘for all x, x has a driver’s licence’). Furthermore, ‘∀x (Fx ∧ Gx)’ and ‘∀x (Fx) ∧ Gx’ are admitted as formulae (different ones), but ‘∀x Fx ∧ Gx’ is not.

Formulae constructed according to these formation rules (applied a finite number of times) are called well-formed. Their meanings are assumed to be well-known; and so are conventions for obtaining instances of formulae by substitution. A few items of special notation will be introduced below as the need arises. Extensions to so-called polyadic predicates (ones of open sentences with multiple variables), and the corresponding extensions to quantification, will not be needed for our purposes.

Interpreting a formula means inserting propositions for proposition letters and predicates for predicate letters, i.e. determining which objects in the universe of discourse the predicates are true of. If the formula becomes true under all interpretations (in all nonempty universes of discourse), it is said to be valid, otherwise it is invalid.

2 The ‘reductionist’ approach to modality

In his discussion of the basic system ‘M_i’ of alethic modalities of possible and necessary truth, von Wright remarks (p 19) that the system presents a close analogy to […] that part of the so-called predicate calculus which contains only one-place predicates and no sentence-variables and no free individual variables’. The analogy is not surprising, von Wright says, for ‘popularly speaking, the possible is that which is true under some circumstances’ and ‘the necessary that which is true under all circumstances’ (which shows the affinity to existential and universal quantification, respectively). These ‘popular’ remarks not only highlight an interesting analogy, but suggest a way of defining possible and necessary truth in terms of quantification over a universe of discourse of ‘circumstances’, thereby reducing modality to familiar notions of quantification.

Indeed, design rationalization logic is our subject of study, but as regards modalities, von Wright’s basic deontic and alethic logics seem easier to cope with for a start. As a prolegomenon to the study of design rationalization logic, let us therefore briefly consider how the ‘reductionist’ approach to modality might work in the context of his alethic system ‘M_i’, since that is the system which prompted the suggestive remarks quoted above.

How can we formalize the notions of a proposition being true under all circumstances, and under some circumstance(s)? We assume a nonempty
universe of discourse is at hand, containing circumstances and nothing else. That a given proposition $P$ is true under all (some) circumstances can be tentatively rendered as ‘$\forall c \ (P$ is true under $c)$’ and ‘$\exists c \ (P$ is true under $c)$’, respectively. Here ‘$P$ is true under $c$’ is an open sentence with one variable ‘$c$’ standing for a circumstance. Unlike, say, ‘$x$ has a driver’s licence’, the open sentence in question is constructed from a proposition $P$. Rather than representing such open sentences simply by predicate letters, we therefore introduce an operator, to be called the predication operator, that takes a proposition as its operand and yields a one-place predicate as result: given a proposition $P$, ‘$[P]x$’ stands for the open sentence ‘$P$ is true under (the circumstance) $x$’. We reserve square brackets for this purpose. Constructs of the form ‘$[P]$’ will play the role of predicate letters.

In more technical terms, the predication operator could be introduced into the framework of predicate logic by adding a formation rule saying that if $\Pi$ is a well-formed propositional formula (one composed solely from proposition letters, the negation sign, the connectives and parentheses), and $\zeta$ is an individual variable (one ranging over the universe of discourse), then ‘$[\Pi]\zeta$’ is a well-formed formula.

Furthermore, the use of predicates generated by the predication operator could be captured, in an axiomatic formulation of predicate logic, or one based on natural deduction, by an inference rule: given a formula $F_1$ that contains a predicate letter $\Theta$, we can infer a formula $F_2$ which is like $F_1$ except that all occurrences of $\Theta$ have been replaced in $F_2$ by ‘$[\Pi]$’, where $\Pi$ is a well-formed propositional formula. Example: from ‘$\forall x \ (Fx) \leftrightarrow \exists y \ (\neg Fy)$’ we may infer ‘$\forall x \ ([P]x) \leftrightarrow \exists y \ (\neg [P]y)$’.

The state (state-of-affairs) of the world we are talking about obviously can influence the truth or falsehood of our propositions (otherwise there would be little point in talking). For instance, the proposition ‘I have a driver’s licence’ happens to be false in a state of the world where it is uttered by me, but it is likely to be true if uttered by the reader; ‘Germany is at war’ is false today but was true in 1942, and so on. We can think of ‘circumstances’ as states of the world, and of the universe of discourse as any selection of such states; e.g. the ones we are in a position to bring about through our actions. (The notion of ‘circumstance’ as developed here is related to that of a ‘possible world’ used in so-called possible-world semantics of modal logic; unlike the latter, however, we need no ‘accessibility relation’ on the universe of discourse.)

Obviously we should not be allowed to claim the truth of a proposition and its negation under the same circumstances. On the other hand, given
the circumstances, either the proposition or its negation should be true (this being a variant of the familiar ‘law of the excluded middle’). Hence the following principle of predication should be respected. It states that under a given circumstance a proposition or its negation holds, but not both (note 1)

\[ \neg [P]z \iff [\neg P]z \] (2.1)

Thus \[ [P]x \land [\neg P]x \] is inconsistent because by 2.1 it is equivalent to the contradiction \[ [P]x \land \neg [P]x \].

The modal operators of possibility and necessity can be defined as follows with respect to a given universe (in analogy to what can be done in a framework of possible-worlds semantics, c.f. Gochet et al. p 38)

‘Necessary \( P \)’ abbreviates ‘\( \forall x([P]x) \)’

\[ \text{[Under all circumstances, } P \text{ holds].} \] (2.2)

‘Possible \( P \)’ abbreviates ‘\( \exists x([P]x) \)’

\[ \text{[Under some circumstance(s), } P \text{ holds].} \] (2.3)

Von Wright defines ‘Necessary’ as an abbreviation of ‘\( \neg \) Possible \( \neg \)’ (Reference 3, p 9). Definitions 2.2 and 2.3 replace von Wright’s definition, because now the corresponding biconditional is provable within a framework of predicate logic

Necessary \( P \) \iff \neg Possible \( \neg P \) (2.4)

Proof (note 2)

A: \( \forall x ([P]x) \iff \exists y (\neg [P]y) \)

follows from a theorem of standard predicate logic. By 2.1, A is equivalent to

B: \( \forall x ([P]x) \iff \exists y (\neg [P]y) \)

and by definitions 2.2 and 2.3, B is equivalent to 2.4

As in von Wright’s system necessity implies possibility:

Necessary \( P \) \implies Possible \( P \) (2.5)

Proof

A: \( \forall x ([P]x) \implies \exists y ([P]y) \)

follows from a theorem of predicate logic (note 3). 2.5 follows from A by 2.2 and 2.3

Von Wright’s axiomatic ‘special principle of possibility’ to the effect that ‘Any given proposition is itself possible or has a negation that is possible’ (op. cit. p 13) is also rendered provable. In symbols it reads
\[ \text{Possible } P \lor \text{ Possible } \neg P \]  
\text{\textit{Proof}}

A: \neg \text{Necessary } P \rightarrow \text{Possible } \neg P
is equivalent to 2.4 proved above.
B: \text{Possible } P \lor \neg \text{Necessary } P
is equivalent to 2.5. According to A, B is equivalent to 2.6

The above examples will suffice to show how modalities can be studied in the well-known framework of standard predicate logic. When it comes to a study of 'the logic of design rationalization' which is less well understood than the various logics of aletic and deontic modalities, I believe a similar reductionist approach is a good strategy, at least for initial explorations.

3 Modalities of preferability in design reasoning

For logical analysis of reasoning involved in rationalization of design decisions, I have proposed three modal operators that modify the meaning of a statement S (or proposition, see note 4); so far the operators have only been informally defined:\footnote{3}

\begin{align*}
\text{Required } S \text{ ('it is required that } S') & \quad \text{(3.1)} \\
S & \text{ is preferable to } \neg S \text{ (note 5) from \textit{all} relevant points of view (of which there is at least one)} \\
& \text{[hence there is no relevant point of view from which } \neg S \text{ is preferred to } S] \\
\text{Good } S \text{ ('it is good that } S') & \quad \text{(3.2)} \\
S & \text{ is preferable to } \neg S \text{ from \textit{most} relevant points of view (of which there is at least one)} \\
& \text{[but there may also be some relevant point(s) of view from which } \neg S \text{ is preferred to } S] \\
\text{Desirable } S \text{ ('it is desirable that } S') & \quad \text{(3.3)} \\
S & \text{ is preferable to } \neg S \text{ from \textit{some} relevant point(s) of view} \\
& \text{[but } \neg S \text{ may also be preferable to } S \text{ from some other relevant point(s) of view]}
\end{align*}

The case of S being 'preferable' to \neg S from no relevant points of view is covered by simple negation: '\neg \text{Desirable } S'.

As suggested by these informal definitions the modal operators can be defined in 'reductionist' terms of quantification over a universe of 'relevant points of view'. The first and the last operator will be defined using universal and existential quantification, respectively. They are intuitively anal-
ogous to ‘Obligatory’ and ‘Permitted’ of deontic logic, respectively, and also to ‘Necessary’ and ‘Possible’ of alethic logic. The ‘Good’ operator, however, has no analogue in the other logical systems; nor is the ‘most’ quantifier treated by standard predicate logic.

The intuition behind the ‘Good’ operator is that in practical design decision-making, some design options are chosen which are not beyond discussion, but which seem, after all, to have more to speak for them than to speak against them. The notion of ‘goodness’ of a design option was introduced because the reasoning testified by the empirical data of the Spangsbjerg case study (a so-called replication protocol\(^9\)) seemed to require, for proper analysis, something ‘in between’ the two extremes of ‘requiredness’ and ‘desirability’.

Furthermore, the three modalities of preferability together allow us to define the *rationality* of a (design) decision to bring about the state-of-affairs described by a statement \( S \). Such a decision is understood here as the *actual preference* of \( S \) over \( \neg S \) (not just as seeing \( S \) as preferable over \( \neg S \)).

A decision to bring about \( S \) is *rational* if and only if the decision-maker(s) know(s) or believe(s) that one of the following cases applies\(^1\,^2\)

\[
\begin{align*}
\text{R1} & \quad \text{Required } S \\
\text{R2} & \quad \text{Good } S \\
\text{R3} & \quad (\text{Desirable } S) \land (\neg \text{Good } \neg S)
\end{align*}
\]

The ‘logic of design rationalization’ we are pursuing is a logic suitable to govern arguments in support of the rationality of design decisions.

4 *Extending predicate logic*

As a step toward such a ‘logic of design rationalization’, we shall define formally the three modal operators introduced above, in analogy to the reductionist approach to alethic logic proposed in section 2. As in section 2 we shall need a ‘predication operator’ and certain ‘principles of predication’, but in addition the notion of ‘goodness’ introduced above requires the ‘most’ quantifier and certain principles ruling it.

The principles of predication and of most-quantification will be used in section 5 in proving theorems corresponding to the patterns of inference encountered in the Spangsbjerg case (note 6). Our aim in the present section is to outline in what ways standard (monadic) predicate logic could
be extended so as to provide a logical apparatus with which we can account for the findings of the Spangsbjerø case study (and which accommodates the ‘logic of design rationalization’ suggested by them). However, a treatment of the extension itself in all technical details (such as axiomatization or development of a decision procedure) is beyond the scope of the present study. We shall limit the present discussion to informal intentional considerations about the validity of the principles suggested.

4.1 The predication operator

We assume a nonempty universe of discourse containing relevant points of view and nothing else. Only one design, in a particular state of evolution, is considered. Let us refer to it simply as ‘the design’. We may think of a relevant point of view, \( x \), as a set \( \{ x_d, x_e \} \) where \( x_d \) is a decision-maker engaged in rationalizing the design, and \( x_e \) is a mental state of \( x_d \) when so engaged.

We introduce a ‘predication operator’ as described in section 2, but one with a different meaning: given a statement (proposition) \( S \), ‘\([S]x\)’ stands for the open sentence ‘\( S \) is preferable to \( \neg S \) from the relevant point of view \( x \)’—or, more precisely, ‘\( x_d \) sees \( S \) as preferable to \( \neg S \) when in the mental state \( x_e \)’.

Note that changing the point of view by changing the decision-maker may, of course, lead to another judgement, but so may a change of the point of view that retains the decision-maker but invokes another mental state. Even in a context where only one decision-maker is involved, any number of relevant points of view may satisfy ‘\([S]x\)’, and any number may satisfy ‘\([\neg S]x\)’.

Note further, that this understanding of the notion of ‘relevant point of view’ and the predication operator does not in itself presuppose or imply any kind of rationality of the decision-making. Rather, it admits the possibility of a decision-maker being torn between opposite preferences, which may be of a purely emotional or intuitive nature. But even so, it makes sense to speak of rationality of decisions, as defined at the end of section 3.

It must be admitted that we have explained one complex concept (‘relevant point of view’) in terms of other complex ones (‘mental state’ of a ‘decision-maker’). It is not our ambition, however, to reduce complex concepts to simple, ‘primitive’ ones. The purpose of our explanation is merely to provide a sufficient intuitive understanding of the universe of discourse to render the proposed formalization plausible.
As was the case in alethic logic, certain principles of predication apply. Thus formula 2.1 (repeated below as 4.1) in the present context states that from a given relevant point of view, a (any) statement is preferable to its negation, or vice versa, but not both.

\[ \neg[S]z \leftrightarrow [\neg S]z \]  \hspace{1cm} (4.1)

To discuss if this should be accepted, let us consider the two constituent conditionals separately. The right-to-left conditional of 4.1 is equivalent to

\[ \neg([S]z \land [\neg S]z) \]  \hspace{1cm} (principle of nonambivalence)  \hspace{1cm} (4.1')

which states the obviously reasonable assumption that one cannot find a statement preferable to its negation and vice versa from the same relevant point of view. The left-to-right conditional of 4.1 is equivalent to

\[ [S]z \lor [\neg S]z \]  \hspace{1cm} (principle of nonindifference)  \hspace{1cm} (4.1'')

which may seem too strong prima facie, with the present reading of the predication operator. Given a statement S which is irrelevant to the design, we might argue that neither S nor \( \neg S \) is preferable to the other from any relevant point of view. For example, a decision-maker engaged in rationalizing a residential development design in Amsterdam, no matter in what mental state, could obviously be indifferent about, say, a statement describing an interior design for the Sydney Opera House. However, this argument relies on the assumption that the statements under consideration include ones that are irrelevant to the design (as well as relevant ones). Nothing is gained by this assumption, from the point of view of formalizing the logic of design rationalization. On the contrary, we can safely assume statements to be relevant as regards the design and its rationalization.

The principle of nonindifference, then, is an assumption of nonindifference on the part of the decision-maker with respect to a design-relevant statement S, when considered from one particular relevant point of view. This assumption does not, however, preclude the possibility of a decision-maker being unable to make a decision to prefer S to \( \neg S \) or vice versa, after considering S from all (or several) relevant points of view. Such a state of indecision might arise, for example, if all the decision-maker could say was 'from some relevant point of view I prefer S to \( \neg S \),' and from some other relevant point of view I prefer \( \neg S \) to S'. In the light of this I feel that the principle of nonindifference is defensible as an axiomatic assumption about the 'relevance' of each individual relevant point of view.

\[ (P \rightarrow Q) \rightarrow \forall z ([P]z \rightarrow [Q]z) \]  \hspace{1cm} (4.2)
The next principle of predication (4.2) amounts to saying that preferability of a statement over its negation (from a given relevant point of view) extends to the consequences of that statement. To see the plausibility of this, assume that, contrary to 4.2, there was some value of \( z \) (let us call it \( \sigma \)) such that \([P]z \) but \( [Q]\sigma \), even though \( P \rightarrow Q \). A contradiction would then arise, for since \( P \rightarrow Q \), we cannot bring about \( P \) without bringing about \( Q \); and this is reason enough to prefer \( Q \) to \( \neg Q \) from the point of view of \( \sigma \), that is, \([Q]\sigma \). Note that from other points of view from which we see \( P \) in a less favourable light than we do from \( \sigma \), we are still free to prefer \( \neg Q \) to \( Q \). Note also that the above contradiction was not established by appeal to a principle of logic, but rather by appeal to our intuitions about preferability and points of view. \( 4.2 \) is an axiomatic assumption on the nature of relevant points of view; not a law of logic.

\[
([P]z \land [Q]z) \rightarrow [P \land Q]z
\]  

(4.3)

The purport of 4.3 is that if two statements are preferred to their negation from the same relevant point of view, then so is their conjunction. This is not obviously plausible. Consider such cases where \( P \) and \( Q \) represent two possible changes of the current state of evolution of the design, each of which has something to speak for it, but which together would be an unhappy move. For example, in designing a two-storey house for a pianist we might consider \( P \): ‘there is a music room upstairs’, and \( Q \): ‘there is a music room downstairs’. A music room on either storey would be fine, but having two would be a waste of space. Another example is this: \( P \): ‘the house has a built-in garage’ and \( Q \): ‘the house does not have a built-in garage’ (where, of course, \( P \) and \( Q \) are logically incompatible). Finally, allowing for considerable but trivial extension of the notion of ‘design’, let us consider a particularly instructive third example: a young man developing amorous relations with two girls, Joan and Jennifer, is making plans for next Friday and contemplates \( P \): ‘I spend Friday night with Joan’ versus \( Q \): ‘I spend Friday night with Jennifer’. Whereas Joan and Jennifer are two nice and sensible girls they are not beyond human feelings of jealousy, so bringing them together might result in a memorable but not altogether pleasant experience (note 7).

What these examples tell us is not to accept such absurd situations as would be represented by the consequent of 4.3 in each example, but rather that two practically or logically incompatible statements cannot both be entertained (considered preferable to their negations) from the same relevant point of view. Thus 4.3 is a further constraint on what may count as a relevant point of view.

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From one relevant point of view we may entertain the idea of a music room on the first floor, and from another the idea of a music room on the second floor, without being compelled in any way to entertain both ideas together. Again from one relevant point of view (say, one of convenience), we may entertain the idea of having the garage built into the house; from another (one of economy, perhaps) we may entertain the idea of not having it built into the house. Finally, the romantic youth may entertain his two options from different relevant points of view (say, those of Joan- and Jennifer-loveliness, respectively) without thinking of a date with both girls simultaneously.

4.2 The 'most' quantifier

We cannot, of course, reduce the 'goodness' of section 3 to standard notions of quantification; but that, I believe, merely proves the complexity of design thinking, rather than flaws in our reductionist approach. We need the 'most' quantifier; and we introduce for it the symbol 'w' (an inverted 'M'—analogous to '∀' which is an inverted 'A' for 'all', and '∃' which is an inverted 'E' for 'exists'). The quantifier phrase 'wx' will be read 'for most x . . . ' or 'for more than half the x . . . ' (and it will be followed by a parenthesized formula, just like the standard quantifier phrases). Such nonstandard quantification has been informally discussed by Johnson-Laird\(^{10}\) (p 137). Only two principles of most-quantification will be assumed.

\[
(\forall t (Ft \rightarrow Gr) \land wx (Fu)) \rightarrow wx (Gu)
\]  

(4.4)

This can be read 'if everything that is F is also G, and most things are F, then most things are G'. The validity is intuitively obvious. The same holds for this principle:

\[
(\forall t (Ft) \land wx (Gu)) \rightarrow wx (Fv \land Gv)
\]  

(4.5)

That is, 'if everything is F, and most things are G, then most things are both F and G'.

4.3 Defining the modalities of preferability

Given a predicate logic thus extended, the introduction of modal operators along the lines of section 2 is straightforward.

\[\text{Required } S \text{ abbreviates } '\forall x([S]x)' \quad \text{[compare 3.1]} \]  

(4.6)

\[\text{Good } S \text{ abbreviates } 'wx([S]x)' \quad \text{[compare 3.2]} \]  

(4.7)

\[\text{Desirable } S \text{ abbreviates } '\exists x([S]x)' \quad \text{[compare 3.3]} \]  

(4.8)
The inference patterns of the Spangsbjerg case

In this section we shall consider the validity of the ten conditional formulae that correspond to patterns of inference encountered in the Spangsbjerg case\(^1\)\(^2\).

The two abductive patterns, ‘D-modal abduction’ and ‘RD-modal abduction’ can be shown to be invalid, like ordinary (‘plain’) abduction (note 8).

\[ ((M \rightarrow E) \land \text{Desirable } E) \rightarrow \text{Desirable } M' \text{ (‘D-modal abduction’) is invalid.} \]  

(5.1)

**Proof**

Let \( \Sigma \) be some particular statement which is both true and required, such that

A: \( \Sigma \land \forall x \left( [\Sigma]_x \right) \)

If D-modal abduction were valid, then all its interpretations would be true. In particular,

B: \( \left( \vdash \Sigma \rightarrow \Sigma \right) \land \text{Desirable } \Sigma \rightarrow \text{Desirable } \Gamma \)

would be true (see note 9). From B we derive K as follows (where no justification is added, each formula follows from the preceding one by some principle of standard logic)

C: \( \left( \vdash \Sigma \rightarrow \Sigma \right) \land \text{Desirable } \Sigma \rightarrow \text{Desirable } \Gamma \)

D: \( \left( \vdash \Sigma \rightarrow \Sigma \right) \rightarrow \text{Desirable } \Gamma \)

E: \( \vdash \Sigma \rightarrow \left( \text{Desirable } \Sigma \rightarrow \text{Desirable } \Gamma \right) \)

F: \( \text{Desirable } \Sigma \rightarrow \text{Desirable } \Gamma \) (from E and the first link of A)

G: \( \exists y \left( [\Sigma]_y \right) \rightarrow \exists z \left( [\Gamma]_z \right) \)

(5.2)

(5.2)

H: \( \exists y \left( [\Sigma]_y \right) \rightarrow \exists z \left( \vdash \left[ \Gamma \right]_z \right) \)

(5.1)

I: \( \forall y \left( \vdash \left[ \Sigma \right]_y \right) \lor \exists z \left( \vdash \left[ \Gamma \right]_z \right) \)

J: \( \exists z \left( \vdash \left[ \Sigma \right]_z \right) \)

(5.1)

K: \( \vdash \forall z \left( [\Sigma]_z \right) \)

(5.1)

but K is false according to the second link of A. Hence B cannot be true, and consequently D-modal abduction is invalid.

\[ ((M \rightarrow E) \land \text{Required } E) \rightarrow \text{Desirable } M' \text{ (‘RD-modal abduction’) is invalid.} \]

(5.2)

**Proof**

Let \( \Sigma \) be some particular statement which is both true and required, such that

A: \( \Sigma \land \forall x \left( [\Sigma]_x \right) \)

If RD-modal abduction were valid, then all its interpretations would be true. In particular

B: \( \left( \vdash \Sigma \rightarrow \Sigma \right) \land \text{Required } \Sigma \rightarrow \text{Desirable } \Gamma \)

would be true. From B we derive K as follows (where no justification is added, each formula follows from the preceding one by some principle of standard logic)
C: \((\neg \Sigma \lor \Sigma) \land \text{Required } \Sigma \rightarrow \text{Desirable } \neg \Sigma\)

D: \((\Sigma \land \text{Required } \Sigma) \rightarrow \text{Desirable } \neg \Sigma\)

E: \(\Sigma \rightarrow (\text{Required } \Sigma \rightarrow \text{Desirable } \neg \Sigma)\)

F: \(\text{Required } \Sigma \rightarrow \text{Desirable } \neg \Sigma\) (from E and the first link of A)

G: \(\forall y ((\Sigma | y) \rightarrow \exists z (\neg \Sigma | z))\) (from F by definitions 4.6 and 4.8)

H: \(\forall y ((\Sigma | y) \rightarrow \exists z (\neg \Sigma | z))\) (from G by 4.1)

I: \(\exists y (\neg \Sigma | y) \lor \exists z (\neg \Sigma | z)\)

J: \(\exists z (\neg \Sigma | z)\)

K: \(\neg \forall z ((\Sigma | z))\)

but K is false according to the second link of A. Hence B cannot be true, and consequently RD-modal abduction is invalid.

The invalidity (in the present formal framework) of the two modally abductive patterns does not mean that using them for practical reasoning should be condemned as ‘incorrect’. Invalidity does not imply implausibility. However, if the assumptions of our framework are accepted, the invalidity of the abductive patterns may be taken as a warning against uncritically relying on them in practical rationalization of decisions. Presumably, our confidence in a conclusion supported by a single modally abductive argument would be increased if another similar but independent argument were added (let alone if a nonabductive argument could be constructed). The failure of some rationalization arguments to prove the second link of condition R3 from definition 3.4 was suggested elsewhere\(^1\) (op. cit. p 273) as one explanation of the observation that design decisions seem tentative and vulnerable to rejection\(^1\). Perhaps frequent reliance on invalid abductive inference patterns (i.e. patterns that do not guarantee true conclusions from true premises) may be added as another.

The remaining eight patterns are all valid. The corresponding conditional formulae are proved below in the manner of ‘natural deduction’: we prove a conditional by assuming its antecedent as a premise, and deriving its conclusion. Proofs will be outlined, often compressing multiple elementary steps into one, and tacitly drawing on theorems and inference rules of monadic predicate logic (e.g. see chapter 3 of Kalish and Montague\(^6\)). Inference steps relying on the principles and definitions of section 4 will be explicitly marked as such.

\[ ((E \rightarrow M) \land \text{Desirable } E) \rightarrow \text{Desirable } M \]

\[^{(5.3)}\] (‘D-modal modus ponens’)

**Proof**

A: \((E \rightarrow M) \land \text{Desirable } E\) (assumption)

B: \(\forall x ([E]x \rightarrow [M]x)\) (from first link of A, by 4.2)

C: \(\exists y ([E]y)\) (from second link of A, by 4.8)
D: \([E]i\)  
E: \([M]i\)  
F: \(\exists z[M]z\)  
G: Desirable \(M\)  

\((P \rightarrow Q) \land \text{Desirable } \neg P \rightarrow \text{Desirable } \neg P\)  
(’D-modal modus tollens’)  
(5.4)

\textbf{Proof}

\begin{align*}
A: & \quad (P \rightarrow Q) \land \text{Desirable } \neg Q \quad \text{(assumption)} \\
B: & \quad \forall x([P]x \rightarrow [Q]x) \quad \text{(from first link of A, by 4.2)} \\
C: & \quad \exists y (\neg Q)y \quad \text{(from second link of A, by 4.8)} \\
D: & \quad \neg Qi \quad \text{(from C)} \\
E: & \quad \neg [Q]i \quad \text{(from D and 4.1 right-to-left)} \\
F: & \quad \neg [P]i \quad \text{(from E and B)} \\
G: & \quad \exists y (\neg P)y \quad \text{(from G)} \\
H: & \quad \exists z (\neg P)z \quad \text{(from H and 4.8)}
\end{align*}

\((P \rightarrow Q) \land \text{Good } \neg P \rightarrow \text{Good } \neg P\)  
(’G-modal modus tollens’)  
(5.5)

\textbf{Proof}

\begin{align*}
A: & \quad (P \rightarrow Q) \land \text{Good } \neg Q \quad \text{(assumption)} \\
B: & \quad \forall x([P]x \rightarrow [Q]x) \quad \text{(from first link of A, by 4.2)} \\
C: & \quad \forall x (\neg Q)x \rightarrow [P]x \quad \text{(from B)} \\
D: & \quad \forall x (\neg [Q]x \rightarrow [P]x) \quad \text{(from C using 4.1 twice left-to-right)} \\
E: & \quad \forall y (\neg Q)y \quad \text{(from second link of A, by 4.7)} \\
F: & \quad \forall y (\neg P)y \quad \text{(from D and E by 4.4)} \\
G: & \quad \text{Good } \neg P \quad \text{(from F by 4.7)}
\end{align*}

\((P \rightarrow Q) \land \text{Required } \neg Q \rightarrow \text{Required } \neg P\)  
(’R-modal modus tollens’)  
(5.6)

\textbf{Proof}

\begin{align*}
A: & \quad (P \rightarrow Q) \land \text{Required } \neg Q \quad \text{(assumption)} \\
B: & \quad \forall x([P]x \rightarrow [Q]x) \quad \text{(from first link of A, by 4.2)} \\
C: & \quad \forall y (\neg Q)y \quad \text{(from second link of A, by 4.6)} \\
D: & \quad \forall y (\neg [Q]y) \quad \text{(from C, by 4.1)} \\
E: & \quad \forall x (\neg [Q]x \rightarrow [P]x) \quad \text{(from B)} \\
F: & \quad \forall x (\neg [P]x) \quad \text{(from D and E)} \\
G: & \quad \text{Required } \neg P \quad \text{(from F, by 4.6)}
\end{align*}

\((P \rightarrow Q) \land \text{Required } P \rightarrow \text{Required } Q\)  
(’R-modal interchange of equivalents’)  
(5.7)

\textbf{Proof \ref{5.7} is a corollary of \ref{5.8} below.}
\((P \rightarrow Q) \land \text{Required } P \rightarrow \text{('R-modal modus ponens')}

\text{Required } Q \quad \text{see note 10)} \quad (5.8)

\text{Proof}

A: \ (P \rightarrow Q) \land \text{Required } P \quad \text{(assumption)}
B: \ \forall x([P]x \rightarrow [Q]x) \quad \text{(from first link of A, by 4.2)}
C: \ \forall y([P]y) \quad \text{(from second link of A, by 4.6)}
D: \ \forall x([Q]x) \quad \text{(from B and C)}
E: \ \text{Required } Q \quad \text{(from D, by 4.6)}

\((P \leftarrow Q) \land \text{Good } P \rightarrow \text{('G-modal interchange of equivalents')}

\text{Good } Q \quad (5.9)

\text{Proof} 5.9 \text{ is a corollary of 5.10 below}

\((P \rightarrow Q) \land \text{Good } P \rightarrow \text{('G-modal modus ponens’ see note 11)} \quad (5.10)

\text{Proof}

A: \ (P \rightarrow Q) \land \text{Good } P \quad \text{(assumption)}
B: \ \forall x([P]x \rightarrow [Q]x) \quad \text{(from first link of A, by 4.2)}
C: \ \forall y([P]y) \quad \text{(from second link of A, by 4.7)}
D: \ \forall x([Q]x) \quad \text{(from B and C, by 4.4)}
E: \ \text{Good } Q \quad \text{(from D, by 4.7)}

\text{(Required } P \land \text{Required } Q) \rightarrow \text{('R-modal adjunction')}

\text{Required } (P \land Q) \quad (5.11)

\text{Proof}

A: \ \text{Required } P \land \text{Required } Q \quad \text{(assumption)}
B: \ \forall x([P]x) \land \forall x([Q]y) \quad \text{(from A, by 4.6)}
C: \ \forall z([P]z \land [Q]z) \quad \text{(from B)}
D: \ \forall z([P \land Q]z) \quad \text{(from C, by 4.3)}
E: \ \text{Required } (P \land Q) \quad \text{(from D, by 4.6)}.

\text{(Required } P \land \text{Good } Q \rightarrow \text{('RG-modal adjunction')} \quad (5.12)

\text{Good}(P \land Q)

\text{Proof}

A: \ \text{Required } P \land \text{Good } Q \quad \text{(assumption)}
B: \ \forall x([P]x \land \forall y([Q]y) \quad \text{(from A, by 4.6 and 4.7)}
C: \ \forall z([P \land Q]z) \quad \text{(from B, by 4.5)}
D: \ \text{Good } (P \land Q) \quad \text{(from C, by 4.7)}

6 \textbf{Related work}

Several authors have contended that some sort of logic should be, or actually is, used in decision-making of design and planning.
A straightforward application of propositional logic has been proposed by Moucka. He considered design 'norms' (requirements) such as "if the bedroom is not connected by a door with the hall then it must have a door into the living room and the living room must not be used regularly for overnight sleeping". Such a composite statement is analysed into atomic ones combined by means of negation and the connectives 'and', 'or', etc. Given a 'norm', all assignments of truth values to the atomic propositions for which the 'norm' comes out true will serve, according to Moucka, as a rough description of a design 'solution'. Conversely, from the descriptions of a body of existing designs one can obtain a concise statement of the underlying 'norms' by reducing the conjunction of the descriptions to a simpler but equivalent form.

Also starting from propositional logic, Johnson has developed a 'logic of speculative discourse' (sections 2–4 of the paper cited), which he seems to intend for use in practical decision-making. In addition to plain truth values he introduces values for truth and falsity 'to be testable at some future time' (hence the name of his logic), and also values 'good' and 'bad' which can be assigned to propositions, alone or in combination with one of the truth values (be it plain or future testable ones). In effect, this amounts to a 14-valued propositional logic. An evaluation procedure is suggested for composite statements of 'rational discourse'; i.e. discourse whose constituent atomic (and hence composite) statements assume one of the 14 values. The handling of normative statements in Johnson's system is interesting because of its simplicity, but probably inadequate for the same reason; for, as Johnson remarks himself, 'to make matters simple, anything not wholly good [is] considered bad', so that if \( X \) is a 'good' statement and \( Y \) a 'bad' one, then \( X \) and \( Y \) is 'a mixture of good and bad, and hence the composite value is bad'.

March proposed his influential 'PDI model' of rational design in 1976. According to the PDI model, designers repeatedly cycle through phases of 'productive' (i.e. abductive), deductive and inductive reasoning. (I have discussed the PDI model more fully elsewhere in connection with the empirical findings of the Spangsbjerg study that seem to clash with March's assumptions.) Abduction was seen by March as the creative element of reasoning in design; 'the only logical operation which introduces any new ideas' (Peirce quoted after March). It is not surprising, therefore, that abduction has received a good deal of attention in the design research literature; both in its 'plain' form as considered by March, and in various forms tinted by deontic modalities.

The PDI model has been debated and augmented in various ways. Roozenburg argues that 'the key mode of reasoning in design may be
called abduction, but only in the sense of innovative abduction’. Innovative abduction is a step of inference from a statement $E$ on a desired effect (‘result’) of the designed artefact, to (1) a rule of the form $C \rightarrow E$, and (2) the statement $C$ expressing what may cause that effect according to the rule. (This is in contrast to plain abduction—‘explanatory abduction’ in Roozenburg’s terminology—where the rule is assumed as a premise along with the effect, and only the cause is inferred.) When performing a step of innovative abduction, Roozenburg explains, we “try to conceive of a new rule (a principle, law, or theory) that allows us to infer the cause” (emphasis added). Whereas this may indeed account for truly creative instances of design, it seems to me that the use of rules in more humdrum cases of ordinary design might be explained in (nonlogical) terms of the designer recalling a relevant rule from a body of background knowledge, as a response to the stimulus of the problem at hand. Once the rule has thus come to mind, reasoning may proceed by plain (or, as I prefer to believe, modal) abduction.

Modal logic or related systems have been proposed, and occasionally used, in quite a few studies of the form of design discourse. An early study by Tzonis et al. relies on deontic modalities for an analysis of architectural design thinking in the period 1650–1800 (and clearly abductive reasoning is encountered; see section 6.1). In a critical discussion of the so-called Argumentative Model of Design, Mann described a ‘standard design argument’ as a variant of deontic abduction: ‘A ought to be because A is instrumental to achieve B; and B ought to be’. The same form of inference was independently proposed and explored in detail by Kim: see discussion in section 6.3. Allor has recommended the language of deontic logic as a suitable formalism for design guidelines (without mentioning abduction). Baljon published a paper in 1982 (based on his 1975 master’s thesis) in which he presents an axiomatic system, an essential part of which is a ‘reconstruction of the concept of ‘desirability’’. His reconstruction is not in modal terms but draws on Bayesian probability theory. However, ‘desirability’ in Baljon’s sense, as we shall see in section 6.2, is strongly reminiscent of our notion of a statement being ‘preferable to its negation’, in terms of which we defined our modalities of ‘requiredness’, ‘goodness’ and ‘desirability’. Recently, Eng referred to von Wright’s observation of the analogy between modal operators and quantifiers (op. cit. page 181) but did not enter into the discussion about modal logic or quantification theory. However, for the support of participatory design discussions, he proposes a computer system in which natural-language statements may be recorded as ‘obligatory’ or ‘permissive’ [i.e. permitted?], and he presents the candidate as an example of a design argument (p 129) which may be interpreted as an instance of both Mann and Kim’s pattern of deontic abduction.
Though obviously not written with design or decision-making in mind, von Wright’s paper, ‘A new system of modal logic’\textsuperscript{25}, represents a potentially interesting approach to the logic of design rationalization. In this paper von Wright sees the logic of probability as an (alethic) modal logic ‘turned into probability-theory by the introduction of a metric for degrees of possibility’ (emphasis added; \textit{op. cit.} p 120). In the present paper we saw a structural similarity between modal logics of possibility (alethic logic) and of desirability (design rationalization); hence construing von Wright’s ‘new system’ as one dealing with degrees of desirability might throw new light on the logic of design rationalization.

However, in the remainder of this section we shall confine ourselves to a brief review of the contributions most closely related to the present study; namely those by Tzonis et al., Baljon and Kim.

\textbf{6.1 Early work by Tzonis et al.}

Tzonis et al. studied the use in architectural reasoning of a deontic obligation operator ‘—P—’ (where ‘—’ is a negation sign and the whole symbol reads ‘it is not permitted that not . . .’; that is, ‘it is obligatory that . . .’). The operator is prefixed to unformalized statements. One example (\textit{op. cit.} p 7) which the authors ascribe to Le Camus de Mezières (1780), reads (with my comments added in brackets)

\[
\begin{align*}
  \neg P & \quad \text{create a character which induces tranquillity} \quad \text{[premise]} \\
  & \quad \text{if shadows are avoided then tranquillity will be} \quad \text{[premise]} \\
  \neg P & \quad \text{avoid shadows} \quad \text{[conclusion]}
\end{align*}
\]

This argument corresponds to a conditional of the form

\[
((M \rightarrow E) \land \Box E) \rightarrow \Box M
\]  
(6.1)

where ‘\(\Box\)’ is a modal operator; in this case ‘—P—’. The argument is structurally similar to the invalid Spangsbjerg pattern of ‘D-modal abduction’ (see 5.1) which can be obtained by inserting ‘Desirable’ for ‘\(\Box\)’ in 6.1. It is not clear from the cited paper by Tzonis et al. whether such an interpretation of Le Camus de Mezières’ argument would be equally feasible.

In a similar manner the authors go on to present another argument based on the writings of Le Camus de Mezières; an argument which appears to be valid, for if ‘—P—’ is represented as ‘Required’ it becomes an instance of ‘\(\text{\textit{R}}\)-modal modus tollens’ (5.6). Further arguments also presented by
Tzonis et al. do not immediately map onto formulae of the rationalization logic we have suggested. The authors do not develop a formal system of logic, which was obviously not their aim; their analysis seems to be more of a linguistic and historical than of a logical nature. It is of interest to us here mainly because, like the present study, it is directly based on empirical evidence.

6.2 Baljon’s axiomatic system

Baljon’s work\(^{22}\) was strictly formal and led him to an axiomatic system of a ‘logic of planning’, though without such direct use of empirical data. His approach was to ‘reconstruct’ formally such obvious core concepts of planning as desirability, in order to subject planning rationality to logical criticism. We cannot hope to do justice to his project in a brief review but shall merely consider a few sample aspects of his system which exhibit interesting relationships to the logic suggested in the present study.

Given a proposition \(p\) (lower case letter in Baljon’s notation) and a speaker (person or an interest group) \(x\), the function value \(D(x, p)\) is a real number that expresses ‘the desirability for \(x\) that \(p\)’ (compare the above remarks on degrees of desirability in von Wright’s ‘new system’). Baljon’s notion of desirability seems related to our notion of preferability as set out in section 4 and embodied in the predication operator. In his axiom 7 he defines a ‘predicat of desirability’ as follows

\[
xD p \equiv D(x, p) > D(x, \neg p)
\]

The left-hand-side of this definition reads ‘for \(x\) it is desirable that \(p\)’. This corresponds closely to our expression ‘\([P]x\)’ (we use the same letter in lower and upper case to denote the same proposition in the two notations), except that the latter is to be understood without recourse to numerical values, and with reference to a ‘relevant point of view’, rather than to a speaker or interest group.

At first sight there seems to exist an analogy between an expression like ‘\(D(x, P) > 0\)’ in Baljon’s system and ‘Desirable \(P\)’ in the present framework. However,

\[
D(x, p) > 0 = D(x, \neg p) < 0
\]

is a theorem of Baljon’s, and this precludes the simultaneous positive desirability (for the same speaker) of a proposition and its negation, whereas in our framework nothing prevents the coexistence of ‘Desirable \(P\)’ and ‘Desirable \(\neg P\)’, even in the presence of only one speaker.
The analogy, as already suggested, is rather one between Baljon’s desirability function symbol and our predication operator: the fact expressed by the above theorem that a statement and its negation cannot both have a positive desirability would seem to correspond to our first ‘principle of predication’ (4.1), that a statement and its negation cannot both be preferable to each other from the same point of view.

An important difference is that probability plays a central role in Baljon’s system, but none whatsoever in our framework. Baljon’s system incorporates Bayesian decision theory, the ‘core’ of which Baljon describes as ‘a mathematical elaboration of the idea, that the desirability of an act is a function of the desirability and probability of all its possible results’ (op. cit. p 186) and summarizes formally in his axiom 13 (not shown here).

Baljon generalizes his desirability function so as to express conditional desirability: ‘D(x, p|q)’ is defined so as to express ‘the desirability for x of p (being true) in case of q (being true)’. No analogous concept exists in our framework—save perhaps implicitly by the assumption that statements are referring to a particular state of evolution of the design; hence their possible desirabilities depend on (the statements describing) that state (section 4.1).

Means-ends relationships are defined in Baljon’s system in probabilistic terms: given a proposition p which describes ‘an end to be aspired’ and a proposition q that describes an activity, q, according to Baljon (p 189) ‘may be qualified as a means to that end’ if and only if $P(p|q) > P(p|\neg q)$ (the probability of p given q is greater than the probability of p given $\neg q$). Surprisingly, such means-ends relationships turn out to be nontransitive, which, according to Baljon, should warn us that traditional design thinking may be flawed. This being as it may, the probabilistic definition itself appears to me somewhat artificial in a design context. At least in principle, a designer can be assumed to decide only about means and ends which it is in his or her power to decide about, and so Baljon’s definition seems to offer an unnecessarily loose coupling between means and ends. On the other hand it can be objected against our approach that the Spansbjerg patterns most clearly expressing a means-ends relationship where the abductive ones which, as we saw, were logically invalid!

Crudely summarized, the chief difference between Baljon’s system and the one proposed here is the reliance upon numeric values for probability and desirability in the one system, and the absence of numbers in the other.

Baljon’s contribution to logic-based design research was an ambitious and systematic pioneering effort. In my view, it deserves not only to become
known as such but also to be considered a subject for future research, albeit research along lines rather different from the ones we have followed here.

6.3 Kim's principle of 'teleological explanation'

As we noted above, Kim's study is seen here as representative of the thinking of Mann\(^{20}\) and Jeng\(^{24}\) as regards their views of deontic abduction in design discourse. Kim\(^{21}\) describes a method for analysis of design intent in terms of means and ends. It is based on a form of argument which he calls 'teleological explanation' (op. cit. p 105), and which can be stated as follows (with slight modifications of the notation to suit the conventions used here)

\[
((M \rightarrow E) \land [E]) \rightarrow [M] \\
\text{(Kim's 'teleological explanation') (6.2)}
\]

Here, \(M\) and \(E\) are statements describing means and end, respectively, and the square brackets obviously constitute what we call a modal operator: given a 'factual statement' \(P\), \('[P]\) is a 'deontic statement' to be interpreted 'it ought to be the case that \(P\)', according to Kim. We see that again we have a pattern of the form 6.1 (see section 6.1).

His intention was not to construct a 'logic of design rationalization', but to build a computerized system that might help its users design for the 'implicit design intent [...]', rather than the explicit design specifications, by tracing up the teleological structure' (p 109). For this purpose the principle of 'teleological explanation' may be an adequate tool of analysis. However, as a theorem of a logic of design rationalization (or a logic of decisions in general) it is dubious.

Kim's use of the word 'deontic' suggests that his principle emerges from 6.1 if 'Op' is related to, or identical with, one of the modal operators known from deontic logic, presumably the one used to express obligation. Under such an interpretation, Kim's principle can hardly be logically valid: even if \(E\) is obligatory and \(M\) brings about \(E\), it is unsafe to conclude that \(M\) is obligatory, because there might be other ways of bringing about \(E\). Likewise, if 'Op' is replaced by any one of the three modal operators we have proposed for our logic of design rationalization, the formula turns out to express an abductive principle.

Kim's principle is problematic for quite another reason, even if we ignore the allusion to deontic logic and stick to his literal interpretation: 'teleological explanation' seems to imply that the end justifies the means. An example (for the sake of clarity chosen from outside the realm of design) will illustrate this point.
A: If I chop off and eat your arm then I’ll be no longer hungry.
B: It ought to be the case that I’ll be no longer hungry.
C: It ought to be the case that I chop off and eat your arm (from A and B)

(note 12). A problem with such a ‘teleological explanation’ (and with the analogous inference patterns discussed by Mann and Jeng) is the lack of distinction between rationalization and decision—between preferability (from a relevant point of view) and actual preference.

7 Conclusions
Extensions of standard monadic predicate logic in two directions have been proposed; firstly, introduction of the ‘predication operator’ so as to capture the notion of the preferability of a statement over its negation from a ‘relevant point of view’, and; secondly, introduction of the nonstandard ‘most’-quantifier. Under the proviso that such extensions admit of the three ‘principles of predication’ and the two ‘principles of most-quantification’ stated and informally justified in section 4, it was shown that the modal ‘logic of design rationalization’ and the associated notion of design rationality (section 3) suggested by empirical findings of the Spangsbjerg case study could thus be given a firm foundation in terms of quantification over a universe of ‘relevant points of view’. More specifically, we saw in section 5 that the modally-abductive patterns of inference encountered in the Spangsbjerg data are demonstrably invalid (hence do not guarantee true conclusions from true premises), whereas the remaining eight Spangsbjerg patterns of inference are provable within the logical framework suggested. Thus standard monadic predicate logic with the proposed extensions may serve as a conceptual machinery suitable for formal logical study of realistic patterns of (rational) design reasoning. Hence, with some right we may claim (the merits of alternative approaches notwithstanding) that the ideas and findings of the present study represent a step towards a formal logic of design rationalization, and, we may hope, consequently an improved understanding of design rationality.

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My best thanks are due to Dr Ir. Cornelis Baljon for his numerous detailed and constructively critical comments on various tentative writings of mine about the material presented in these pages. He drew my attention to the work of Tzonis et al., he pointed out weak spots in my reasoning (see note 7 below), and his suggestions—after a prolonged and heated debate by correspondence—ultimately induced me to scrap over 30 pages of manuscript and rewrite the entire exposition from scratch. Despite all this, his view of logic in design and mine seem as divergent as they were initially,
and Baljon cannot, of course, be held responsible for any flaws or imperfections that might still remain in this paper. Valuable suggestions for improvements of the paper were offered by two anonymous referees, and by Professor Norbert Roozenburg in his capacity of editor.

Notes
1 The left-to-right conditional of 2.1 is equivalent to ‘\([P]z \lor \neg [P]z\)’, that is, ‘either the proposition or its negation holds under the given circumstance’. Furthermore, 2.1 is equivalent to ‘\([P]z \leftrightarrow \neg [P]z\)’ whose left-to-right conditional says that ‘if the proposition holds under the given circumstance, the negated proposition does not’; whereas the right-to-left conditional of 2.1 itself says that ‘if the negated proposition holds under the given circumstance, then the proposition itself does not’.
2 This and the following proofs are outlined without the full details of formal derivation in a particular formalization of predicate logic.
3 Namely ‘\(\forall x (Fx) \rightarrow \exists y (Fy)\)’, which reflects the standard assumption of a nonempty universe of discourse, cf. Quine pp 116, 117, 129, 137, 147, 173.
4 In the context of designing, I find it natural to use ‘statement’ in lieu of ‘proposition’ to emphasize that the states-of-affairs of the world spoken about are usually fictitious, or not yet at hand. From a logical point of view, however, there is no difference.
5 Strictly speaking, a statement cannot be preferable (nor actually be preferred) to another statement, but the states-of-affairs described by the statements can be. For the sake of readability, however, we shall understand this tacitly.
6 Most of the principles in question were in fact discovered by constructing formal proofs of the theorems in section 5 and introducing plausible principles of predication and most-quantification whenever needed to justify a proof step.
7 I owe this example to Dr Ir. Cornelis Baljon (private communication 20 April 1995), who presented me with it in a rather different context, namely in order to demonstrate the invalidity of a formula which in our present notation can probably best be written as (Good \(P \land \) Good \(Q\)) \(\rightarrow\) Good \((P \land \neg Q)\). At that time I had not worked out in details the reductionist approach to the proposed logic of design rationalization, nor a clear notion of its modal operators. Such was my confusion that I believed this formula to be valid, and I am indebted to Dr Baljon for vigorously prompting my ‘second thoughts’.
8 Roughly speaking, plain (i.e. nonmodal) abduction is the inference of a statement \(C\) from premises of the forms \(C \rightarrow E\), and \(E\), with the corresponding (invalid) conditional: \((C \rightarrow E) \land E\) \(\rightarrow\) \(C\). It may be
thought of as ‘reasoning backwards’ from effect $E$ to cause $C$, via the rule $C \rightarrow E$. True premises do not guarantee a true conclusion. The term ‘abduction’ stems from Peirce\textsuperscript{15,18}.

9 The reader may feel perplexed by the subformula ($\neg \Sigma \rightarrow \Sigma$) which reads, somewhat enigmatically, ‘if $\Sigma$ isn’t true, then it is true’. The formula (which is equivalent to $\Sigma$) is introduced for proof-technical reasons only, namely by inserting the statement $\neg \Sigma$ for $M$ and the statement $\Sigma$ for $E$. If D-modal abduction were valid, it would come out true even under this, deliberately contrived, interpretation. (But it does not, as the proof shows.)

10 This is not a principle encountered in the Spangsbjerg case; it is included here for its general interest, and as a means of proving Spangsbjerg principle 5.7.

11 Like 5.8, this is not a Spangsbjerg principle, but is included for its general interest, and as a lemma supporting 5.9.

12 This example of ‘teleological explanation’ is inspired by another offered by Kim (pp 107, 108 of his paper); in slightly modified notation it reads

$$((\text{eating} \rightarrow \neg \text{being hungry}) \land (\neg \text{being hungry})) \rightarrow \text{[eating]},$$

where again square brackets surround statements which ‘ought to be the case’.