Geometric Configuration and Graphical Representation of Spherical Tensegrity Networks

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Abstract
The term "Tensegrity," that describes mainly a structural concept, is used in building design to address a class of structures with very promising applications in architecture. Tensegrity structures are characterized by almost no separation between structural configuration and formal or architectural expression (Liapi 2000). In the last two decades structural and mechanical aspects in the design of these structures have been successfully addressed, while their intriguing morphology has inspired several artists and architects. Yet, very few real world applications of the tensegrity concept in architecture have been encountered. The geometric and topological complexity of tensegrity structures that is inherent to their structural and mechanical basis may account for significant difficulties in the study of their form and their limited application in building design. In this paper an efficient method for the generation of the geometry of spherical tensegrity networks is presented. The method is based on the integration of CAD tools with Descriptive Geometry procedures and allows designers to resolve and visualize the complex geometry of such structures.

Keywords
Tensegrity Networks, Visualization, Geometric Configuration

1 Introduction

Tensegrity, according to Buckminster Fuller who coined the term, stands for "structural integrity," and defines primarily a structural concept (Fuller 75). The term tensegrity is also used to describe coherently a distinct class of structure that present very unique structural, mechanical and morphological characteristics. Tensegrity structures consist of bars and cables. The bars, which act as compression members, are connected only to cables, and not to each other, while the cables undertake only tension and form a continuous closed system. Internal tension is what holds bars and cables together, and allows a tensegrity structure to rigidify at a specific geometric configuration. As a result, in most applications of the tensegrity concept in structures, bars look like they float in a network of cables, which gives to them their morphological and visual identity. Because of this specific feature, Keneth Snelson, who was the first to conceive them, named them "structures of floating compression" (Snelson 65).

Tensegrity networks provide an alternative to massive building structures, since they can be lightweight, do not require anchorage, and are inherently deployable (Hanaor 98, Motro 2001). Potential applications of the tensegrity concept in building design include large space covering structures for stadiums, swimming pools etc., traveling exhibits, movable hospitals, provisional shelters and temporary storage facilities.

Morphological aspects of tensegrity systems have been investigated by George Emmerich, who invented the "autotendant," (self-tensioned structure), at the same time that Fuller came up with his "tensegrity" patent (Emmerich 96). In the last two decades structural and mechanical aspects of tensegrity systems have been addressed by various researchers who identified their advantages for specific applications (Hanaor 92, Motro 92, Skelton 97, Sultan 01).

Despite the potential applications of tensegrity networks in architecture, their use is limited due mainly to their complex geometry that accounts for significant difficulties in their configuration, representation, design and analysis. Difficulties in the geometric configuration of tensegrity networks derive from the configuration of the units in conjunction with the method by which units are attached to each other to form a network. Characteristic features in the geometry of units and networks are presented in the following sections.

1.1 Geometric configuration of tensegrity units

The type of tensegrity networks that are most appropriate for application in building design are the double-layer tensegrity grids which typically occur from the assembly of simple tensegrity units such as tensegrity prisms or truncated pyramids.

A tensegrity prism is a skew prism, or more precisely an antiprism, formed by cables along the edges of the prism and with bars along the diagonals of the side faces in a consistent right handed or left-handed sense. In the prestressed state, when all cables are in tension, the two parallel bases of the prism are rotated in respect to each other by an angle that is dictated by the requirement for stability of the shape. The angle between the two base polygons of the tensegrity prism is unique for each polygon of n sides and is given by the formula: a = 90-180/n (Hanaor 98). According to this, for a tensegrity unit of a triangular base the angle a is 30 degrees, for a tensegrity unit of a square base is 45 degrees etc (Figure 1).

The same rules apply for tensegrity units in which the two parallel bases are similar but of different size. The closest regular polyhedron to this type of tensegrity unit is the truncated pyramid.

2.2 Tensegrity networks

The form of a tensegrity network depends on the geometric configuration of the composing units and the way units are connected to each other. In order to maintain bar independence throughout

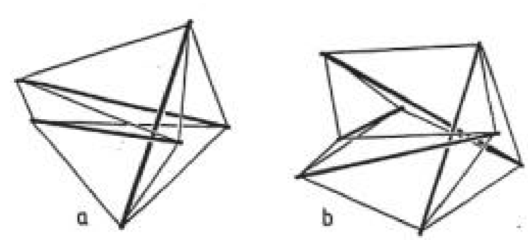


Figure 1. Tensegrity units of triangular (a) and square base (b)

the structure several patterns for connecting the units have been identified (Emmerich 88, Hanaor 92), (Figures 2,3). Tensegrity prisms of triangular or square base can be used as building blocks for the development of flat tensegrity networks, while tensegrity-pyramids can be used to generate curved networks. (Hanaor 92)

In a real world situation, the span may be predetermined depending on space limitations or other architectural needs. The height, or structural depth, may be another constant determined by architectural or structural requirements. A particular size and proportions of individual units may be desired, or a certain amount of side overlap between the upper or lower bases of adjacent units may also be given. The curvature of the structure is a result of the relationship between these parameters. Various combinations and radii of curvature may need to be explored before the initial geometric configuration is reached.

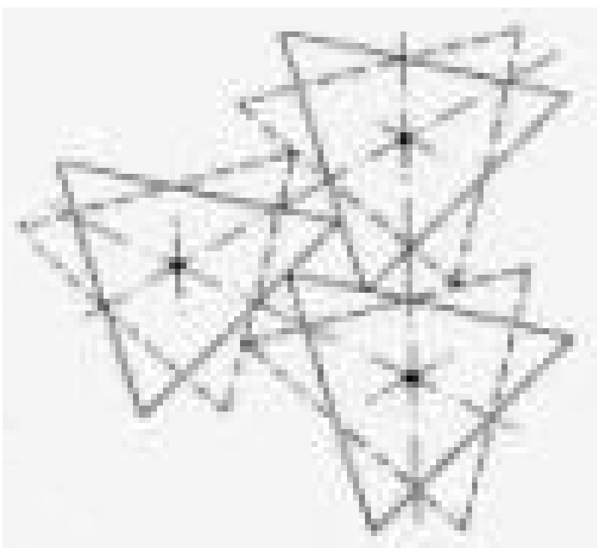


Figure 2. Connecting tensegrity units of triangular base

Physical models still remain the best way to explore preliminary forms of structures during the early design phase. However building models of tensegrity structures involves a very tedious process since minor inaccuracies in member lengths can result in structures with no rigidity. Using flexible parts to represent tension elements renders model building a little easier since the structure acquires its rigidity and geometric configuration by the stretching of flexible members (Figure 4). Generating several models until the desired shape is established, even if flexible members are to be utilized, is a very time demanding process and rather inefficient. In addition analytic methods for the structural performance of tensegrity networks assume easy availability of the initial geometry. Acquiring numerical values for the flexible and rigid members of the structure and their angular relationships from the model alone is not a reliable approach, especially when precision is required.

Although the rules for joining units so that the structure rigid members will remain disjointed in the cable network are clearly set, no algorithm has been developed that ties all the parameters together and that will allow us to obtain actual values for some of the parameters when the others are already assigned a value. So the questions that have not been addressed yet relate to the manner in which the proportions and size of the composing units, as well as the connection pattern and unit overlap on each base , affect the configuration of the network, or how such algorithms can be used to generate digital models that will allow

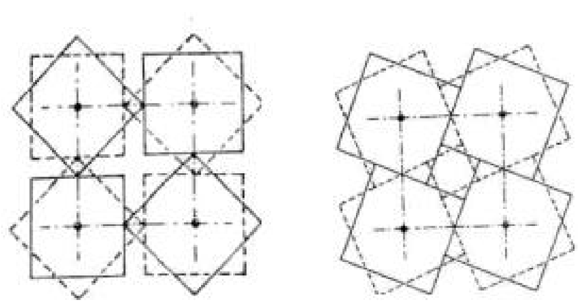


Figure 3. Connecting tensegrity units of square base

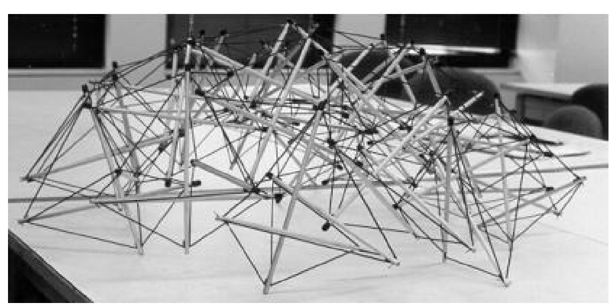


Figure 4. Model of a tensegrity structures composed of units of triangular base (built by the author)

designers to explore morphological variations of tensegrity structures.

In the following sections the geometric approach that led to an efficient method for form exploration, representation and visualization of such structures is presented first. A step by step graphical procedure that combined constructive geometry with CAD tools is also presented, as well as suggestions for improving the method.

2. Geometric Considerations in the Representation of Tensegrity Units

Unlike regular space trusses tensegrity networks are composed of tensegrity units which are antiprisms and therefore do not fall within the range of regular polyhedra. In addition their connection pattern needs to conform to several constrains to ensure cable continuity and bar independence throughout the entire structure; it therefore requires that units do not connect end to end, but, by a partial overlap of their upper and lower base cables. Hence the configuration of tensegrity networks cannot be addressed as a regular space filling problem.

The main issue therefore is to determine the geometric basis of the problem and to come up with a geometric construction that takes into account the interdependence of the various parameters and constrains of the problem. Towards this effort, a procedure that simplifies the representation and study of the configuration of tensegrity units is presented first. Significant observations that derive from this procedure and which address issues in joining units to form curved structures, are also discussed. For this study a pyramidal tensegrity unit of square base has been used. One of the reasons for selecting this unit, is its biaxial symmetry. It is anticipated that, if an effective method for the resolution of the geometry of the spherical network, composed of tensegrity units of square base, is found, the same method could be expanded to study and explore single curvature structures; tensegrity units with biaxial symmetry seem to be the most appropriate for the construction of single curvature structures .

2.1 Geometric Simplifications

The study of the geometry of a typical tensegrity unit shows that : a) all side cables and bars lie on asymptotic axes, b) the overlapping constraints apply to cables of both bases c) bases are rotated in respect to each other, c) changes in the height of the unit, or the ratio of the size of the bases, do not affect the angular relationship between the two bases but the length of the side cables and bars and their angular relationship do change.

Based on the above observations a simplified representation of a typical unit that keeps only the upper and the lower base, and leaves out side-cables and bars, has been used in several stages in the process towards the generation of the geometry of a tensegrity network.

Another construction, that is proven to be critical importance, further simplifies the configuration of a tensegrity units by inscribing it within the closest simpler Euclidean solid. For a regular prismatic unit, a cylinder reduces the parameters of a tensegrity unit to radius and height, and for a truncated pyramidal unit, the truncated cone reduces parameters to upper and lower radius and height. Representing a pyramidal tensegrity unit with a regular cone, instead of a truncated one, allows designers to consider a whole family of tensegrity units which have the same upper base, the same base angle in orthogonal projection, but different heights (Figure 5).

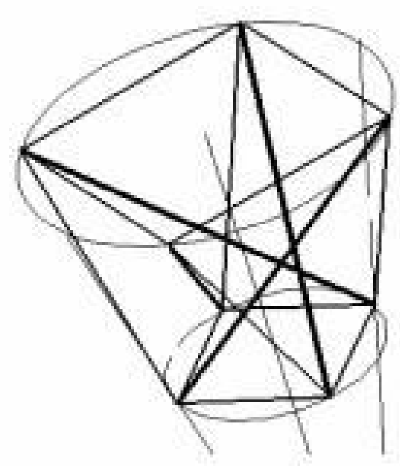


Figure 5. Inscribing a tensegrity unit in a cone

2.2 Observations based on geometric simplifications

Considering the geometry of the cone inscribing a tensegrity unit of square base, the lines that connect the tip of the cone and the vertices of the smaller base of the tensegrity unit, will also meet the base of the cone at 4 points that form a square inscribed in the circular base of the cone and is rotated 45 degrees in respect to other base of the unit.

The construction of a triangular plane, drawn by the projection of a side of the lower square on the base of the cone, and the tip of the cone, is a very useful application of the previous observation (Figure 6).

In Euclidean 3D Geometry, this plane also represents the geometric locus that provides the length and exact position in space of the sides of the smaller square base of any tensegrity unit of the same upper base that fits within the cone. To phrase it differently, any intersection of this plane with a plane parallel to the base of the cone, yields a segment, that represents the side of the smaller base of a tensegrity unit that fits within the cone and the height of which is given by the distance of the intersecting plane from the base of the cone.

Another important construction that occurs from the concept of the cone that encloses the unit, and which is critical for the study of the overlapping geometry of two adjacent units, is discussed below:

By inscribing in cones two adjacent tensegrity units, placed parallel to each other and by overlapping upper bases, we receive in the top projection two intersecting circles that share a common cord. A significant observation here is that the common cord bisects the overlapping section of the sides of each base polygon (Figure 7). This helps in understanding how units will relate to each other in a curved configuration. More specifically the first problem that occurs when one tries to resolve the geometry of two adjacent units that contribute to a curvature, is to determine a plane in respect to which the two units need to rotate, so that the axis of the two units will intersect while respecting the overlap constrains. Without the concept of the cone, determining such a plane and its angle in respect to original position of the two tensegrity units is very difficult:.

The geometry of the two intersecting cones, with coplanar bases, reveals that the plane defined by the axis that passes through the common cord and is perpendicular to the plane of the bases of the cones before rotation, fulfills this requirement. Indeed when a unit, mirrored with respect to this plane, will yield the other unit, and the axis of the two units that occur intersect on the axis that passes through the common cord and lies on the mirror plane. At the same time by rotating the two cones, or the two units, by the same angle towards this plane, the projections of their sides on a plane perpendicular to the one they rotate, overlap. It is interesting to note here that in reality the two units, only got one common point, which is the middle point of the overlapping section. (Figure 8).

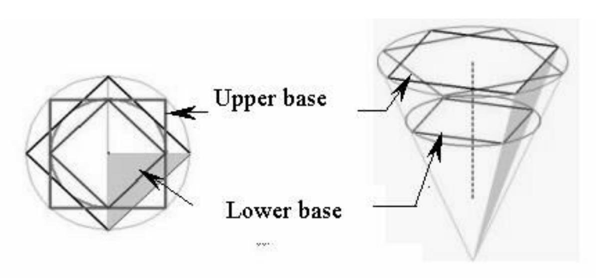


Figure 6. Projecting the lower base on the upper base perspective and top views

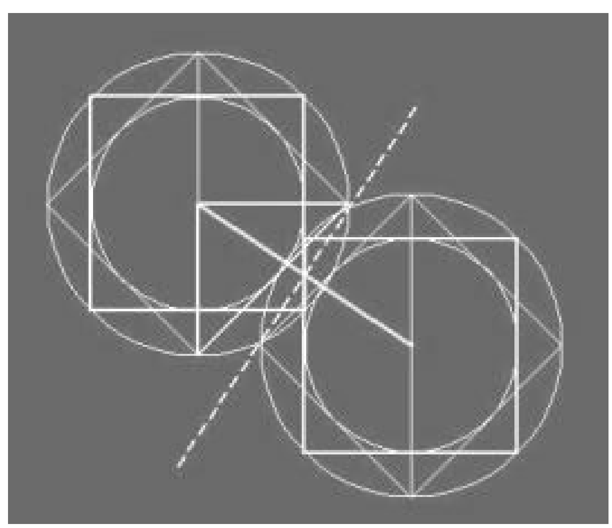


Figure 7. Geometry of adjacent tensegrity units with coplanar bases (top view)

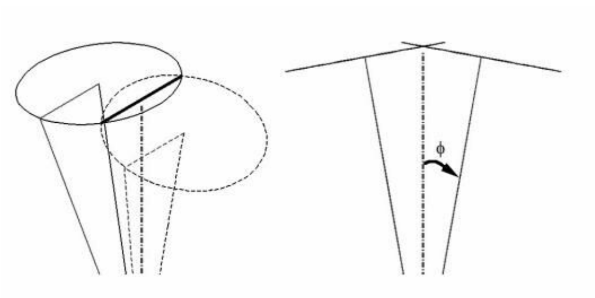


Figure 8. Rotation of adjacent tensegrity units

Therefore this construction also suggests that in the actual structure several adjustments to the lengths of the base cables of the units need to be done so that, when connected towards a curved configuration, they will continue to physically overlap by the given amount. More details about this issue will be provided in an upcoming publication by the author.

3 Graphical Generation of a Spherical Tensegrity Network

For the development of a spherical tensegrity network a graphical method has been developed based on the constructions and observations discussed in the previous section.

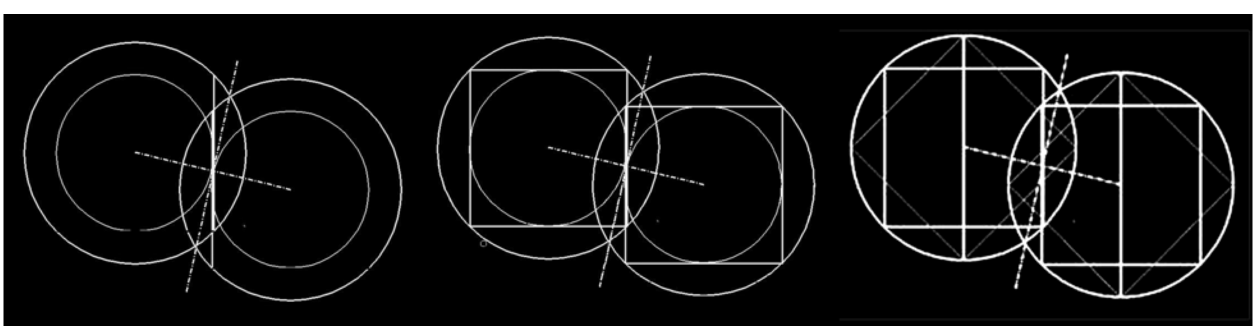
Assuming that in several stages during the study of the geometric configuration of the structure, the concept of intersecting cones replaces the concept of adjacent units with overlapping sides, a geometric construction that provides the exact position of the tensegrity units within the intersecting cones is required. This construction is presented first. The process that generates the entire network follows.

3.1 Placing adjacent tensegrity units within intersecting cones.

For each regular polygon that represents the base of a tensegrity unit, the circumscribed and inscribed circles are uniquely defined. This implies that when the circumscribed circle, that coincides with the base circle of the intersecting cones, is give, the inscribed circles to the base polygons can be constructed. Figures 9 and 10 illustrate the process:

Since the two intersecting cones are placed first, the distance between the centers and common cords are given. From the middle point of the common cord a tangent to the inscribed circle is drawn. This tangent, when extended on both sides until it hits the circumscribed circle, defines a cord in the circle, which yields one side of the square base of the unit. (Figures 9a, 9b) When extended in the other direction gives the side of the adjacent square (Figure 9c). Since from the middle point of the common cord two tangents to the inscribed circles can be drawn, two different sets of overlapping squares can be obtained (Figure 9c). Also, since the requirement for stability of tensegrity units with square base specifies a rotation of 45 degrees between upper and lower base, then a similar construction will yield the lower bases of the tensegrity units (Figure 10 a).

Given the height of the unit, the lower base can be placed in its correct position within the cone. Bars and cables can be also added for the completion of the structure (Figure 10 b)



a *b* *c*

Figure 9. Steps in the geometric construction of tensegrity units that fit within two intersecting cones

3.2 Determining the overlap of the sides of the lower bases

In order to obtain the curvature of a tensegrity network the axis of all units in the network need to intersect at a fixed point in space which will be the center of the curvature of the structure. In addition to this, upper and lower sides of adjacent units need to partly overlap. Because of the rotation of the bases of a typical unit, determining the overlap of the lower bases, when the upper base overlap and the curvature are given, cannot be resolved as a regular space packing problem.

In the previous section (2.2) it has been shown that the curvature of the structure is obtained by rotating the two cones around a plane defined by their common cord and the center of the spherical structure. The construction described in the following section will assist in the understanding of the interdependence of parameters involved in the geometric configuration of tensegrity structures and will provide a method for determining the overlap between the lower bases:

As shown earlier, the top view of the intersecting cones indicates that when the sides of the lower bases of the units do not overlap, their projections on the base of the cone (which represent the edges of the triangular planes), that lie on the upper level, do not overlap either, but remain parallel to each other (Figure 11a). By rotating the cones in respect to the plane that has already been defined, the triangular plane within each cone will rotate with it. Since the two triangular planes represent the geometric locus of all lower base sides of any tenegrity unit that be inscribed into the cone, at the point where the projections of the lower base sides on the upper base overlap, (or where the two parallel edges of the triangles overlap) the sides of the lower squares of the units will overlap as well. The overlapping lower squares can be easily constructed (Figure 11 b, 11c) . The center of curvature can be also established from this construction.

The proposed method for obtaining the initial geometry of tensegrity networks which in essence relies upon the construction of the two overlapping cones draws the following conclusions:

When the height of the truncated cone is known, that is the height of the tensegrity unit, which coincides with structural depth of the network, the cones need to be rotated until the base of the trapezoid, that replaces the triangle in the case of the regular cone hits the main axis. If the lower overlap is known then, one can examine if the selected overlap and depth of structure will give a surface of acceptable curvature. Inversely the depth of the unit can be graphically obtained after a desired curvature is set. A third alternative is to set the curvature, the structural depth and the amount of overlap between the greater bases, and to graphically obtain the smaller bases of the

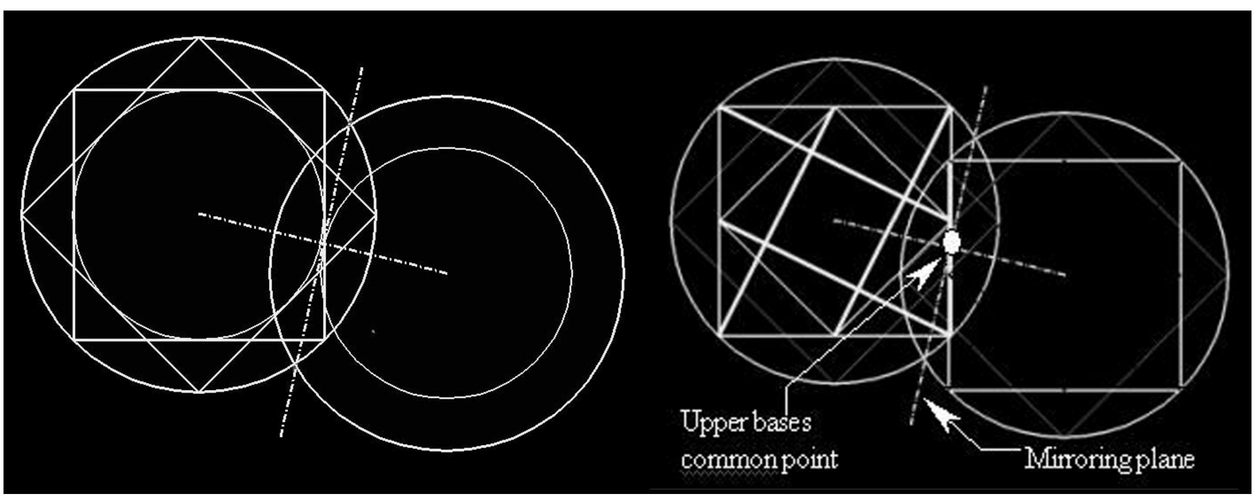


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Figure 10. Construction of lower base and additional side members of the tensegrity units

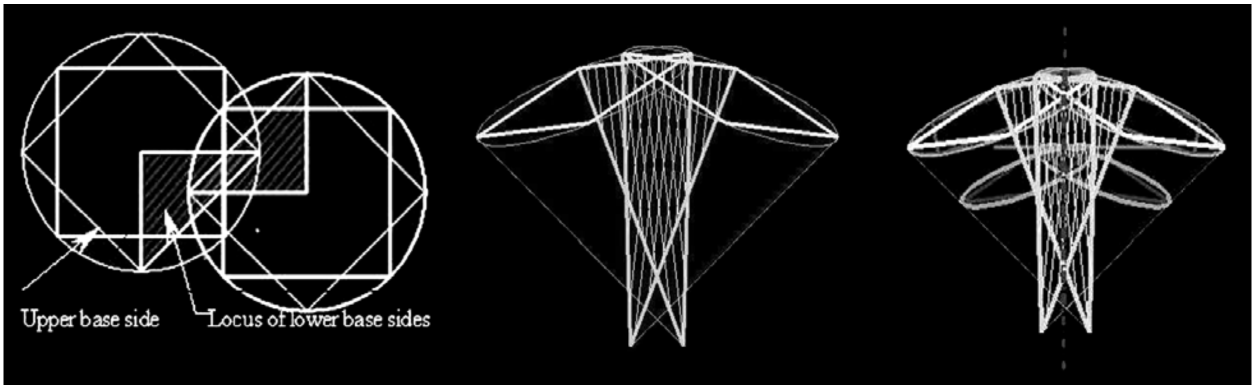

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Figure 11. Steps in the geometric construction of the overlap of the sides of the lower bases

cones, and thus the lower base of the tensegrity unit.

When the height of the truncated cone is known, that is the height of the tensegrity unit, which coincides with structural depth of the network, the cones need to be rotated until the base of the trapezoid, that replaces the triangle in the case of the cone that is not truncated, hits the main axis. From this point and beyond different degrees of overlap can be established. If the lower overlap is known then, one can examine if the selected overlap and depth of structure will give a surface of acceptable curvature. Inversely the depth of the unit can be graphically obtained after a desired curvature is set. A third alternative to set the curvature, the structural depth and the amount of overlap between the greater bases, and to graphically obtain the smaller bases of the cones, and thus the lower base of the tensegrity unit.

The same basic geometry can be used to receive boundary conditions, that is geometric limitation beyond which units do not intersect. A more thorough discussion of boundary conditions and constraints that occur from the geometric formulation will be presented in an upcoming publication by the author.

3.3 Generation of the entire network of units

Once the relative position of two adjacent units with overlapping sides is found, 2 more units can de generated by following similar mirroring procedures. (Figure 12)

The next step involves the generation of the spherical grid. The generation of the grid assumes some regularity in the subdivision of the spherical dome. Different geometric approaches can be followed, such as dividing the sphere using concentric greater circles, using parallel circles, or using parallel circles in one direction and greater circles in the other. A subdivision that is based on concentric greater circles has been chosen for this project (Figure 13). For a curvatures with a large

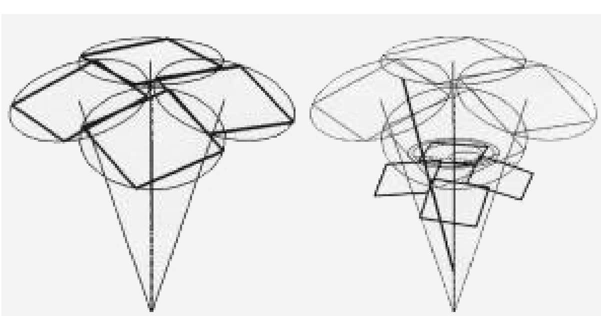

Figure 12. Generation of four units

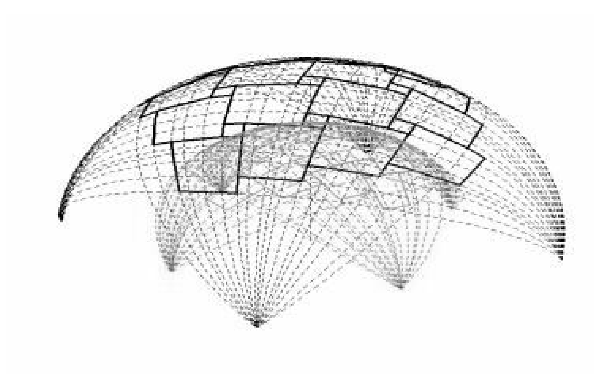

Figure 13. Generation of the entire network

radius, that is when the structure represents only one small section of the sphere, and the units are of small size relative to the size of the structure, then identical units can be used. For higher curvature and bigger units, different sizes of units need to be considered.

From the digital model that occurs from the proposed construction, angular measurements as well as measurements or radii and distances can be made. Coordinates at the nodes can be also obtained directly from the software database. A further elaboration on the method requires some adjustments on the geometry of the square bases of the units, which in the curved configuration are no longer planar.

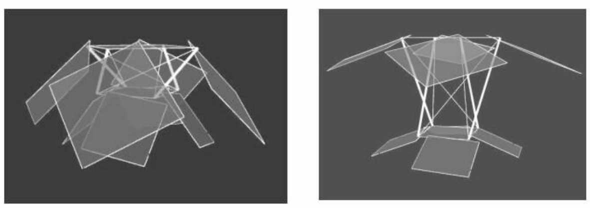

Figure 14. Studies on the morphology of a tesegrity dome

The entire process has been generated using a commercial CAD software package. Software features such as 3D representation and manipulation of primitives, easy access to auxiliary coordinate systems, generation of surface, that facilitate the process, can be found in most CAD software. A parametric description of the basic unit, as well the development of macro equations for automating some of the steps in the procedure could be used to improve the efficiency of the method.

A 1/2c scale model of a structure composed by 16 units of square base, designed and constructed by the author, validated the efficiency and accuracy of the method (Figure 14) Several digital models in which the interdependence of the various parameters that determine the morphology of the structure were developed before the actual structure was built (Figure 15).

4 Conclusions

Geometric constructions following basic Euclidean geometry theorems, and CAD. procedures have been effectively combined to develop a method for the generation of spherical tensegrity structures. The proposed method allows design-

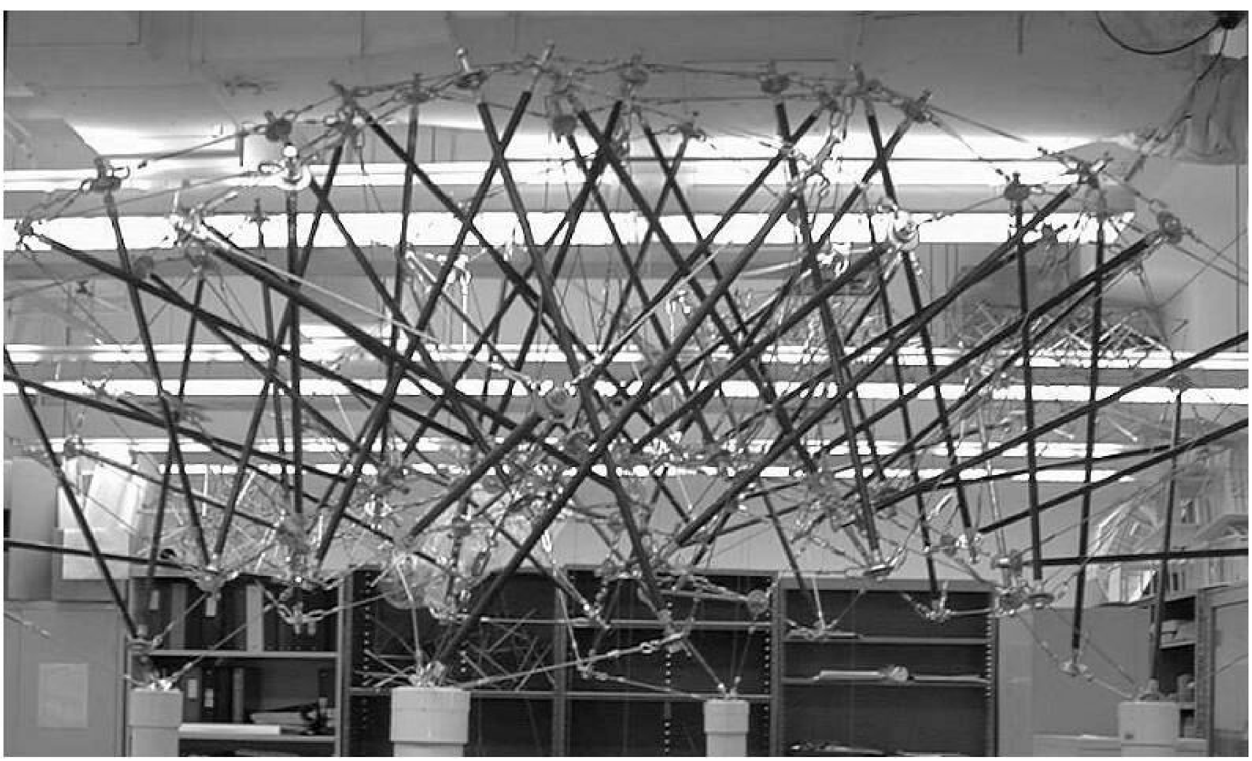

Figure 15. Model of a tensegrity dome composed by sixteen units of square base

ers to explore and visualize various morphologies that respond to specific architectural requirements. A ½ scale model of a spherical structure has validate the efficiency of the method.

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