Evolutionary Automata for Suburban Form Simulation

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Abstract: The paper outlines a research project to develop a dynamic simulation of suburbanization processes. The approach to simulating suburban form relies on modelling different interacting processes on various scales. Two layered models are implemented, the Socio-Economic and Zoning model and the Suburban Form model, respectively by means of cellular automata and genetic programming. The Socio-Economic and Zoning model simulates exogenous factors and endogenous processes of large-scale suburban dynamics. The model approximates the area by means of a rectangular grid to the scale of hundred meters. The Suburban Form model uses a smaller grid, to the scale of meters, and is three-dimensional. The resulting dynamic, 3D, fine-scale model will create scenarios of suburban growth, allowing evaluation of their consequences on built environment and landscape.

1. INTRODUCTION

The paper outlines an ongoing project to develop a dynamic simulation of suburbanisation processes in Europe, through a collaborative research effort between the European Commission's Space Application Institute and the Department of Architectural Design at the Politecnico di Torino.

Urban form is the unstable result of complex interactions of site, history, economy and milieu. In suburban settings this interplay is faster and straighter than in city cores. In city cores, density in itself makes change often difficult to track, slow to accomplish, and, more important, highly influenced by individual site-specific conditions such as ownership, real estate, memorial and artistic values, occupation etc. The “thickness” of the compact city core thus makes it difficult (and maybe useless) to simulate its change, as the model, in order to be accurate, should take in account so many non-typical aspects, that it would become something similar to Borges’s one-to-one map of the world. Suburban edges growth occurs in simpler conditions, where growth factors can be reasonably typified into general categories and thus reliably modelled. Its peculiar features are low-density, mixed land use, incremental growth, fragmentation, multiplicity, and a considerable autonomy of the many, anonymous local actors (Boeri, Lanzani and Marini, 1993). Even when “rules” (plans, codes, laws) are applied – as it is in most part of Europe – the actual outcome in a built landscape is largely unforeseen, and maybe undesired. This is due to the constant interplay between rules and local actors, who freely “interpret” – actually re-write – any rule to adapt it to their own needs and wills (Boeri, 2000).

We can even portray suburbia as an “auto-organising” system (Lanzani, 1991). We mean by this that its change is neither mechanically directed from the outside as in a hierarchic system, in which the large scale determines the small scale (Holland, 1975), nor completely self-standing, with no actual interplay occurring between inside and outside. The notion of auto-organisation rests on the analogy between the city/territory and a biological organism, deeply rooted in the history of architecture (Kruft, 1985). The analogy traditionally leads to a mechanist or organic interpretation of the city/territory, according to the dominant interpretation of biological life. Cognitive sciences, stressing the importance of individuals versus the positivist dominance of species, according a central role to describing dynamic processes rather than states, relations rather than scopes, offer new keys to understanding of emerging territorial assets, by means of a deeply reviewed old analogy (Maturana and Varela, 1980).

Autonomy implied in the notion of territory as an auto-organising system is a serious challenge to urban planning and design: perhaps the most radical since planning and design have become specific disciplinary, technical and professional fields. It may be of some use to remember that our activity field is an historical construct, whose cultural roots can be traced to the XV century Italian Renaissance (Choay, 1986), but which became generalised, structured and socially recognised in the XVIII and XIX century’s processes of modern State organisation (Picon, 1988), reaching its peak in ‘50es post-war-reconstruction and ‘60es welfare societies (Picon and Desportes, 1997). A constant and profound relationship (dependence) on power and state has somehow inscribed hierarchy and authority in the genetic code of architecture and urban planning, making the final user (the citizen, the inhabitant, the visitor) into a voiceless quantity, incapable of individual choice (Tosi, 1994) (Hall, 1988). When asked by Doctor Jaoul’s little daughter about her room in the father's new house, Le Corbusier answered: "Go and play, little darling, *I* know what *you* need."

The difference between Doctor Jaoul's daughter and today's actors is that the latter can do, and actually do, without Le Corbusier's advice, shaping their own built environment.

Simulating suburban change means taking a step forward and taking into account the wide range of possible results a self-organising system can produce, and may also be a step forward in regaining a role in suburban growth for architects, planners and designers.

2. WHY A MODEL, WHAT FOR A MODEL?

Let's take an example. Driving from Kent to Veneto (from London to Venice) through Flanders, Netherlands, Baden-Wüttenberg, non-alpine Switzerland and sub-alpine Brianza, we would observe the recurrence of a new urban (or suburban) form of settlement, made up of small single-family houses (cottages, fermettes, boerderettes, villette) independent or clustered, commercial facilities, storage centres, small enterprises, showrooms, car sellers, interchange parking, sport grounds, leisure parks, high-tech farming etc, all of them aligned in strips of various depth along ring roads and intercity roads (Figure 1). The mix might locally be slightly different, and quality of buildings tends to decrease going southwards, but the result is roughly the same everywhere. Largely un-planned and incremental, this kind of linear suburban growth is just one of new urban types that underline the role mobility, accessibility - and thus infrastructure - have acquired in contemporary urban networks (ITATEN, 1996) (Baart and Metz, 2000).

Figure 1. "all of them aligned in strips of various depth along ring roads and intercity roads."

2.1 The Problem

How did this all happen? Not due to intentional planning. Infrastructure, of course, was planned (and in some cases, designed). Urbanisation was planned. Zoning was planned. Facilities were foreseen. Buildings where allowed, and designed. But everything happened in separated fields, with separated rules and knowledge. The final outcome was never taken into account as a whole, as a landscape. Neither where the role of local actors are taken into account. In fact, usually infrastructures where not meant to be urbanised. They became urbanised due to the constant pressure of a multitude of different actors and interests, any of them acting alone to get their fair share of (sub) urbanisation: families, small firms and mall chains, to mention the most frequent.

Could it have been avoided? We guess not. We otherwise cannot understand why it happened in countries with so different administrative systems, many of them really efficient in enforcing plans and rules. Could it have made into a different landscape? We guess it could have been.

What we learn from the strips' case is: first, that unexpected side- and cross-effects are more important than correctness of original choices (what happens on the sides of the highway is more crucial than the highway's section or trace); second, that far more actors than expected have been able to play a key-role, although their small-scale individual actions where not coordinated into a clear, large-scale strategy.

2.2 The Model

The capability of modelling different possible paths of suburban growth would allow planners and designers a pristine evaluation of the consequences of design and planning strategies, in their whole complexity. The model should not only allow simulation of possible future states - by simulating the multiplicity and diversity of local actors' reactions to design and planning choices - but also recursive interaction with policy, planning and design making by means either of successive corrections and adaptations, or of radical changes of strategy if needed (choice → simulation → evaluation → adaptation/change → simulation …). This means leaving the current "curative" approach of "damage containment" for a more effective preventive integrated design and planning strategy.

Modelling suburban phenomena is unfeasible, at least with the mechanicistic or reductionist apparatus. The magnitude of the phenomena, the interplay of the causes and effects, and the role of local actors in the comprehensive process is such that it requires new intellectual and experimental instrumentation.

We need an holistic approach to model suburban dynamics, considering the interplay between individual behaviours of local actors (e.g. the decision to settle, the choice of a location, the seek for visibility or, on the contrary, privacy, the choice of a building typology and construction system, the necessity for adaptation and further change when facing new needs, etc.) and global processes (e.g. general planning, socio-economic trends, market dynamics, infrastructure, mobility and accessibility etc.), up to regional scale and vice versa. Therefore major interventions are applied from the top-down, for instance planning and zoning actions, major infrastructure or facilities interventions, as in fact it still.

Furthermore, the model has to consider three-dimensional phenomena. Contemporary suburbanisation processes have clearly shown the limits of a bi-dimensional attitude, reducing planning and urban design to the mere outlay of zoning areas. But 3D modelling would also allow a more direct interplay between planners and designers on one side, decision-makers, citizens and users on the other. Their visual evidence would subtract evaluation and decision from the elite hands of technicians, making by means of a communicative language an open public discussion and participation possible. This should result in a further increase of efficacy, as public debate should no longer get stuck in positional conflict about "what" (are we for or against the new highway?) but evolve into open negotiation about "what/how" (what happens if the highway is built? and if not? who is damaged? are there possible compensations? can intersections be changed? can design be improved? …) (Bobbio, 1999). Far from disparaging the technician's competence, such a process would enhance its key role in fostering new and creative solutions, offering to public debate the wider possible range of assets.

Our approach to simulating suburban form relies on modelling different interacting process at various scales. In the view of the validation with real European scenarios, we are implementing two layered models, the Socio-Economic and Zoning model and the Suburban Form model, respectively by means of cellular automata and genetic programming.

3. SOCIO-ECONOMIC AND ZONING MODEL

The Socio-Economic and Zoning Model (SEZ) simulates exogenous factors and endogenous processes of large-scale suburban dynamics. The model approximates the area by means of a rectangular grid at the scale of hundred meters, i.e. square cells of 100 m side.

The space of the model is anisotropy: at each cell is associated a vector representing respectively actual use, location, accessibility and zoning status of the area, making it more or less suitable for development.

The dynamics is simulated by means of Cellular Automata: the state of any cell is related to the states of the neighbouring ones.

3.1 Background on Cellular Automata

Cellular automata were originally introduced by von Neumann (1963 and 1966) and Ulam (1974) as a possible model of biological systems.

For the purposes of the SEZ model, a cellular automaton is a discrete dynamical system. Space, time, and the states of the system are discrete. Each site –called a cell– in a regular spatial lattice or array –the grid– can have any one of a finite number of states. The states of the cells in the lattice are updated according to the rules. That is, the state of a cell, at a given time, depends only on its own state one time step previously, and the states of its nearby neighbours at the previous time step. All cells on the lattice are updated synchronously. Thus the state of the entire lattice advances in discrete time steps (Batty and Xie, 1994) (Cecchini, 1999) (Couclelis, 1997) (Engelen, White and Uljee, 1997) (Wagner, 1997) (Engelen,. Geertman, et al., 1999).

The mathematician's definition (Wolfram, 1994) of a cellular automaton is: a regular lattice of sites, where each site takes on k possible values, and is updated in discrete time steps according to a rule that depends on the value of sites r in some neighbourhood around it.

Conventionally, d=dimension, k=states per site, r=radius. In the SEZ model are assumed d=2, k=24 and r=8.

A d-dimensional cellular automaton takes as its underlying space the lattice S^Z (Z=integers, infinite in both positive and negative directions) where S is a finite set of k elements. The dynamics are determined by a global function

$F : S^Z \rightarrow S^Z$

whose dynamics are determined "locally". A "local (or neighbourhood) function" f is defined on a finite region

$f : S^{2r+1} \rightarrow S^Z$

The global function F arises from f by defining

$F(c)_i = f(c_{i-r}, \ldots, c_{i+r})$

3.2 Applying Cellular Automata to Socio-Economic and Zoning Model

In essence, the application of cellular automata to the SEZ model determines the status of an individual cell according to the states of the cells within its neighbourhood. The neighbourhood consequence is evaluated for each of the states per site, which the cell could be converted to. At run time the cellular automaton generates attraction or repulsion ("forces" dynamics) for each cell according to the associated status and transition rules. Since the dimension of the cell is a hectare, in the SEZ model the neighbourhood radius is 0.8 km. This size of the neighbourhood properly simulates micro-scale suburban dynamics, while larger or macro-scales can be simulated by means of coarser granularity of the model, i.e. larger cells. That is why, it is considering layering various cellular automata models of the same urban area, but at various scales and modelling different phenomena.

The model is developed by the European Commission's Space Application Institute in collaboration with RIKS Institute, and is validated with historical data series over the last 40 years for several European cities (Figure 2).

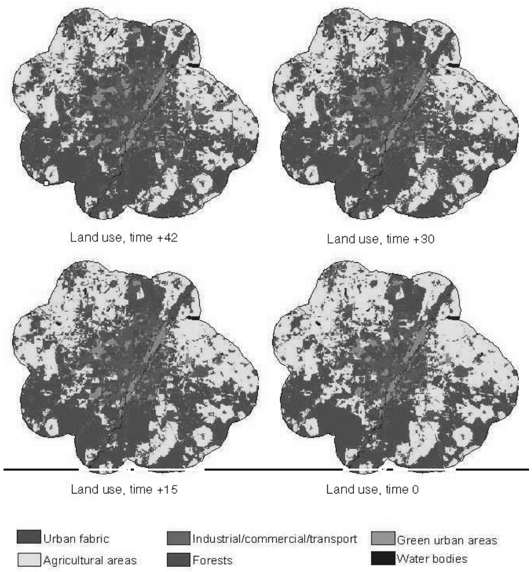

Figure 2. Munich, simulated land use over the last 42 years.

4. SUBURBAN FORM MODEL

The Suburban Form model (SF) uses a thinner grid than the SEZ model, at the scale of meters, and a three-dimensional grid instead of a bi-dimensional one. At each cube of the grid is associated its state (e.g. residential, industrial, commerce, tertiary, services, agricultural…). The 3D grid cell or cube is also referred to as a *voxel* (volumetric elements) (Jones, 1989).

The push to change, from the initial state, is an input from the SEZ model, measured as a delta in density for each land use: greater the demand-density of a certain destination for a specific area, greater is the "pressure" to change the existing status of the area (Figure 3).

The model pursues the maximisation of use and economic values for local actors.

A set of transition rules defines the constraints the model has to fulfil. They are global, apply to the entire model, or specific, domain dependent. These rules are:

– Building and zoning regulations (i.e. maximum heights, minimum distances etc.);
– Accessibility (e.g. secondary or private roads, entrance/s),
– Observed usual behaviours of local actors (e.g. search for visibility, use of simplified construction grids etc.);
– Typological and constructive recurrences (e.g. residential and industrial types, prefab building spans etc.);
– Specific criteria apply when the input from the SEZ model overcomes the capability of "greenfield" development by converting agricultural/unused areas and/or expanding existing buildings. These criteria shift from incremental (plot-by-plot) to radical change of whole areas, trying, for instance to aggregate fragmented plots or to intensify building by adopting more easily rented types. The greater the requirements from the SEZ model, stronger the pressure to change the present use and form of the area.

The matching of the global and specific criteria is an NP-hard optimisation problem (Garey and Johnson, 1979): finding the exact solution of the problem for a real urban scenario is prevented by exponential explosion. The SF model implements genetic programming to discover and learn best procedures to pursue the optimisation functions (Broughton, Tan, and Coates, 1997) (Jagielski and Gero, 1997).

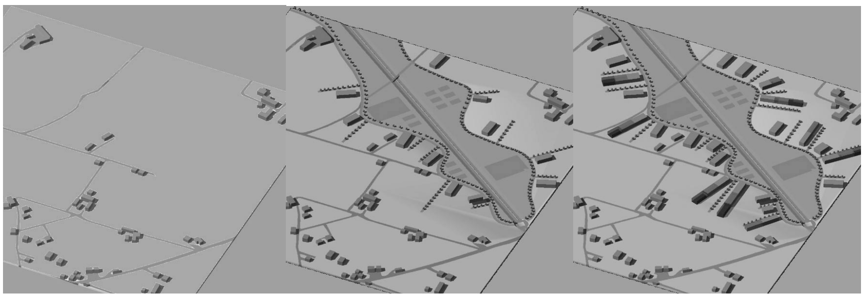

Figure 3. Suburban form simulation at time 0 (left), +5 (centre) and +15 (right).

4.1 Background on Genetic Programming

Genetic Programming (GP) is a biologically inspired methodology that automatically generates a computer program from a high-level statement of a problem's requirements. It was developed by John Koza (1992, 1994 and 1999), starting from the genetic algorithm described in Holland's book *Adaptation in Natural and Artificial Systems* (1975). Koza suggests that the desired program should evolve itself from the initial statement/s through the evolution process. Instead of writing the program to fulfil the requirements and pursue the goals of the SF model, GP methodologies search the space of possible computer programs for best ones, according to the defined goals. A population of executable computer programs is created, satisfying the given requirements. The solutions provided by the individual programs are evaluated, one against the others: worst programs are abandoned, while the best ones further "reproduce" themselves according to genetic laws, i.e. sexual recombination (crossover), mutation, gene duplication or gene deletion. The reproduction and evaluation process is repeated over and over, until the results, the goals of the model are satisfactory approximated.

General GP methodology can be summarised in the following three steps (Koza, 1994):

1. Randomly create an initial population of individual computer programs.
2. Iteratively perform the following substeps on the population of strings until the termination criterion has been satisfied:
    - Assign a fitness value to each individual in the population using the fitness measure.
    - Create a new population of strings by applying the following three genetic operations. The genetic operations are applied to individual string/s in the population selected with a probability based on fitness (with reselection allowed).
        - Reproduce an existing individual string by copying it into the new population.
        - Create two new strings from two existing strings by genetically recombining substrings using the crossover operation at a randomly chosen crossover point.
        - Create a new string from an existing string by randomly mutating the character at one randomly chosen position in the string.
3. Designate the string that is identified by the method of result designation (e.g., the best-so-far individual) as the result of the genetic algorithm for the run. This result may represent a solution (or an approximate solution) to the problem.

4.2 Applying Genetic Programming to Suburban Form Model

The comprehensive suburbanisation processes are simulated disaggregating them into subproblems, functions in the GP terminology, each considering an individual urbanisation objective or task. These functions are divided into four categories: plot aggregate, road, building, and parking.

The functions are competitively applied to the initial area in a concurrent process. Namely the functions in the SF program concurrently work on the area, and eventually its adjacent plots, until the building and its facilities are completed. The suburbanisation process, generated by each individual computer program, is evaluated to determine the fittest. Until the goals of the model are satisfactory approximated, the evolution process is iterated and a new generation of computer programs is created by means of reproduction, crossover and mutation. Since a constrained syntactic structure is involved, crossover is performed so as to preserve this syntactic structure in all offspring.

4.2.1 Initial Area

An initial area is the given input to the GP program. This area has a shape, from cadastral maps, a possible new destination and an incremental density (from the SEZ model).

4.2.2 Plots Aggregate

Of the initial area it is evaluated whether the new destination and density are compatible with (1) eventually existing buildings (2) the area extent. If not, the “plot aggregate” function is applied trying to enlarge the area aggregating adjacent plots, minimising the required financial investment.

4.2.3 Road

Accessibility to the area is considered in relation to the (new) destination and density. The “Road” function tries to maximise the accessibility creating new roads, minimising the economic cost, i.e. the land occupation vs. minimum path to building entrance/s and eventual facility area/s.

4.2.4 Building

The “Building” function inserts a new building in the area. This function is the root of a subtree of functions maximising respectively (1) building and zoning regulations (i.e. maximum heights, minimum distances etc.), (2) usual behaviours of local actors (e.g. search for visibility, use of simplified construction grids etc.), (3) typological and constructive recurrences (e.g. residential, tertiary or industrial types, prefab building spans etc.).

4.3 Domain Knowledge and Genetic Programming

GP offers techniques to search the space of computer programs for the fittest, but this search is “blind”. Thus it is advantageous to incorporate some knowledge to reduce or lead the search. Because the space of possible computer programs which GP methodologies explore is huge, the search can take too long to pursue a satisfactory solution. Furthermore GP can miss promising solutions, which design practice teaches us are effective.

We are working to incorporate two kinds of knowledge, respectively on the suburbanisation processes, and on spaces and objects.

The knowledge on the process can be represented into the formalisation of the problem, which GP has to search, in variation operators known as useful, or into the performance index in the form of known behaviours, for instance behaviour of local actors or typological and constructive recurrences. Incorporating such knowledge focuses the genetic search, producing a more effective exploration of the space of computer programs.

The knowledge on spaces and objects is given directly by the architect or planner, whose experience can suggest promising solutions. These solutions are often the result of a creative synthesis, but often without awareness of the underlying creative process. To incorporate this knowledge in the SF model, we are pursuing direct designer’s intervention during the GP process. The designer acquires more direct control of the GP process through:

- display of the solutions provided by the individuals programs of the current generation,
- pause of the genetic process,
- modification of a design solution/s or the creation of a new one,
- continuation of the GP process with a new generation considering the modification manually introduced.

To implement these four steps, the system should integrate:

- visual clustering of the individuals, as they could sum to hundreds of thousands for each generation,
- an editor to interact with the form and its attributes,

– a map the changes from the editor inside the internal GP representation, which is an executable program.

Our present approach is associating the user's commands in the editor directly to executable statements, which the GP can immediately reuse.

5. CONCLUSIONS

To model suburban processes we are facing some core problems posed to modern science: the magnitude of the phenomena, the interplay of causes and effects and the role played by local actors and phenomena in the comprehensive process. The title "*evolutionary automata"* states our exploration in instrumentations for simulating the peculiar processes of production of space in a self-organising territory, by means of cellular automata and genetic programming. This integration is still undergoing.

The SEZ model is in the calibration phase, and is proving good results for several European cities. The analysis of the validation and calibration processes is behind the scope of the present paper (cf. White, Engelen, et al., 1999).

The SF model is still in an "accumulative" phase, where knowledge-rules on typological and constructive recurrences (e.g. residential, industrial types, commercial etc.) are added to the model. The model still has to be put through the experimental cycle: run the model, analyse results, compare them with reality... Genetic programming is proving its capability of synthesise-design suburban environments, entailing the site, accessibility and building, considering topology, dimension and layout issues. For prototypal applications, where it is relevant the capability to quickly change and experiment with the rules and parameters of the model, GP is demonstrating demanding for both the users, because translating high-level rules into executable structures requires a major conceptualisation effort, and for the computers, because GP search is very CPU demanding for the creation of the population and the generations.

On the other hand, it is remarkable the capability of GP to generate innovative solutions and to discover regularities in the space problem. We expect experimentation with a greater number of cases to shed light on rules underling the evolution of suburbanisation dynamics with validity transcending the specific cases.

If GP will demonstrate, through the calibration and validation phases, an acceptably accurate instrument for 3D dynamic and interactive simulation, it would offer an effective tool for decision-making and design improvement at an intermediate scale, between the region and the single plot. The scale where built landscapes are shaped, planning decisions get physical and have to interact with local actors, environment becomes a specific site. The scale where the current crises of planning and urban design facing multiplicity and autonomy is taking place.

6. REFERENCES

Baart, T., and T. Metz, 2000, *Atlas van de verandering*, NAI Uitgevers, Rotterdam.

Batty, M. and Y. Xie, 1994, "From cells to cities", *Environment and planning B*, 21, pp. 31-48.

Bobbio, L., *La democrazia non abita a Gordio*, Franco Angeli, Milano.

Boeri, S., 2000, *USE – Uncertain States of Europe*, in: *Mutations*, Centre Arc en Réve, Bordeaux.

Boeri, S., A. Lanzani and E. Marini, 1993, *Il territorio che cambia*, Abitare Segesta, Milano.

Broughton, T., A. Tan, and P.S. Coates, 1997, "The Use of Genetic Programming in Exploring 3D Design Worlds", in: Junge, R. (ed.), *Conference Proceedings of CAAD Futures 1997*, München, pp. 885-915.

Cecchini, A., 1999, *Meglio meno, ma meglio automi cellulari e analisi territoriale*, FrancoAngeli, Milano.

Choay, F., 1986, *La regola e il modello*, Officina, Roma.

Couclelis, H., 1997, "From cellular automata to urban models: new principles for model development and implementation", *Environment and Planning B*, 24, pp. 165-174.

Engelen, G., R. White and I. Uljee, 1997, "Integrating constrained cellular automata models, GIS and decision support tools for urban planning and policy-making", in: Timmermans H. (ed.), *Decision Support Systems in Urban Planning*, E&FN Spon, London, pp. 125-155.

Engelen, G., S. Geertman, P. Smits and C. Wessels, 1999, "Integration of GIS and Dynamic Modelling for Regional Planning, in: Geertman, S., S. Openshaw and J. Stillwell (eds.), *Geographical information and planning*, Springer, Berlin.

Garey, M. and D. Johnson, 1979, *Computers and Intractability*, W.H. Freeman, San Francisco.

Hall, P., 1988, *Cities of tomorrow*, Basil Blackwell, Oxford.

Holland, J., 1975, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor.

ITATEN, 1996, "Indagine sulla trasformazione del territorio italiano", in: A. Clementi, G. Dematteis and P.C Palermo (eds.), *Le forme del territorio italiano*, Laterza, Roma-Bari.

Jagielski, R. and J.S. Gero, 1997, "A Genetic Programming Approach to the Space Layout Planning Problem", in: Junge, R. (ed.), *Conference Proceedings of CAAD Futures 1997*, München, pp. 865-884.

Jones, C.B., 1989, "Data Structures for Three-Dimensional Spatial Information Systems in Geology", in: *International Journal of Geographical Information Systems*, 3, 1, pp. 15-31.

Koza, J., 1992, *Genetic programming: on the programming of computers by means of natural selection*, MIT Press, Cambridge (Ma).

Koza, J., 1994, *Genetic programming II: automatic discovery of reusable programs*, MIT Press, Cambridge (Ma).

Koza, J., 1999, *Genetic programming III: darwinian invention and problem solving*, Morgan Kaufmann, San Francisco.

Kruft, W. A., 1985, *Geschichte der Architekturtheorie*, Beck, München.

Lanzani, A., 1991, *Il territorio al plurale*, Franco Angeli, Milano.

Maturana, H. R., and F. J. Varela, 1980, *Autopoiesis and cognition. The realization of the living*, Reidel, Dordrecht.
Picon, A., 1988, *Architectes et engenieurs au siècle des lumières*, Presses de l'École National des Ponts et Chaussées, Paris.
Picon, A., and M. Desportes, 1997, *De l'espace au territoire*, Presses de l'École National des Ponts et Chaussées, Paris.
Tosi, A., 1994, *Abitanti*, il Mulino, Bologna.
Ulam, S., 1974, "Some ideas and prospects in biomathematics", *Ann. Rev. Bio.*, 255.
von Neumann, J. 1963, "The general and logical theory of automata", in: von Neumann, J. *Collected Works*, *Design of computers, theory of automata and numerical analysis*, Taub, A. H. (ed.), Pergamon, Oxford.
von Neumann, J. 1966, *Theory of Self-Reproducing Automata*, Burks, A. W. (ed.), University of Illinois Press, Urbana.
Wagner, D., 1997, "Cellular automata and geographic information systems", *Environment and Planning B*, Vol 24, pp. 219-234.
White, R., G. Engelen, I. Uljee, C. Lavalle and D. Erlich, 1999, Developing an Urban Land Use Simulator for European Cities, JRC Technical Paper, Ispra.
Wolfram, S., 1994, *Cellular Automata and Complexity*, Addison-Wesley Publ., Reading (Ma).