The Design Space of Schematic Palladian Plans for Two Villa Topologies

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Abstract
Given the plan topology for the layout of the rooms of a Palladian villa, we ask what is the design space of possible Palladian plans. What does this space look like in terms of dimensions and proportions? Two plan topologies are examined to throw some light on this question. One is that of the Villa Angarano while the other is that of the Villa Badoer. A Maple program was written for each topology to solve the equations for room proportions, to cycle through the possible proportional spacing of the underlying Tartan grid, and to plot out the possible plans in the design space. The programs eliminate from the design space those alternatives that violate basic Palladian constraints, such as no room having an aspect ration greater than 2:1, and such additional constraints that we and other authors have found. A selection of the plotted output of plans in the design space for each topology is presented as well as three-dimensional plots showing the number of plan alternatives in different regions of the critical parameter space, namely, the length and aspect ratio of the plan. We believe that this is the first instance of enumerated Palladian plans which goes beyond topology to examine issues of dimension and proportion. One conclusion is that one cannot scale a Palladian plan topology to fit any set of overall dimensions.

Keywords
Palladio, Design Space, Villa, Plans, Rule-Based Generation

1 Introduction

In 1978 Stiny and Mitchell developed a shape grammar, a set of graphical rules, to encapsulate the style of Palladian floor plans in designs generated by the grammar. This is probably the first attempt to apply the concept of shape grammars to architectural style. (Stiny and Mitchell, 1978). Stiny and Mitchell proved the viability of shape grammars for representing style by demonstrating that several known Palladian villas can be generated by the grammar and that some possible villa plans generated by the grammar were also convincingly Palladian. They did not prove or demonstrate that every plan generated by the grammar was convincingly Palladian. As Hillier has pointed out

"In the last analysis, architectural theory is a matter of understanding architecture as a system of possibilities, and how these are restricted by laws which link this system of possibilities to the spatial potentialities of human life. At this level, and perhaps only at this level, architecture is analogous to language. Language is often naively conceptualized as a set of words and meanings, set out in a dictionary, and syntactic rules by which they may be combined into meaningful sentences, set out in the grammars. This is not what language is, and the laws that govern language are not of this kind. This can be seen from the simple fact that if we take the words out of the dictionary and combine them in grammatically correct sentences, virtually all are utterly meaningless and do not count as legitimate sentences. The structures of language are the laws which restrict the combinatorial possibilities of words, and through these restrictions construct the sayable and the meaningful. The laws of language do not therefore tell us what to say, but prescribe the structure and limits of the sayable. It is within these limits that we use language as the prime means to our individuality and creativity." (Hillier, 1996)

This quotation from Hillier frames the issues to be explored in this paper by means of a detailed study of two particular Palladian plan typologies, namely the typology of the villa Angarano and that of the villa Badoer. We will be looking at the issue of constraints ("the laws which restrict the combinatorial possibilities") and how this reduces the "system of possibilities", that is to say, the design space of possible Palladian plans for those typologies. Here we are using the term, design space, in the sense that it is used in information science as the collection of design alternatives. What we would like to demonstrate in this paper is what the design spaces for these two typologies are like: How many designs do they each contain and what do they look like? Providing a better understanding of design spaces is of great importance to future computer based design systems which might assist a designer with stylistic guidance in laying out designs and in detailing without the designer having to worry about every detail. We do not think that rule-based generative design systems will be able to design without human intervention simply because the design space is usually very large. Consequently, the job of the designer is to select from a vast design space. It is through the provision of constraints over and beyond those that define a particular style that a design for a specific site and client comes about (one can say that, in effect, what designers normally do is to select from a vast design space my means of trained intuition).

Now Stiny and Mitchell's shape grammar is a set of shapes and rules for the spatial relationships between the shapes. To use the language analogy, one can say that the shape grammar contains both the words of a language and the syntax. The grammar includes some constraints, in the form of labels and markers, which restrict when the rules can be applied. One can think of these constraints as local constraints because they influence the emerging design locally where a rule is applied. In view of the local nature of the built-in constraints one cannot expect these constraints to say anything about the overall design. It may well be that there should be some constraints operating at the overall level of a design to distinguish Palladian from non- or neo-Palladian designs. It is also at this overall level that a designer would choose one design over another in the design space to select a design most suitable for a particular client and site. Mitchell has described this process as one of applying predicates to determine if designs are acceptable (Mitchell, 1990). Now the question is whether or not the Palladian shape grammar should include some constraints operating at an overall level to limit the design space

of possible plans to Palladian plans. Does it include non-Palladian plans?

To answer this question, consider briefly the structure of Stiny and Mitchell's Palladian grammar. The grammar begins by laying out a tartan grid. As Wittkower showed in 1949 (Wittkower, 1949), the plans of Palladian villas can be thought of as being underlain by a tartan grid along which the walls align. Palladio, in the Four Books of Architecture, always laid his plans out with the axis of symmetry running vertically. Given that all Palladian plans have a large room, the *sala*, on the axis of symmetry, the underlying tartan grid must have an odd number of cells running horizontally. Possible grid dimensions that Palladio has used are 5x3, 5x4, 3x5 and 5x6, where the first number is the horizontal number of cells. Next Stiny and Mitchell concatenate some cells and possibly shift some walls off the grid. All the rule applications up to this point define the topology of the spaces. On this basis Stiny and Mitchell have enumerated all the possible Palladian plan topologies for 3 x 3 and 5 x 3 topologies (Stiny and Mitchell, 1978a). After the basic topology of the room layout has been derived, additional rules are applied for the following (Mitchell, 1990):

- Exterior wall definition and detailing
- Principal entrances and wall inflections
- Exterior columns
- Windows and doors
- Terminators (determine when rule application ceases).

A commendable feature of the Stiny and Mitchell shape grammar is that it is parametric, meaning that it is independent of the actual dimensions of the plan and the spacing of the underlying tartan grid. It therefore provides a clear overview of the design space of possible Palladian plans in terms of plan topology without the obfuscation that would be caused by throwing in specific dimensions. Stiny and Mitchell implied that the addition of dimensions "following Palladio's well known rules for proportions" had no implications for the design space of possible plans. (Mitchell, 1990a).

More than a decade after Stiny and Mitchell developed their Palladian shape grammar, Hersey and Freedman developed a computer program that can generate Palladian (including some non-Palladian) plans and corresponding elevations. Unlike the Stiny and Mitchell approach, Hersey and Freedman include specific dimensions right from the start by a specification of the length and width of the overall plan. A specific layout is then generated by a series of vertical, horizontal and combined vertical/horizontal splits. The proportions used for the splits are based on the proportions and the probability with which Palladio used them. Hersey and Freedman's software does not therefore lend itself to systematically enumerating possible plans but it does show, if one generates enough plans, what Palladio was likely to do. It immediately becomes clear that the Palladian shape grammar does not contain sufficient constraints to generate only Palladian plans. Indeed, even at the level of topological possibilities, it is clear that some, such as plans with rooms that span the entire plan are not Palladian. More importantly, however, Hersey and Freedman show that inclusion of dimensions brings out the need for these constraints, some of which are local in nature and some of which affect the overall plan:

- No room dimensions is less than seven Vicentine feet
- The ratio of the smallest to the largest room area is 9 or less.
- No room should have an aspect ratio greater than 2:1
- Rooms on the axis of symmetry tend to be larger than adjacent ones
- Other rules that they discovered are:
- The total number of rooms in a plan is equal to or less than 20.
- The total number of interior rooms without windows is equal to or less than 4.

Several additional constraints become evident when using the Hersey and Freedman software (Seebohm, 1991). For example, there should not be a series of rooms, much smaller than the *sala*, encircling the *sala*. The *sala* must be on the perimeter of the plan (for light) or at most one room removed from the perimeter so that the adjacent room can serve as a vestibule of or recessed portico. Close observation of Palladio's plans has shown, as well, that Palladio did not insist on the room proportions that are recommended in his Four Books of Architecture (i.e. 1:1, 1:v2, 4:3, 3:2, 5:3, 2:1) as pointed out by Hersey and Freedman (Hersey and Freedman, 1990) and by March who

showed that Palladio used ninety different ratios in the Four Books of Architecture (March, 1998). The only real constraint seems to be that he rarely exceeded an aspect ratio of 2:1 (Even this ratio is occasionally exceeded as March has shown). Nor does he necessarily use sequences of dimensions that form musical ratios as Wittkower had proposed in 1949 (Wittkower, 1949). Instead, it seems highly likely, that the sequences that were used had many other connotations from forming various harmonic ratios to number sequences with cosmological implications as recently shown by March (March, 1998). As evidence for the importance of number sequences based on harmonic and geometric means one need only look at the pages and pages that Daniele Barbaro devoted to this subject in the commentary on Vitruvius that he worked on with Palladio (March, 1998a).

It seemed to us that, in the light of the importance of dimensions and the emergence of additional constraints, that it would be desirable to enumerate the complete design space for some specific villa typologies on the assumption that the design space of all typologies would be unfathomable. In addition, we felt that it would be desirable to somehow classify the designs to clearly characterize the design space. Considering that

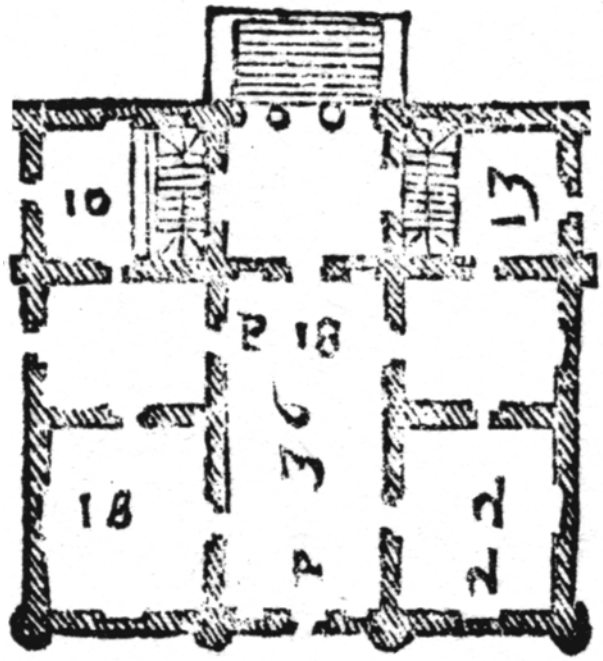


Figure 1. Plan of the Villa Angarano from the Four Books of Architecture

Hersey and Freedman's software does not lend itself to systematic enumeration, we set out to write some software of more limited objectives that would focus specifically on the typologies of the Villa Angarano and the villa Badoer. The detailed study was undertaken as the final thesis in his program of Independent Studies at the University of Waterloo by the second author (Chan, 2000).

2 Two Plan Typologies

The Villa Angarano was built around 1548 at Bassano del Grappa to the North West of Venice (Holberton, 1990). It has been completely altered from the original shown in the Four Books of Architecture. From the plan of the villa shown there one can calculate the exterior dimensions of the villa using the room dimensions and assuming that the walls are two Vicentine feet thick. The resulting exterior dimensions are 62 Vicentine feet in horizontal width and 55 feet in length. The plan of the villa is reproduced below:

This is a two story villa implying that the stairs lead to the second story as well as to the basement. Unlike most of the villas, this one does not show a podium into which the basement would normally extend. The large room on the first floor would most likely have served as an entrance hall for receiving visitors while an identical room above would have been the *sala* for formal gatherings. The plans of each floor would have been identical. The mid-sized rooms would have served multiple purposes, not designated for specific functions as rooms in North America or Europe. They could have been used as sitting rooms or libraries. The smallest rooms at the back on the ground level, with their doors leading to the porticos that lead to the adjoining *barchesse* or barns, seem to serve as vestibules. The same rooms on the floor above could have been small studies or even bedrooms. Often Palladian villas have three sizes of rooms, the *sala* the largest, then the *stanze*, rooms serving as sitting rooms, then *camere*, serving as bedrooms and finally, *camerini*, the smallest rooms. These latter were used as storage rooms, washrooms or servants' rooms. In this villa one size of room, the *camere* or the *camerini* are missing.

The Villa Badoer was built between 1556/7 and 1563 at Fratta Polesine (Rovigo) to the South West of Venice (Holberton, 1990a). It still exists in the form it was built complete with the surrounding *barchesse* which form an enclosed court or *cortivo* in front of the Villa. This Villa sits on a podium derived from an earlier foundation. It is a one-story villa with a basement in the podium and a mezzanine floor above the main floor. Such mezzanines were used for the storage of grain and for housing servants. The basement accommodated the kitchen and related storage. The calculated exterior plan dimensions are 74 feet in width horizontally and 48 1/2 feet in length. The plan of the built villa is as shown in the Four Books of Architecture, except for the rear portico and stairs which were not built and the interior walls which are less than two feet thick.

3 Solving for Possible Plans

3.1 Objectives

What we wanted to do is to start where Stiny and Mitchell left off by investigating what would happen if, given the length and width of a plan, one solved for all the possible Palladian plans that satisfy known constraints. We would not go so far as to add wall inflections, door and window openings and entrance details, because our objective is to take a specific plan typology (Villa Angarano and Villa Badoer) and to tabulate all the possible dimensioned schematic plans. That is to say, we wanted to enumerate the design space of all possible schematic plans for a particular topology.

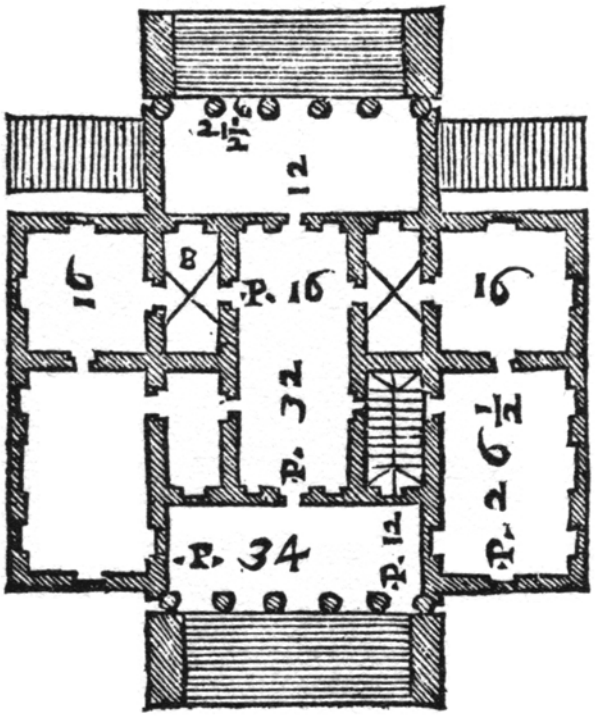


Figure 2. Plan of the Villa Badoer from the Four Books of Architecture

3.2 Assumptions

Considering that Palladio did not use his recommended ideal room proportions in many cases, we have assumed that a continuum of room proportions is acceptable as long as the aspect ratio of length to width is equal to or less than 2:1. Thus the problem of solving for specific dimensioned schematic plans is not that of solving for all the possible plans with rooms having the exact ideal proportions by means of a set of simultaneous linear integer equations (Mitchell, 1990). We assume that we can approximate the set of possible schematic plans for a particular length and width of the plan by cycling through the possible integers defining the proportional spacing between the cells of the underlying grid. Thus, for the plan topology of the Villa Angarano below, we can generate all the possible schematic plans by cycling through the integer values M and N for the vertical spacing and the values, F, G, and H for the horizontal spacing, where the integers range between 1 and 5. The use of integers

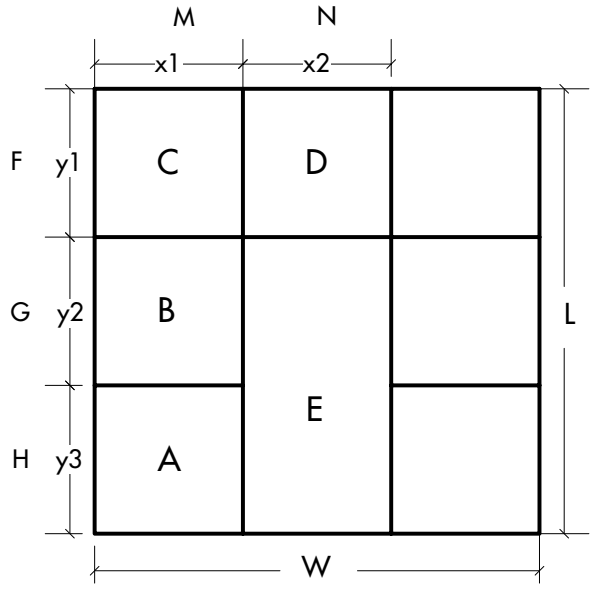


Figure 3. Plan topology of the Villa Angarano

for the values of M, N, F, G, and H is, of course, an approximation to the actual ratios found in Palladio's villas but we felt that the differences are not visibly noticeable. Certainly, the differences are not noticeable in plan drawings (Hersey and Freedman, 1990a). Small variations from these integer proportions can be looked at as variations on or shifts from essentially the same proportional scheme. By using only integer proportions for grid spacing, the set of possible schematic plans is kept finite and not too large. This in turn allows us to determine the effect of plan dimension and aspect ratio on the finite number of possibilities.

Note that by looking at schematic plans, we are ignoring wall thickness. Adding wall thickness would not change our analysis because we can think of our grid spacing as the spacing between the walls. Including wall thickness would mean that we would have to compensate the width and length of the plan by accounting for the number of wall thicknesses that contribute to the length and width. For the small schematic plans presented below, wall thickness is not visually noticeable.

3.3 Methodology: Villa Angarano

We cycle through all the likely values of the integers, M, N, F, G, and H for a given width and length of the plan (or, in our case, a given length and aspect ratio, P = length/width). In fact, we cycle through the integers three times, once for the location of the stairs in each of the three horizontal bands of rooms. These integers and the length and aspect ratio of the plan are the independent variables. We then set up a system of simultaneous equations to solve for the dependent variables which are x1, x2, y1, y2, y3, a, b, c, d, and e where the a, b, c, d, and e are the aspect ratios of the rooms. We solved the system of 10 equations in 10 unkowns with Maple, the software program for symbolic mathematical computation. The solutions to the equations are presented in Appendix A. A Maple program was written to cycle through the integer proportions and to plot small schematic plans and to print the associated parameters. As the program cycles through every room, every room in every plan is checked to see if it meets certain constraints. As well, some relationships between rooms are checked for other constraints. In addition to cycling through different proportional spacing, the Maple program for this villa was also written to cycle through all possible locations of the stairs (These are not shown in the schematic but can be located in any of the three rooms in the suite of rooms to either side of the *sala*. For symmetry reasons, Palladio always placed stairs symmetrically about the central axis, although in practice both sets of stairs were not always built.)

3.4 Methodology: Villa Badoer

The plan topology of the Villa Badoer is as shown below (Figure 4).

For this typology we cycle through the integers M, N, O, Q, R, and S for a given width and aspect ratio, P. Again these integers and the length and aspect ratio of the plan are independent variables. Simultaneous equations were set up in Maple to provide solutions for the dependent variables y1, y2, y3, x1, x2, x3 and a, b, c, d, e and f, where the last six are the aspect ratios of the rooms. The solutions are shown in Appendix B. As for the villa Angarano, the program for this villa topology cycles through the possible locations of stairs in rooms C and D.

3.5 Constraints

The constraints that were applied are as follows:

- The *sala* is the largest room, namely, room E in both villas (There is probably a maximum size for this room but we have not had to rely on such a constraint).
- The aspect ratio of any room is equal to or less than 2:1 except as noted.

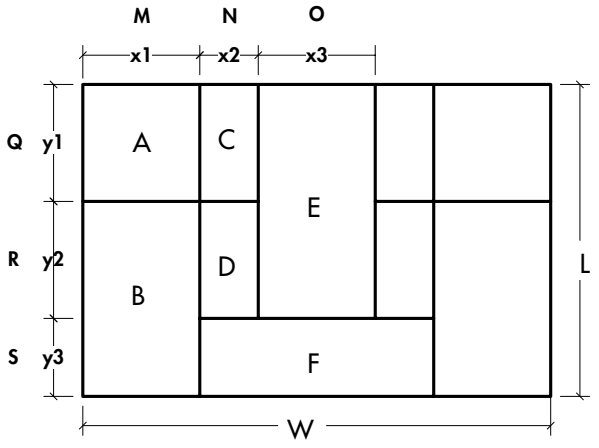


Figure 4. Plan Topology of the Villa Badoer

- The width of the cells in the underlying grid lying on the axis of symmetry are wider than adjacent cells (A long, thin *sala* can result if the width was not constrained this way).
- No room has dimensions that are less than 7 feet.
- No room is smaller than 40 square feet
- Rooms surrounding the *sala* are not identical in shape (We have not allowed two identical rooms side by side because that occurs only in two early villas, the villa Godi and the villa Gazotti and one later one, the villa Mocenigo).
- The ratio of the area of the largest room, the *sala*, to that of the smallest room is 9:1 (Hersey and Freedman, 1990b).
- The y dimension of the *sala* is at least 1.6 times the y dimension of an adjacent recessed portico.
- In the Villa Angarano typology, the length of the stairs is the same as the y dimension of the room.
- Rooms with stairs are between 11 and 19 feet in the y dimension (the length of the stair) and at least twice as wide as the stairs whose width is taken as 7 feet (based on stairs in the Four Books of Architecture)

4 Results

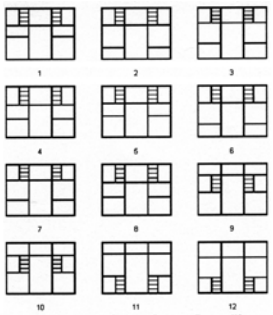
4.1 Plans from the Design Space of the Villa Angarano

The program for enumerating the schematic plan alternatives and removing cases that did not satisfy the constraints was written to plot schematic plans and to list the relevant parameters for each plan beneath as shown in the Figure 5 below. Note that when cycling through the integers M, N, F, G, and H it is easy to obtain configurations which are multiples of another plan such as 2:4:2 which is the same as 1:2:1. Such duplicates are culled from the enumerated set of schematic plans. Also note that in the parameter data, the dimensions for the room with the stair do not include the stair, just the space that remains. All the plots are scaled so that the vertical dimension is always the same in order to fit the large number of alternatives on a reasonable amount of paper and still be legible. Hence they do not show the relative sizes.

We tried to provide some structure in the display of the schematic plan alternatives to make the design space for each plan size and overall aspect ratio visually more comprehensible. We tried displaying by sorting the plans by the width of the middle row of rooms and, secondly, by the position of the stairs. The sampling of plans shown below is a small selection from a set that ranges in the width of the plan from 10 to 200 feet in 10 foot increments and in aspect ratio from 1:20 to 1:2 in 1:20 increments.

There seems to be no clustering of the design space to make it visually more comprehensible in any obvious way beyond sorting plans by absolute size and overall aspect ratio and within plans as we have done. Room proportions change continuously in discreet steps from one plan to the next. One possibility that has been suggested to us is to do a cluster analysis. We could try looking for clusters based on groupings of certain combinations of plan size, overall aspect ratio and the aspect ratios of the various rooms. It is not clear, however, why any of these parameters would operate in concert.

One could probably make the design space more comprehensible by reducing this set further with additional constraints. For example, Palladio was very particular about the amount of change in the size of rooms as one went from one room to the next. He liked, for example, to have transitions such that the long side of one room becomes the short side of another room. Of course, if one looks at these schematic plans as a designer with a specific client and site in mind, then the alternatives would be reduced very quickly. For example orientation to light would have a significant effect because it was the custom to have small rooms face South so that they could be easily heated in winter and large rooms face North so that they would stay cool in summer.



plan	num	M	N	F	G	H	A	B	C	D	E	X1	X2	Y1	Y2	Y3
1	1	1	1	3	2	4	1	1/2	75/58	3/4	3/2	50/3	50/3	25/2	25/3	50/3
2	2	1	1	3	4	2	1/2	1	75/58	3/4	3/2	50/3	50/3	25/2	50/3	25/3
3	3	1	1	3	4	3	27/40	9/10	135/116	27/40	63/40	50/3	50/3	45/4	15	45/4
4	4	1	1	4	3	4	9/11	27/44	450/319	9/11	63/44	50/3	50/3	150/11	225/22	150/11
5	5	1	1	4	3	5	15/16	9/16	75/58	3/4	3/2	50/3	50/3	25/2	75/8	125/8
6	6	1	1	4	5	3	9/16	15/16	75/58	3/4	3/2	50/3	50/3	25/2	125/8	75/8
7	7	1	1	4	5	4	9/13	45/52	450/377	9/13	81/52	50/3	50/3	150/13	375/26	150/13
8	8	1	1	5	4	5	45/56	9/14	1125/812	45/56	81/56	50/3	50/3	375/28	75/7	375/28
9	9	1	1	2	3	4	1	75/58	1/2	1/2	7/4	50/3	50/3	25/3	25/2	50/3
10	10	1	1	3	4	5	15/16	75/58	9/16	9/16	27/16	50/3	50/3	75/8	25/2	125/8
11	11	1	1	2	4	3	75/58	1	1/2	1/2	7/4	50/3	50/3	25/3	50/3	25/2
12	12	1	1	3	4	3	135/116	9/10	27/40	27/40	63/40	50/3	50/3	45/4	15	45/4

Figure 5. Schematic Plans for the Villa Angarano Typology (P = 3/4, width = 50 feet, stair width = 7 feet)

4.2 Three-dimensional Plots of the Total Number of Possibilities for the Villa Angarano

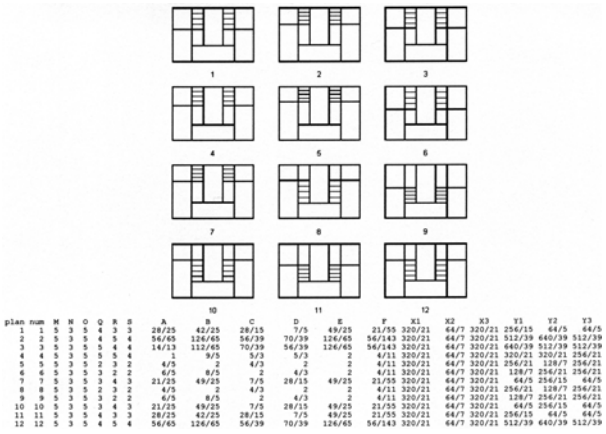
By plotting number of schematic plan alternatives versus the width and aspect ratio of the plan one can see in a dramatic way, how the overall dimensions affect the number of possibilities. A major influence on the number of possibilities is the fact that the stairs have to fit the length of a room. Figure 6 shows such a plot (It was obtained by exporting the number of plan alternatives for each value of overall length and aspect ratio to a spreadsheet and then using the built-in plot functions). The maximum number of possibilities is 120 for plan with a length of 60 and an aspect ratio of 3/4.

4.3 Plans from the Design Space of the Villa Badoer

As for the Villa Badoer, a Maple program was written to cycle through the integer values for the proportional spacing between the grid lines. Again, multiples with essentially the same proportional spacing were removed. For this scheme possible schematic plans for the width of plan ranging from 10 to 400 (in 10 foot increments) with the aspect ratio, P, ranging from 0.05 to 2 were attempted. An interesting result for this scheme is that it is only possible to obtain schemes with the aspect ratio of room F less than 2:1 for plan dimensions of 60 x 45 feet and 70 x 52.5 feet for a total of four plans (In Palladio's villa Badoer as shown in the Four Books, the aspect ratio of this room, the recessed portico, is 34:12, that is, almost 3:1). We therefore relaxed the aspect ratio constraint on room F when plotting possible schemes, a selection of which is shown in Figure 7.

4.4 Three-dimensional Plots of the Total Number of Possibilities for the Villa Badoer

Figures 8 and 9 show plots of the number of possible schematic plans versus the width of the plan and the aspect ratio (The peak values are 59 and 611, respectively). In Figure 8 the proportion of room F is constrained not to exceed 3.1 while in Figure 9 the proportions of this room are unconstrained. If we remove all constraints except the one requiring all rooms to have an aspect ratio of less that 2:1 we obtain a design space characterized by Figure 10 where the peak value is 4052 possibilities. Note that only requiring room aspect ratios to be less than 2:1 still leads to an op-



plan	num	M	N	O	Q	R	S	A	B	C	D	E	F	X1	X2	X3	Y1	Y2	Y3
1	1	5	3	5	4	3	3	28/25	42/25	28/15	7/5	49/25	21/55	320/21	64/7	320/21	256/15	64/5	64/5
2	2	5	3	5	4	5	4	56/65	126/65	56/39	70/39	126/65	56/143	320/21	64/7	320/21	512/39	640/39	512/39
3	3	5	3	5	5	4	4	14/13	112/65	70/39	56/39	126/65	56/143	320/21	64/7	320/21	640/39	512/39	512/39
4	4	5	3	5	5	5	4	1	9/5	5/3	5/3	2	4/11	320/21	64/7	320/21	320/21	320/21	256/21
5	5	5	3	5	2	3	2	4/5	2	4/3	2	2	4/11	320/21	64/7	320/21	256/21	128/7	256/21
6	6	5	3	5	3	2	2	6/5	8/5	2	4/3	2	4/11	320/21	64/7	320/21	128/7	256/21	256/21
7	7	5	3	5	3	4	3	21/25	49/25	7/5	28/15	49/25	21/55	320/21	64/7	320/21	64/5	256/15	64/5
8	8	5	3	5	2	3	2	4/5	2	4/3	2	2	4/11	320/21	64/7	320/21	256/21	128/7	256/21
9	9	5	3	5	3	2	2	6/5	8/5	2	4/3	2	4/11	320/21	64/7	320/21	128/7	256/21	256/21
10	10	5	3	5	3	4	3	21/25	49/25	7/5	28/15	49/25	21/55	320/21	64/7	320/21	64/5	256/15	64/5
11	11	5	3	5	4	3	3	28/25	42/25	28/15	7/5	49/25	21/55	320/21	64/7	320/21	256/15	64/5	64/5
12	12	5	3	5	4	5	4	56/65	126/65	56/39	70/39	126/65	56/143	320/21	64/7	320/21	512/39	640/39	512/39

Figure 7. Schematic Plans for the Villa Badoer Typology (P = 2/3, width = 64 feet, stair width = 7)

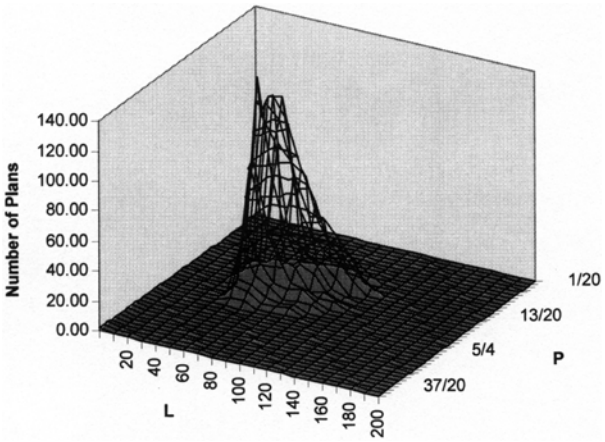


Figure 6. The Design Space of the Villa Angarano

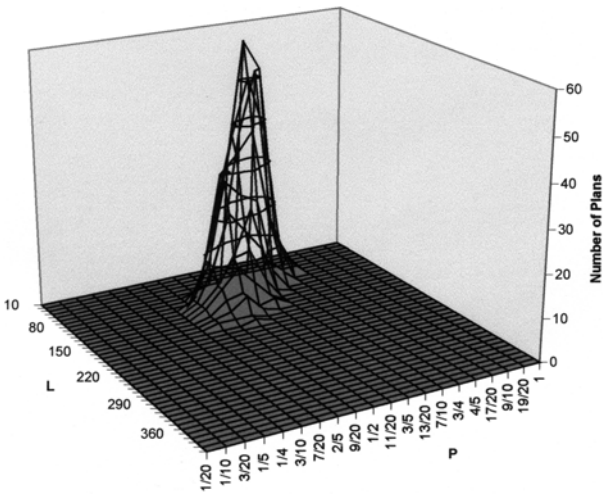


Figure 8. The Design Space of the Villa Badoer, F = 3:1

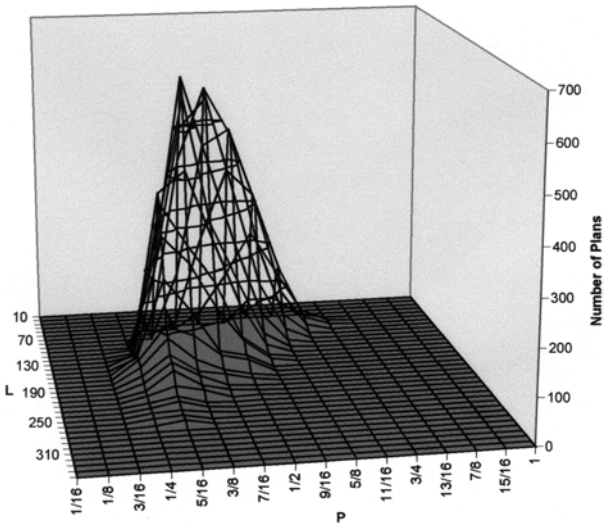


Figure 9. The Design Space of the Villa Badoer, F unconstrained

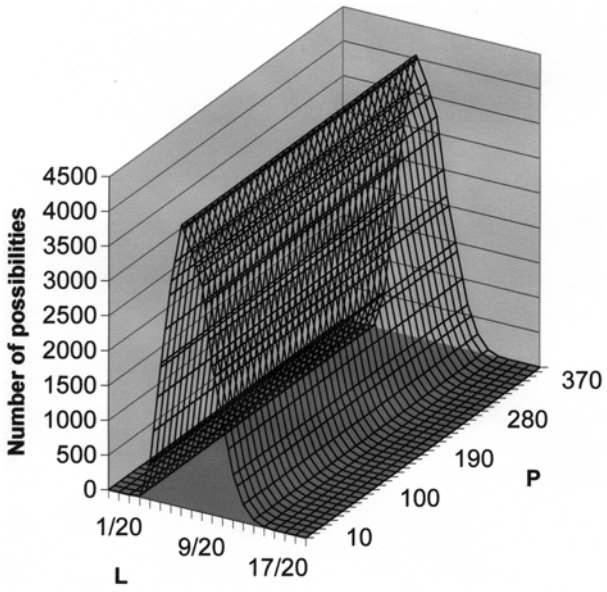


Figure 10. The Design Space of the Villa Badoer, unconstrained except for room aspect ratio < 2:1

timum overall aspect ratio and a limited range of acceptable overall ratios but no limit on plan size.

5 Conclusions

This paper has explored what happens to the design space of schematic Palladian plan topologies when dimensions are inserted for the overall plan and the spacing of the underlying grid. Previously it was assumed that once dimensions are inserted that one solves for all possible plans that have rooms with Palladio's recommended proportions. We now know that Palladio mostly did not use his recommended proportions. We have shown the resulting design space of possible schematic plans when the restriction on room proportions are removed and other restrictions (constraints) are applied. Inserting actual dimensions reduces the design space considerably from what would be the case without dimensionally dependent constraints such as stair dimensions, ratio between room areas of the largest and smallest rooms and minimum room dimensions. For example, by comparing Figures 9 and 10, we see that at the optimal plan dimensions, dimensionally dependent constraints reduce the number of possibilities for the Villa Badoer to about 600 from about 4000. Corresponding figures for the Villa Angarano are 120 and 848. (Without any restrictions on proportions the number of possibilities for the Villa Angarano is 6555). Sufficiently far from the optimal plan dimensions the number of possibilities reduces to zero. For these reasons one cannot scale a Palladian plan and expect it to satisfy dimensionally dependent constraints. Each topology has associated with it a range of possible plan dimensions and optimal plan dimensions for the maximum number of possibilities.

We have attempted to provide some new insights into the question of constraints. While the syntax of shape grammars embodies some local constraints in the form of predicates, labels and markers, they do seem to require additional constraints operating at the level of the overall design generated by a shape grammar. Several such constraints, building on the work of Hersey and Freedman, are identified for the Palladian shape grammar of Stiny and Mitchell. We have emphasized the role of constraints in this paper because of Hillman's point in the introduction that without them we have, by analogy to language, syntactically correct but meaningless architecture. In formal terms this is equivalent to saying that we have buildings without a consistent style. Without these constraints the Palladian grammar does not necessarily lead to Palladian plans.

We also wish to emphasize that there is nothing absolute about stylistic shape grammars. They are a snapshot of a style used by an architect at a particular time. There is nothing to say that the ar-

chitect might not adhere to the grammar at a later time. We know, for example, that Palladio exceeded the 2:1 proportion for rooms adhered to in most of his villas and that once, later in his career, he placed two identical rooms side by side. Even the rule that the *sala* should be larger than all the other rooms comes close to being broken in the Villa Ragona where the area of the *sala* is 319 square feet and the area of a stanza is 306 square feet.

Acknowledgements

We wish to thank Professor Vera Golini for her co-supervision of the second author's thesis that lead to this paper. Professor Golini's support, encouragement and rigor in reviewing the thesis was greatly appreciated. We are also grateful for the reviewers' helpful comments.

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Appendix A Villa Angarano: Solutions for room proportions and dimensions

$i = m / n$

$j = f / g$

$k = g / h$

$x1 = (i\ L)/(1 + 2i)$

$x2 = 1/ (1 + 2\ i)$

$y1 = (j\ k\ p\ L)/(j\ k + k + 1)$

$y2 = (k\ p\ L)/(j\ k + k + 1)$

$y3 = (p\ L)/ (j\ k + k + 1)$

$a = (p\ (1 + 2\ i)\ /(i\ (j\ k + k + 1))$

$b = (k\ p\ (1 + 2\ i)) / (i\ (j\ k + k + 1))$

$c = (j\ k\ p\ (1 + 2\ i)) / (i\ (j\ k + k + 1))$

$d = j\ k\ p\ (1 + 2\ i\ )/(j\ k + k + 1)$

$e = (p\ (k + 2\ k\ i + 1 + 2\ i)) / (j\ k + k + 1)$

Appendix B Villa Badoer: Solutions for room proportions and dimensions

$h = m / n$

$i = n / o$

$j = q / r$

$k = r / s$

$y1 = (j\ k\ p\ L)\ /(j\ k + k + 1)$

$y2 = (k\ p\ L)\ /\ (j\ k + k + 1)$

$y3 = (p\ L)\ /\ (j\ k + k + 1)$

$x1 = (h\ i\ L)/\ (2\ \ h\ i + 2\ i + 1)$

$x2 = (i\ L\ )\ /\ (2\ h\ i + 2\ i + 1)$

$x3 = 1\ /\ (2\ h\ i + 2\ i + 1\ )$

$a = y1/x1$

$b = \ (y2 + y3)/x1$

$c = y1/x2$

$d = y2/x2$

$e = (y1 + y2)/x3$

$f = y3/(2\ x2 + x3)$