Representation and Reasoning About Shapes: Cognitive and Computational Studies in Visual Reasoning in Design

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Abstract. This paper describes some recent cognitively-based and computationally-based research on representing and reasoning about shapes. The cognitive studies are based on protocol analyses of designers and indicate that visual reasoning in design involves drawings of shapes and their relations in the generation of unexpected results. The computational studies are concerned with the development of qualitative representations of shapes that can be used to reason about shapes. Two representations are described: half-planes and landmark-based qualitative codes. Reasoning using these representations is presented.

1 Introduction

The way we understand and interact with the world is largely through our visual sense. Our external visual world is seen as being composed of shapes and their relations. Our visual ideas are influenced by our understanding of this external visual world. The role of designing is to suggest changes in the external world we sense. A large class of designing involves reasoning with and about ideas concerned with shape and form. The external representation of such ideas plays a fundamental role in visual reasoning. Therefore, it is not surprising that there is increasing interest in both the roles that external representations of shape play and the ways in which shapes can be represented for a variety of expected and unanticipated purposes. This paper describes recent research into both the cognitive roles of sketching in visual reasoning and the computational means of representing shapes for visual reasoning.

The first part of the paper presents the results of a protocol study of a designer, focussing on the aspect of his work that concerns the particular role the shapes in his sketches played in the development of his ideas. The second part of the paper presents two computationally-based qualitative representations of shapes, representations that build on previous work. The claim is made that any model of shape representation and reasoning needs to be qualitative based on the evidence provided by cognitive studies.

2 Cognitive Study of Shape Sketching in Design

Designers, particularly those concerned with form, use sketches of shapes during the design process. One commonly held view is that sketching of shapes is a means of representation whose role is to act as external memory. Whilst there is considerable evidence to support this hypothesis, it appears that the role of sketching extends far beyond this initial hypothesis.

Freehand sketches are believed to encourage discoveries of unintended features and consequences [1], [2]. Making a depiction on paper forces some organization and specificity in terms of visuo-spatial features [3], regardless of whether or not the sketcher pays attention to them. For example, a depiction necessarily takes some shape and occupies an area of a certain size on paper, even though these visual features may not be intended by the sketcher. When a sketcher makes a new depiction, intending it to hold a spatial relation to some existing depictions, it will automatically produce spatial relations between the new depiction and other existing depictions that the sketcher does not intend. These implicit visuo-spatial features, in turn, may be discovered in an unexpected way by later inspection.

Our previous study of architects' sketches presented evidence that their thoughts of functional issues in an architectural design task are situated actions; they are born from the process of drawing on paper and perceiving the visuo-spatial features of depictions [4]. Then, a question arises. What aspects in the acts of drawing sketches and perceiving features in them enable a designer to invent important design issues and requirements of a given problem? By "invent" we do not mean that the issue or requirement has been generated for the first time in history, i.e. historical-invention in Boden's terminology [5], or that a designer has generated it for the first time in his or her life, i.e. psychological-invention. What is meant here is that a designer has generated the issue or requirement for the first time in the current design task, in a way situated in the design setting. We call this unexpected discovery a "situated-invention (S-invention)". The remainder of Section 2 briefly describes a study of an architect in terms of the unexpected discoveries made from shape depictions and draws from work carried out jointly with Masaki Suwa and Terry Purcell [6].

2.1 The Study

We asked an architect to carry out a design in a session that lasted for 45 minutes. He was given the task of developing the conceptual design of a museum on a given site in a natural environment in the suburb of a large city. The architect was encouraged to draw sketches on tracing paper. His sketching activities were videotaped. We then used a retrospective protocol during which he reported on what had been on his mind for each stroke of his pencil during the design session, while at the same time he was able to watch the videotape of his actions [6]. Figure 1 shows some the architect's sketches during this design session.

The S-invention of design issues and requirements appear in protocols as the acts of setting up goals to bring them into reality. We found in his protocols many instances of goals of this kind. For example, in a relatively early phase of his process, he decided that visitors to this museum should experience a cheerful and pleasant feeling even before getting to the main building from the parking lot. As another example, he talked about the necessity of service area for the buildings, such as for the delivery of goods and garbage collections. Functional requirements of this sort, the former being psychological and the latter practical, were not given to him as initial requirements, but rather emerged during the process. We coded from the protocols instances of goals to invent functional issues and requirements.

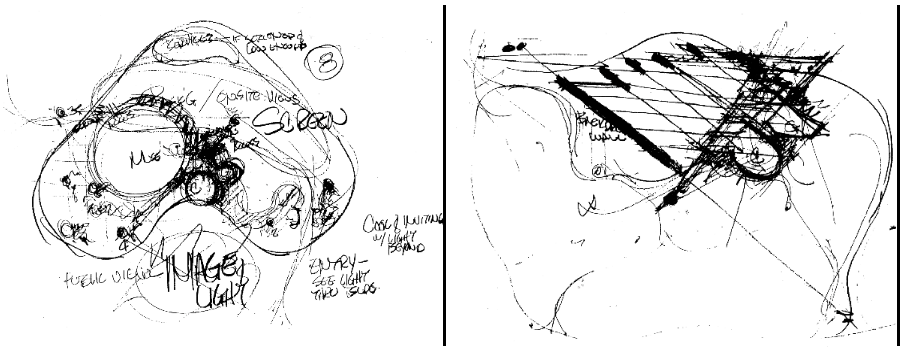


Fig. 1. Two pages of sketches from the designer indicating the level of complexity of the depiction.

Here, we will concentrate on shape reasoning, in particular on those aspects of shapes that contribute to unexpected discoveries – the S-invention of design goals.

2.2 Unexpected Discoveries from Shape Depictions

We have defined unexpected discovery as a class of perceptual actions. An unexpected discovery is a "new" perceptual action that has a dependency on "old" physical action(s) [6]. This means that if an architect traces over or pays attention to the existence of previously drawn element(s), and notices visual features of or relations among those elements, we say that the perceptual action is an instance of unexpected discovery.

There are three semantically distinct types of unexpected discovery. Table 1 summarizes the three types, and Table 2 shows the kinds of physical actions relevant to the definition of unexpected discovery.

The first type of unexpected discovery is the discovery of a visual feature such as shape, size or texture of a previously-drawn element. It is defined as a "new" perceptual action that has a dependency on an "old" physical action. If the architect traces over a circular line that was originally a simple indication of an area for a function, e.g. entrance hall in the main building of a museum, and now begins to attend to its circular shape for the first time, this is an instance of unexpected discovery of the first type.

The second type is the discovery of a spatial or organisational relation among more than one previously-drawn element. It is defined as a "new" perceptual action that has a dependency on more than one "old" physical action. If an architect traces over an element and at the same time pays attention to another element near the first element, and notices the proximity between the two elements for the first time, this is an instance of unexpected discovery of the second type.

TABLE 1. Three distinct types of unexpected discoveries (from [6])

type	definition		description
	behavior	dependent on	
type 1	"new" attention to a shape, size or texture	a single "old" physical action	discovery of a visual feature of an element
type 2	"new" attention to a relation	more than one "old" physical actions	discovery of a spatial or organisational relation among elements
type 3	"new" attention to an empty space among elements	implicit	discovery of an implicit space that exists in between elements

TABLE 2. Types of "old" physical actions (from [6])

name	description of action
touching/tracing	touch or trace over a previously drawn element on a sketch
copying	copy a previously drawn element on a new sheet of paper from the sheet underneath
inspecting	pay attention to the existence of a previously drawn element

The third type is the discovery of a space that exists in between previously drawn elements. This is the perception of figure-ground reversal, one of the characteristics of human perception. The definition of this type is an exception; its dependency on "old" physical actions is implicit. Of course, when an architect mentions the discovery of a space of this sort, he or she must be paying attention to the existence of surrounding elements. But, it is not clear which of the surrounding elements he or she is actually paying attention to. This implicitness did not hamper the codings of this type in our protocol study because, while reporting, our architect pointed to the area of an implicit space on the screen where his sketching activities were being replayed.

2.3 Cognitive Uses of Shapes in Design

Empirically it has been shown, and as many scholars have previously argued (for example [7] and [8]), that drawing sketches of shapes during conceptual design plays a crucial role in the birth of ideas. Design sketches serve as a medium through which a designer makes visuo-spatial reasoning; a designer externalises newly formed but still vague ideas in the form of less rigid and ambiguous depictions on paper in the form of shapes. By inspecting those externalised shapes, the designer finds useful clues to refine them, which motivates him or her to draw again. In Schön's terminology, the designer is having a "reflective conversation with his or her idea" [9]. It is due to cyclic behaviors based on re-examination and re-interpretation of shapes and their meanings that design ideas develop from their conception to maturity.

3 Qualitative Shape Representation and Reasoning

Given the primacy of sketching of shapes at the conceptual stage of designing it is important to be able to provide computational support to designers. However, computer-aided design (CAD) has focussed largely on the detail design stage rather than this stage. As a consequence the computational tools have been concerned with numerically-based descriptions of shapes in order to produce graphical images. The problem with numerically-based representations is that they make assumptions which cannot be met during the conceptual design stage. The primary assumption that cannot be met is the availability and significance of the numerical values that are used to define the shapes. Sketches of shapes at this stage are conceived of as classes rather than instances and require a qualitative representation.

Sketches are normally composed of aggregations of lines that form a contour of some shape in two- or three-dimensional space. Even though sketches and shapes are similar in their usage, there are some important differences. Sketch is a more general term referring to any aggregation of lines with some design significance. Shape, however, is more specific term, which is a set of closed and connected lines either with rectilinear or curvilinear line segments where vertices and/or nodes are located in defined positions on a two-dimensional plane. Shape exists at either the sketch level or the detailed design level with different denotations. A shape at the detailed level denotes a unique design solution to a specific design problem with all its numeric data. A shape at the sketch level, however, denotes a group of related categories of shape classes with any necessary numeric data. The latter focuses on the pictorial salience and shape patterns that are depictions of following elements:

Thus, the description of sketches in any symbolic scheme may be treated as the problem of describing distinctive shape characteristics at the categorical level. Since these shape characteristics of sketches can be treated as features, the representation of sketches becomes the issue of recognising, capturing, and representing these shape features as discrete symbols. The description of salient pictorial features of sketches as symbols and as patterns of symbols provides the basis for a computational paradigm for the qualitative representation and reasoning about shapes.

3.1 Logic-Based Representation of Shapes

3.1.1 From Halfspaces to Logic

Here we define a logic formalism for representing shapes as objects based on half-spaces. This formalism was introduced in Damski [10] as halfplanes for shapes in 2D and extended to 3D objects [11]. It also applies to halfspaces with both straight and non-straight boundaries. We briefly describe its development and demonstrate its applicability. Full details may be found in the references listed above.

A halfspace is one of two parts of a volume defined as a set of points. Each set of points defines one halfspace, as shown in Figure 2. We will present some basic definitions for the halfspace representation.

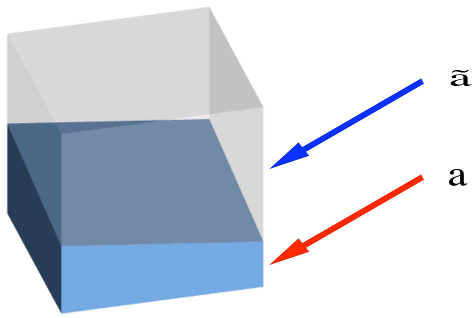


Fig. 2. Halfspace a in $\boldsymbol{U}$ and its complement, ã.

Definition 1 (Universe of discourse): The universe of discourse $\boldsymbol{U}$ is a set of points $p(x,y,z)$ in a reference space. There is only one universe of discourse

Definition 2 (Halfspace): A halfspace H is a non-empty set of points $p(x,y,z)$, $H = \{ p(x,y,z) : f(x,y,z) > 0\}$ where $p(x,y,z) \in U$. Given a halfspace a, there is only another halfspace ã which is a set of points $p(x,y,z)$ which belongs to U and does not belong to a. An example of a halfspace is the shaded space shown in the Figure 2. We need to arbitrarily allocate the bounding plane to one or other of the halfspaces.

Definition 3 (Object): An object O_s is a set of points $p(x,y,z)$ in U formed by the unions and intersections of halfspaces.

Definition 4 (Volume): A volume V_s is the minimal set of points $p(x,y,z)$ in U that can not be divided into a smaller set by any halfspace.

Theorem 1: Every volume is formed by the intersection of all halfspaces in $\boldsymbol{U}$.

Theorem 2: Objects are a union of volumes.

Definition 5 (Drawing): A drawing D is defined as one universe of discourse U which contains at least one halfspace.

Since halfspaces divide the universe of discourse into two spaces, it is possible to assign a truth value True and False to each space and use it as a grounded predicate in classical logic. The mapping from set description into logic is defined as:

Halfspace: For a given halfspace *a* we define the predicate *hs(a)* with truth value □ and □*hs(a)* for the halfspace ã, where □ can be True or False. By convention, we assign the truth value True to the halfspace where its name lies. Thus *hs(a)* in Figure 2 is True.

Volume: According to Theorem 1 a volume V_l is expressed logically by the formula: $hs(a_1)$ □ $hs(a_2)$ □ ... □ $hs(a_n)$, for a given *n* halfspaces in a drawing *D*.

Object: According to Theorem 2 an object O_l is expressed logically by the formula V_1 V_2 ... V_m for a given *m* volumes in the drawing *D*.

Drawing: Drawing is a set of Wff - well formed formulas (according to its definition in first-order logic) using the symbols *hs(x)* for halfspaces, *v(x)* for volumes and *o(x)* for objects.

This new logic representation can be used to reason about objects and their relationship.

Theorem 3: Given two objects O_{s1} and O_{s2} and their logical correspondences *o(s1)* and *o(s2)*, if O_{s1} □ O_{s2} then *o(s1)* □ *o(s2)*.

Definition 6 (Reference): A reference ***R*** is a formula in the following format: *v(x)* □ *hs(y)*, where a given volume is inside an halfspace.

Definition 7 (Model): A model ***M*** is a set of all possible references for a unique halfspace and its negation. A model is *minimal* $\boldsymbol{M}_{\min}$ if it has the smallest number of references in a drawing.

Definition 8 (Interpretation): Given a volume V_1 defined by the formula: $hs(a_1)$ □ $hs(a_2)$ □ ... □ $hs(a_n)$, an interpretation ***I*** of V_1 is an assignment of truth values to $hs(a_1)$, $hs(a_1)$, ... $hs(a_n)$ and no $hs(a_i)$ is assigned both True and False.

Definition 9 (Visible): A volume V_1 is said to be *visible* under a model ***M*** iff for any interpretation ***I*** in which V_1 is True, ***M*** is also True. In logic, that says V_1 is a logical consequence of ***M***, denoted as ***M*** *models* V_1.

For a given *n* halfspaces, it is possible to have 2^n different volumes. Logically, a model ***M*** can constrain this upper limit to a lower value. The definition of *visible* is important because we can distinguish what volumes can be "seen" in a particular geometrical drawing (model ***M***) from all possible volumes (upper limit).

Theorem 6: Given *n* halfspaces, there are 2^n possible volumes.

In order to illustrate these definitions, Figure 3 shows three halfspaces *a*, *b* and *c*. The volume V_1 is described by the formula: *hs(a)* □ *hs(b)* □ *hs(c)*. The volumes V_1, V_5 and V_7 are examples of *visible* volumes. Since we have three halfspaces, there are 8 (i.e. 2^3) possible volumes. In this particular example only 7 volumes are visible. The volume not visible is: □*hs(a)* □ □*hs(b)* □ □*hs(c)*.

An object is a combination (disjunctive normal formula) of volumes. For example, an object O_1 can be V_1 V_5, which can be expanded to:

(*hs(a)* □ *hs(b)* □ *hs(c))* (□*hs(a)* □ *hs(b)* □ *hs(c))* which can be simplified further to: *hs(b)* □ *hs(c)*

Finally, the *model* $\boldsymbol{M}_1$ which describes this topology is given by the formula F_1.

□*hs(b)* □ □*hs(c)* □ □*hs(a)* Formula F_1

We can infer that, given *hs(x)* as non-empty halfspaces and the model $\boldsymbol{M}_1$, the visible volumes are V_1, V_2, ..., V_7. As more formulas are added to the model, the fewer the visible volumes we get. For example, in Figure 3 if we move the boundary of *c* to the right until the volume V_1 disappears there will be a need for an additional formula in $\boldsymbol{M}_1$, as shown in F_2.

hs(a) □ *hs(b)* □ □*hs(c)* Formula F_2

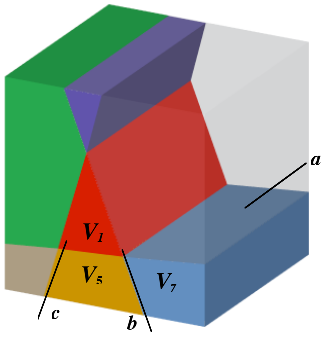


Fig. 3. Halfspaces *a*, *b* and *c* in ***U***.

The new model $\boldsymbol{M}_2$ is F_1 □ F_2. Now the formula *hs(a)* □ *hs(b)* □ □*hs(c)* for V_1 has the truth value False, which means it is a non-visible (empty) volume. The model $\boldsymbol{M}_2$ is:

(hs(b) □ *hs(c)* □ □ *hs(a))* □ *(*□ *hs(b)* □ □ *hs(c)* □ *hs(a))* Formula F_3

It now becomes a simple matter to construct logic operations that result in defining all the possible topological connectivities between objects as well as directional relations.

3.1.2 Example of Logic-Based Representation

Consider Miro’s “Girl in a Hat” shown in Figure 4. We are now able to represent it using our halfplanes.

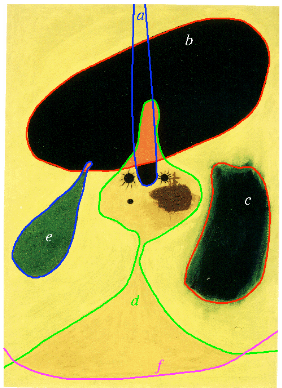


Fig. 4. Jean Miro's "Girl in a Hat"

The hat is now represented as:

$$hs(b) \wedge \neg hs(c) \wedge hs(f)$$

Whilst the girl's head and body are:

$$\neg hs(c) \wedge hs(d) \wedge \neg hs(e) \wedge hp(f).$$

The representation of such free-form shapes is difficult in a numerically-based formalism and would usually involve B-splines or similar that do not lend themselves to any form of reasoning directly. From the representation above we can reason about a variety of relations between all the objects that go to make up this painting [11].

3.2 Feature-Based Qualitative Representation and Reasoning

3.2.1 Qualitative Representation

We shall form our features from semantic regularities in a qualitative representation of shapes that is founded on well-known methods. There is a large body of literature on qualitative representation and reasoning. Here, inter alia, we use Freeman's chain coding scheme with landmark-based qualitative codes [12], [13], [14]. Landmarks occur where there is any change in the value or type of a qualitative code. We show the encoding of shape characteristics in terms of three discrete stages of representation [15].

(i) physicality → symbol (P → S)

This is the phase where our qualitative representation scheme operates. Our approach is to represent characteristic physicality of a shape through three basic shape attributes, called Q-codes, and encode them into qualitative sign values:

1. vertex angle at a landmark (A)
2. relative length of edges at a landmark (L)
3. curvature of a boundary segment (K).

A qualitative encoding scheme has been devised for these shape attributes using the Q-codes shown in Table 3 [16].

TABLE 3. Definition of Q-codes.

	A-code	L-code/K-code
Numeric value range	0 □ □ □ 2□	- □1,k □
Landmark set	{0, □}	{- , 0, }
Interval set	{[0,0],(0,□),[□,□],(□,0)}	{(- ,0),[0,0],(0,+)}
Q-code set	$\{A_{nil},A_{+},A_{0},A_{-}\}$	$\{L_{-},L_{0},L_{+}\}$, $\{K_{-},K_{0},K_{+}\}$

(ii) symbol □ regularity (S □ R)

As a result of the Q-code representation, the physicality of a shape is described as a sequence of symbols which is assumed to denote some characteristics of the shape. Some of these characteristics are easy to identify from the structural regularities in the encodings, while others are rather complicated because they appear in more complex patterns of encodings.

A sequence of Q-codes that represents a shape attribute is termed a Q-word. A symbol sequence describing a closed and connected shape contour is termed a sentence (Q-sentence). Any repeating structurally pattern in terms of symmetry, iteration and alternation [17] is termed a phrase (Q-phrase). Thus, analogically, the symbol sequence can be conceptualised as a hierarchy of Q-codes □ Q-words □ Q-phrases □ Q-sentences and so on. The transformation from symbol sequence (unstructured) to regularities (structured) brings three interpretation possibilities.

Firstly, the repetition in symbol structure is a distinctive characteristic that is easily recognised as a form of syntactic regularity. Repetitions are normally categorised into three basic types, namely, iteration, alternation, and symmetry [17].

1. *Iteration*: repetition of symbols or a pattern of symbols in a regular interval (example: aaa..., ababab..., abcabcabc...).
2. *Alternation*: repetition of symbols or a pattern of symbols in an irregular interval (example: abcabdeabfgab..., abcabdabe...).
3. *Symmetry*: repetition of symbols or a pattern of symbols in a reflective way (example: abcdcba, abcddcba).

If the syntactic repetition is recognised, then the sequence of symbols is thought to have a regularity that is related to specific shape patterns.

Secondly, a pattern of symbol sequences can be identified as denoting specific categories of shape classes that are well-known or familiar in contour. These are shape patterns with specific labels, examples are shown in Figure 5.

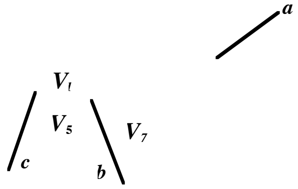


Fig. 5. Some well-known shape patterns.

Thirdly, shape pattern regularities of symbol sequences can only be identified by investigating the patterns and occurrences of words (Q-words) when they do not match any pre-existing label.

(iii) regularity $\rightarrow$ feature (R $\rightarrow$ F)

When syntactic regularities are identified from the symbol sequences they become shape features. These shape features are labeled either by matching with an existing feature knowledge-base or by creating a new mapping to specific design semantics.

In addition to finding shape features, we can use those features as the basis for reasoning about shapes. We can, for example, determine categorical information about groups of shapes. Once the regularities of syntax patterns have been identified, each word is categorised into one of the five atomic shape feature categories: protrusion, indentation, iteration, alternation and symmetry. Based on identified words, commonalities of feature characteristics are determined by comparing matchings and mismatchings of each shape's features to the description of their category that is defined by the aggregation of all its common shape features. Comparisons can now be made either of a shape to the category or of one shape to another.

3.2.2. Similarity Measures Based on Category

We use the similarity measure equation proposed by Estes [18]:

$$Sim(\mathrm{A,B}) = \mathrm{t}^{k} \times \mathrm{s}^{N-k}$$

where:
A, B: shape feature category or shape category
k: number of matching shape features
N: total number of shape features in the category
s: total number of mismatching shape features
t: total number of matching shape features.

The following tables show some of the similarity measures of the 12 sketches for alternative conceptual layout designs for a museum in Texas by the architect Louis Kahn, Figure 6. These similarity measures can be used to assist in the understanding of the similarities and differences of these conceptual layout designs.

Indentation Category

Table 4 shows the similarities in the indentation category represented as (A_+ n*(A_-) A_+) for values of n from 1 to 3.

TABLE 4. Similarity of indentation category.

	A	B	C	D	E	F	G	H	I	J	K	L
(A_+ A_- A_+)	t	t	t	t	t	t	t	s	t	t	t	t
(A_+ 2*A_- A_+)	t	t	t	t	t	t	t	t	t	t	t	t
(A_+ 3*A_- A_+)	s	s	s	s	s	t	s	t	t	t	t	t
	t^2s^1	t^2s^1	t^2s^1	t^2s^1	t^2s^1	t^3s^0	t^2s^1	t^2s^1	t^3s^0	t^3s^0	t^3s^0	t^3s^0

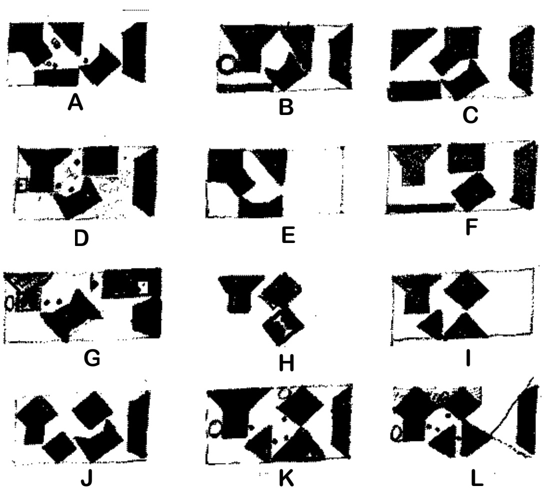


Fig. 6. Kahn's sketches for the planning of a museum complex [19].

Protrusion Category

Table 5 shows the similarities in the protrusion category represented as $(A_-\ n{*}(A_+)\ A_-)$ for values of n from 1 to 5.

TABLE 5. Similarity of protrusion category.

	A	B	C	D	E	F	G	H	I	J	K	L
$(A_-\ A_+\ A_-)$	s	s	t	s	s	s	t	s	t	t	t	t
$(A_-\ 2{*}A_+\ A_-)$	t	t	t	t	t	t	t	s	s	t	t	t
$(A_-\ 3{*}A_+\ A_-)$	t	t	s	t	t	t	t	t	t	s	t	s
$(A_-\ 4{*}A_+\ A_-)$	t	s	s	t	t	s	t	t	t	t	t	t
$(A_-\ 5{*}A_+\ A_-)$	s	s	t	s	s	s	s	s	s	s	s	s
	t^3s^2	t^2s^3	t^3s^2	t^3s^2	t^3s^2	t^2s^3	t^4s^1	t^2s^3	t^3s^2	t^3s^2	t^4s^1	t^3s^2

Alternation, iteration and symmetry categories are considered only for words up to length 6. It is assumed that the shapes from these sketches are not sufficiently complex to require an analysis of words longer than this. All word lengths are measured in Q-codes. For this analysis the only Q-code used was the A-code. We can produce tables of the similarities in alternation, iteration and similarity categories.

We can summarise the similarity measures of the 12 sketches to the five shape feature categories in terms of matching shape features as:

1. indentation category: (3: F I J K L); (2: A B C D E G H);
2. protrusion category: (4: G K); (3: A C D E I J L); (2: B F H);
3. alternation category: (38: J); (33: A H); (30: B C G); (29: F); (26: D); (25: I); (24: K L); (23: E);

4. iteration category: (13: F J); (11: A D L); (10: C); (9: B E); (8: K); (5: G I); (4: H); and
5. symmetry category: (21: F L); (20: J); (18: A D); (16: B); (15: C H K); (14: E I); (10: G).

From these results we can infer the following:

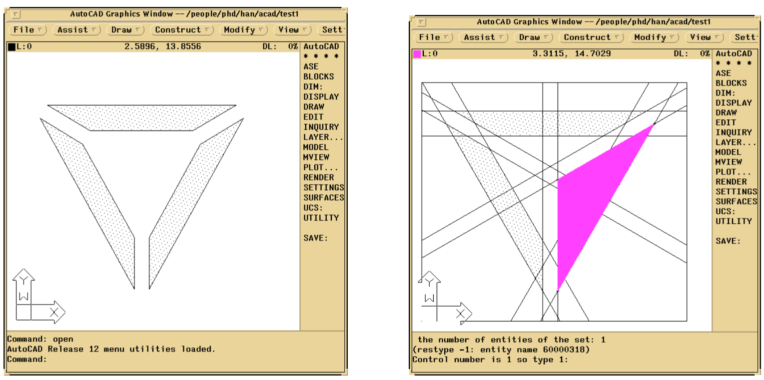
1. which sketches are more similar to the shape classes defined either by each shape feature category or some combination of shape feature categories; and
2. we are able to compare one sketch to two or more other sketches to assess the similarity of design ideas in terms of shape features. Comparisons can be made either by a single shape feature category or by a combination of two or more shape feature categories.

4 Discussion

In Section 3 we presented some work on the qualitative representation of shapes. We have not dealt with two important issues in visual reasoning in designing: emergence and shape patterns.

Visual emergence is the representation of shapes and features which were not intentionally placed there and as a consequence were not initially represented. There is a vast body of literature in Gestalt psychology (for example [20], [21]) but a smaller body concerned with symbolic and computational modeling of visual emergence. Symbolic and sub-symbolic models of visual emergence have been developed that cover all the embedded figure cases and many of the illusory figure cases [22], [23]. Such models have been tested using experiments in which human subjects were asked to identify emergent figures. The results of these experiments demonstrate a very high level of performance from these models.

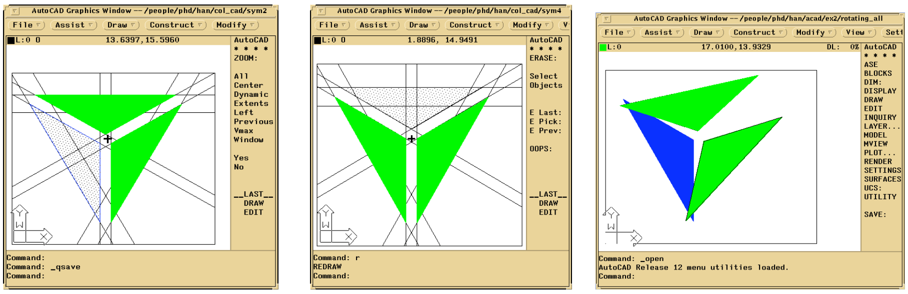
Consider the plan shown in Figure 7(a). Figure 7(b) shows one emergent triangle derived from that plan.



(a) (b)

Fig.7. *(a) Shape as drawn, and (b) one emergent shape found by the system (from [24]).*

Figures 8(a) and (b) show two emergent reflective symmetry relationships found from emergent triangles. Figure 8(c) shows the emergent triangle and the emergent reflection relationship used in subsequent designing. The designer has chosen one emergent triangle and repositioned and rotated it about its base node and as a consequence a triangle that has a reflective symmetry relationship with it also rotates. This example demonstrates that it is possible to build computer models whose results match those expected by human designers.



(a) (b) (c)

Fig. 8. *(a) and (b) Emergent reflection relationships found that use the initial emergent triangle, and (c) using both the emergent triangle and the emergent reflective symmetry relationship in subsequent designing (from [24]).*

Of increasing interest is the ability to represent and manipulate patterns produced by a multiplicity of individual shapes. Figure 9 depicts a range of patterns observable from the floor plans of architect designed buildings and spaces.

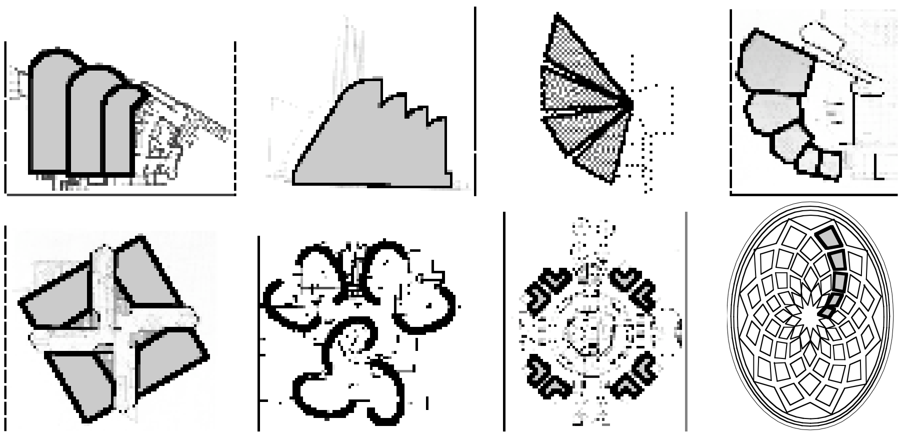


Fig. 9. *Patterns from shapes in floor plans of buildings and spaces.*

New representation languages are needed to represent shape patterns symbolically [25]. With such qualitative representations it becomes possible to carry out a variety of reasoning tasks. The determination of the style of a composition of shapes through a representation of the common characteristics of the symbolic representation is one such task of interest [25], [26].

Much of conceptual designing involves shapes. Formal methods of representing shapes and their relationships provide a basis for the development of computational tools to support designing. The ability to reason using these representations provides opportunities to enrich those representations through the addition of increasingly abstracted characteristics. From the simple representation of shapes it is possible to locate and represent emergent shapes, emergent relationships, emergent patterns and even the style of a composition of shape elements.

Designers use shapes in various ways during their designing. The ability to represent shapes and their relations forms the basis of both models for understanding designing processes and computational support tools for reasoning about shapes during designing.

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