A SHAPE COMPACTNESS MEASUREMENT INDEXING WITH FUZZY MULTICRITERIA DECISION MAKING APPROACH

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Abstract. Compactness of a district or zone is determined by considering its appearance and the area of dispersal of the district (Altman, 1998) and it is used as a characteristic to describe shape (Shiode, 1998; Knight, 1997). However, existing compactness measurement used to assess district plans in redistricting applications are vague and imprecision although there are more than thirty over Euclidean or non-Euclidean measurement methods. Therefore, this paper presents an integrated shape compactness measurement indexing with a fuzzy multicriteria decision making approach (FMCDM) to enhance the measurement of shape compactness. An experiment is conducted in order to verify the practically of the proposed model to improve existing compactness measurement method.

1. Introduction

Compactness includes a mathematical analysis of the shape and regularity of the spatial shape. Laurini & Thompson (1992) mentioned that shape refers to the structure of the mode of arrangement related to function or the appropriateness and effectiveness of purpose. It is important in control of the district shape or district boundary complexity because irregular geographical boundaries may cause oddness in district shape in various applications such as election zone rearrangement, sales territory management for business and forest resources planning applications.

Compactness measurement for district compactness and continuity assessment gives an index for assessment of compact district or zone (Bong, 2001b). However, existing compactness measurement used to assess district plans in redistricting applications are vague and imprecision although there are more than thirty over Euclidean or non-Euclidean measurement methods (Altman 1998; Bong 2001a). Frequently, decision making in this district assessment involves uncertainty of the data acquired and the variety of evaluation tools. In addition, numerical measurements in compactness measurements give uncertain and vague value for accurate assessment of district compactness and continuity. Altman (1998) uses an axiomatic analysis to join formal measures of compactness to analyze the consistency of the Euclidean compactness criteria and he proved that most compactness indices satisfy one shape axiom, but violate others. Furthermore, Dominique & Michel (1999) showed different methods for getting fractal dimension may change due to different condition. Thus, existing compactness measurement methods perform differently to different circumstances and single methods alone is not effective enough because each of the method may suffer from its limitation.

Therefore, section 2 presents an integrated shape compactness measurement indexing with a fuzzy multicriteria decision making approach (FMCDM) to enhance the measurement of shape compactness. The generation of the integrated compactness index is based on the synthesis of the concepts of fuzzy set theory, Analytical Hierarchical Process (AHP), α-cuts concept, index of optimism of decision maker to estimate the degrees of satisfaction of the judgments on a district plan and their attitudes toward risk. We use the fuzzy set theory and Analytical Hierarchy Process to formalize the gray area because it provides fuzzy numbers that are easy to use in expressing qualitative assessments of the Decision-Makers, DM (Deng 1999, Chen 1996, Mon 1995, Yeh 1997). An experiment is conducted in order to verify the proposed model in section 3. Results and analysis from the experiment showed the effectiveness of the integrated indexing with the ability to cope with fuzziness and multiple aspects of compactness measurement. Lastly, the paper outlined potential improvement and recommendations for future use.

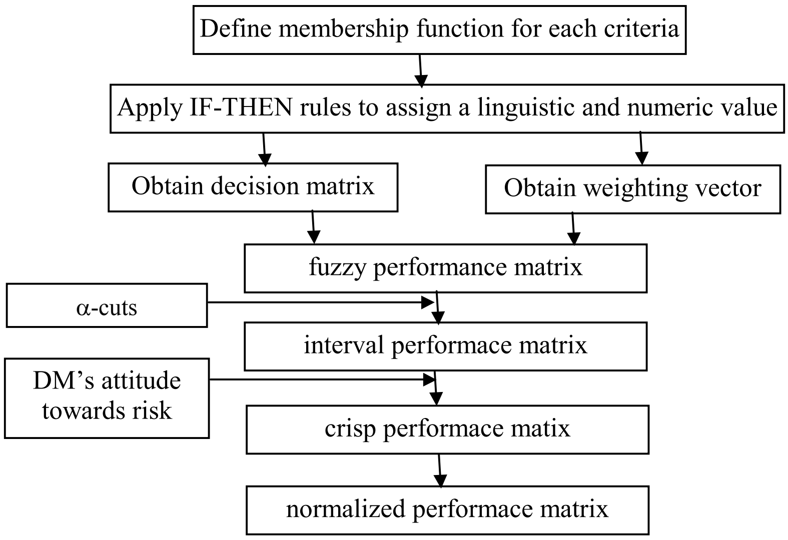

Figure 1: Architect Design for the constructing the proposed integrated compactness measurement indexing

2. Design For Integrated Compactness Measurement Indexing With Fuzzy Multicriteria Decision Making

The design for constructing the integrated compactness measurement for this paper is summarized in Figure 1. Firstly, we apply fuzzy set theory in constructing rulesets for each criteria of the integrated compactness measurement before we integrate multiple criteria by using Fuzzy Analytical Hierarchy Process. The ruleset plays an important role to determine the optimality of district compactness. We define the rules by linguistic terms and their membership function. Linguistics terms are defined for each of the selected criteria and their weighing vector (Herrera and Verdegay, 1995). In addition, we use the definition of the interval of confidence at level α (α-cuts). This helps characterize the triangular fuzzy number as

$$\forall \alpha \in [0,1], \quad \bar{A}_n = [a_1^n, a_3^n] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3] \qquad (1)$$

The method used will give an effectiveness level (x) to each of the selected criteria (C). The effectiveness levels belong to a set of linguistic terms that contains various degrees of preference required by the DM. This study use linguistic terms x (effectiveness, x) = {Very Poor (VP), Poor (P), Fair (F), Good (G), Very Good (G)}, defined in Figure 2. A membership function, which assigns to each linguistic term with a grade of membership, is associated with the fuzzy set in [0, 1]. When the grade of membership for a linguistic term is one, it means that the linguistic term is absolutely in that set and verse vice. Borderline cases are assigned values between zero and one.

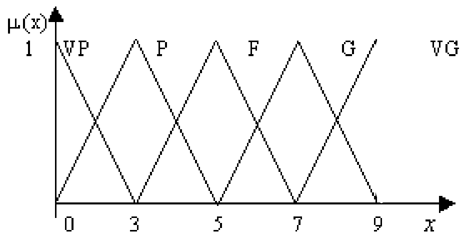

Figure 2: Membership functions, μ(x) and the linguistics terms for effectiveness (x) of each criterion (C)(Bong, 2001b)

Considering the different importance of criteria, different weights are determined for each criterion. By pairwise comparison of the relative importance of criteria, the pairwise comparison matrix is established based on pairs of criteria *Ci*, and *Cj*. The comparison scale ranges from 1 to 9. To facilitate the making of pairwise comparison, triangular fuzzy numbers defined in Table 1 are used. A triangular fuzzy number $\bar{x}$ expresses the meaning of 'about x', where $1 \leq x \leq 9$, with its membership function. Fuzzy number $\bar{x}$ used by Deng (1999) is revised here to better reflect the decision situation involved.

Table 1: Linguistic terms used by the decision matrix (Deng, 1999)

Linguistic term	Very Poor (VP)	Poor (P)	Fair (F)	Good (G)	Very Good (VG)
Fuzzy number	$\bar{1}$	$\bar{x}$			$\bar{9}$
Membership function	(1, 1, 3)	$(x-2, x, x+2)\ for\ x = 3,5,7$			(7, 9, 9)

By using the fuzzy numbers defined in Table 1, a fuzzy reciprocal judgment matrix for the decision matrix for m criteria and n alternatives is given as

$$X = \begin{bmatrix} x_{11}\ x_{12}\ \dots x_{1m} \\ x_{12}\ x_{22}\ \dots x_{2m} \\ \dots\ \dots\ \dots\ \dots \\ x_{n1}\ x_{n2}\ \dots x_{nm} \end{bmatrix} \quad (2) \qquad W = (w_1, w_2, \dots, w_m) \quad (3) \qquad Z = \begin{bmatrix} w_1 x_{11} + w_2 x_{12} + \dots + w_n x_{1m} \\ w_1 x_{12} + w_2 x_{22} + \dots + w_n x_{2m} \\ \dots\ \ \dots\ \ \dots\ \ \dots \\ w_1 x_{n1} + w_2 x_{n2} + \dots + w_n x_{nm} \end{bmatrix} \quad (4)$$

where x_{ij} represent the linguistic assessments of the performance rating of alternative $A_i(i=1,2,...n)$ with respect to criterion $C_j(j=1,2,...m)$. The weighting vectors for the evaluation criteria is given directly by the DM or obtained by using pairwise comparison of the AHP as defined in Table 1. Then, the model obtains a fuzzy performance matrix (5) representing the overall performance of all alternatives, with respect to each criterion by multiplying the weighting vector with the decision matrix.

By using a α-cut (1) on the performance matrix (4), an interval performance matrix can be derived as in (5), where $0 \le \alpha \le 1$. The value of α represents the DM's degree of confidence in his/her fuzzy assessments regarding alternative ratings and criteria weights. A larger α value indicates a more confident DM. Incorporated with the DM's attitude towards risk using an optimism index α, an overall crisp performance matrix is calculated in (6) by $z_{i\alpha}^{\lambda'} = \lambda z_{ir}^{\alpha} + (1-\lambda) z_{il}^{\alpha}, \lambda \in [0,1]$.

$$Z = \begin{bmatrix} [z_{1l}^{\alpha}, z_{1r}^{\alpha}] \\ [z_{2l}^{\alpha}, z_{2r}^{\alpha}] \\ \dots \\ [z_{nl}^{\alpha}, z_{nr}^{\alpha}] \end{bmatrix} \quad (5) \qquad Z_{\alpha}^{\lambda'} = \begin{bmatrix} z_{1\alpha}^{\lambda'} \\ z_{2\alpha}^{\lambda'} \\ \dots \\ z_{n\alpha}^{\lambda'} \end{bmatrix} \quad (6) \qquad z_{i\alpha}^{\lambda} = \frac{z_{i\alpha}^{\lambda'}}{\sqrt{\sum_{i=1}^{n} (z_{i\alpha}^{\lambda'})^2}} \quad (7)$$

λ= 1, λ=0.5, and λ= 0 are used to indicate that the DM involved has an optimistic, moderate, or pessimistic view respectively. An optimistic DM is apt to prefer higher values of his/her fuzzy assessments, while a pessimistic DM tends to favor lower values. After the facilitation of the vector matching process, a normalization process in regard to each criterion is applied to (7), resulting in a normalized performance matrix expressed. The values of N_{α}^{λ} indicate the degree of preference with respect to the alternatives for fixed α and λ, respectively where $\alpha \in [0,1], \lambda \in [0,1]$. Indeed, this value is the Integrated Compactness Indexing (ICI), which considers all the compactness measurements or evaluation criteria earlier. The larger the value, the more compact the shape and the less complex is its boundary, thus the more preference the alternative.

3. Experiment

An experiment is conducted in order to verify the practically of the proposed model to improve existing compactness measurement method. Specifically, we determine two different compactness measurement methods to consider the compactness criteria and they act as two different separate criteria. They include the non-Euclidean measurement based on fractal dimension (FD) and the Euclidean measurement based on area-perimeter ratio (EM). The higher the fractal dimension, the more complex it is for the district boundary. Subsequently, the district compactness is determined by the value on the fractal dimension. Thus, we choose to use box counting dimension to calculate the fractal dimension. First, we need to cover a zone by 'r'-size boxes and determining how many boxes of a particular size 'r' intersect the image (Knight, 1997). Thus, the number of boxes of size 'r' is needed to cover the zone is given by:

$$N(r) = \frac{i}{r^D} \tag{8}$$

$$\log(\text{box count}) = a + b \log(\text{box size}) \tag{9}$$

Then, the size 'r' is changed to progressively smaller sizes and the corresponding numbers of non-empty boxes are counted N(r). The logarithm of N(r) versus the logarithm of 1/r gives a line whose gradient corresponds to the box dimension. The intercept is represented by a, and b is the slope. The fractal dimension D is represented by the absolute value of the slope b. Meanwhile, the EM is based on equation below:

$$compactnes\ s = \frac{Perimeter^{\ 2}}{Area} \tag{10}$$

The third criterion selected is the size of the district or the zone. Three criteria are considered with respective weight value to produce integrated compactness measurement indexing, ICI.

Figure 3 shows the comparison result for the each compactness measurement index for the two of the selected criteria, EM and FD and the proposed indexing, ICI normalized from circle (polygon 2). It is clear that ICI could perform well not only to react the area and perimeter control but also for the boundary complexity. On the other hand, FD could not assess the polygon compactness very well because it gave a blur and imprecise index.

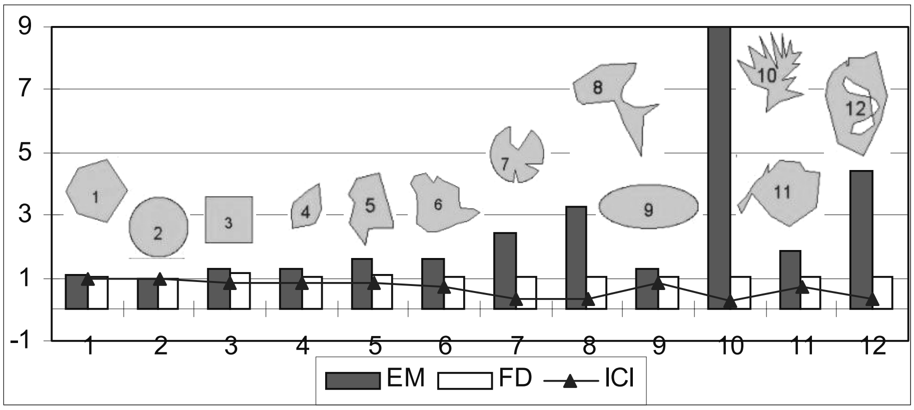

Figure 3: Comparison on different compactness measurement index

4. Conclusion

The paper has been completed successfully despite all the constraints being posed to. The most profound accomplishment among all is the success in obtaining Integrated Compactness Indexing by incorporation fuzzy multicriteria decision making for compactness measurement index. Though the information sources are limited, it is enough to prove the applicability of the developed redistricting algorithm to incorporate shape compactness information into redistricting technique. The algorithm is simple and yet efficient as compared to other more complicated methods. The result is satisfactory and comparatively better than other traditional approaches as well. The success of definition, modeling, and incorporation of the tertiary information also highlighted the applicability of Fuzzy Multiple Criteria Decision Making approach in compactness assessment.

5. Recommendation

The research can be extended especially in analyzing the relationships (correlation) and interdependence between various shape criteria. The detail analysis on their relationships may help their consideration in the model to enhance the result of the redistricting technique. Presently, researches are mutually exclusive and no detail study has been done to analyze the their relationships.

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