MATHEMATICS FORMATION AND CREATIVE DESIGN

A. Tinnirello,♦, R. Voget*,♦, S. De Federico♦*

*Cátedra Matemática

Facultad de Arquitectura, Planeamiento y Diseño

Universidad Nacional de Rosario

♦Departamento Ciencias Básicas-Facultad Regional Rosario

Universidad Tecnológica Nacional

e-mail: atinni@citynet.net.ar, rvoget@frro.utn.edu.ar , sdefederico@frro.utn.edu.ar

***Key words**: Architectonic Spaces, creative design, simulation, mathematics, geometric structure, fractal.*

Abstract

One of the earliest human beings desires has been to inhabit a place that may assemble beauty and functionality. Architecture and Design have been the disciplines in charge to formalize it. Their study, as well as the way they are taught have been adapted to fit the needs; and the velocity of their transformation is even greater as well as the link between design, art and technology. Creativity, in it basic generic entity, is the capability to solve appropriately and originally phrased architectural problems which involves not only space, act, environment and semiotics but with everything related to make the project able to be constructed and inhabitable.

New technologies are based on complex algorithms which, by the use of simulators, achieve to produce complexity works that would have been unbelievable twenty year ago. These algorithms have a strong mathematical basis and allow to generate other working methods so as to create wonderful geometrical objects. The study of this New Geometry requires to explore and expand this field of knowledge in the Architecture studies. In order to analyze and use complex design systems to generate non linear experimental models, it is necessary the Mathematical contribution, not only at the University education stage but also at the professional life.

This New Mathematics adequately focused, is able and must be an essential ally to creative design which is born with an exercised imagination in the formation stage; therefore it must aid to establish a space where knowledge and ability for architectural work can be created, synthesized and experimented.

This work tries to encourage students and in relation to Geometry promotes the following aspects: (i) Inspection of new architectural spaces, (ii)Comprehension of the geometrical structure, (iii) Originality and common sense, (iv) Relation between Geometry and design of construction constitutive elements,(v) Insertion of man in the space, (vi) Conditioning of design to human body dimensions, (vii) Fractal geometries.

According to what has been expressed, this proposition acquires a fundamental significance to develop a spatial vision of geometrical shapes in students, in order to stimulate the understanding of the existing relation between abstract geometrical elements and their real applications in Architecture, Geometry and Design and Art. Besides, the purpose of this work has the aim to approach knowledge at the architectural design process and to the study of shapes and mathematical models that such designs sustain , and ultimately demonstrate the importance of an academic organization that involve teachers from different disciplines.

I. Introduction

Due to development in modern technology a new look arise for geometry studies, the scale concept has changed so the relationship between the sciences of complexity and design and art has shifted througout the last years.

All the geometry shapes are a graphical representation of a mathematical equation and the succesive transformations applied, so the comprehension of the geometrical structure is associated with an experimental work which allows students to acquire the ability to develop all the posible geometrical elements, working forward the real application in design, promoting the construction of constitutive elements, studying their shapes and searching the mathematical sustain.

This work is possible in a lab class where the students explore, create and experiment the spacial vision by the use of simulators to make a project able to be constructed.

II. Motivation and objectives

The objective of this work is to present new teaching technologies that, in a sense, could be part of the educational aspect of a “workshop class”. The discussion of hands-on teaching methodology is taken beyond the general principles of active learning and learning by doing, and on the basis of teaching experiments at lab class a set of thecniques are presented to shift the focus from the teacher to the learner.

Any teaching methodology in the field of design must enable students to construct concepts and develop attitudes.The constrution of concepts requires empirical and reflective abstraction. These are students' activities from which he or she draws information on objects or phenomena, projecting them onto a higher logical plane (e.g., moving from action to representation, from the operation to the object of thought, from contents to forms) reconstructing and reorganizing on the higher plane what he or she has observed/constructed on the lower plane.

Thus the point of departure in the construction of a concept works on a creative propposed design, based on any interest transformation, aiming students mental activities, knowing transformations and the generating power, recognising the great examples of Architecture and stablishing the relationship between these examples and the choice design.
It may appear difficulties in the analisys and elaboration of artistic representation in which movements exists. In several ocasions the issue is the visualization of the movements geometric forms included in a representation. The problem worsen when in artistic representation exists sliding, axial symmetry composition and parallel translation to symmetry axes. Symmetry is the plane isommetry that presents students learning problems.

II a. Comments on methodologies

With the aim to promote students to search a variety of examples from Art and Design this project follow up complemented activities:

a) Visualization of architectural examples from buildings, designs and arts.
b) Seeking for forms, shapes, symmetries and the relationship between the geometry form and simulated objects of plane and space.
c) Describing and illustrating about the possible ways reached by the visualized forms by succesive transformations.
d) Generating new shapes varying parameters in the mathematical model found.
e) Comparing these results with the known graphics studied during the developments of the course.

II b. Computer training

Computer packages are introduced to familiarize students with computers applications at an early stage, to make them develope and appreciate the advantage of using these powerful tools. Simple problems can be formulated, increasing the difficulties using simple programming assignments algorithms and generating a gradually process where students arrive at complex works that would have involved time-consuming efforts without computer support.

In the initial part of this session students learn to identify the most commonly and simple programming assignments with a guide tutorial help to generate working methods to produce wonderful geometrical objects.

III-Practical Sessions

A pattern, whether in nature or art, relies upon three characteristics: a unit, repetition, and a system of organization. Symmetry is a fundamental organizing principle in nature and in culture. The analysis of symmetry allows for understanding the organization of a pattern, and provides means of determining both invariance and change.

In this project, the word *pattern* denotes a regularity in some dimension. The simplest examples are repeated, visual units ordered with translational –linear- or rotational symmetry. Patterns also exist in a scaling dimension, where similar forms occur at different magnification. When geometric self-similarity is defined on a hierarchy of scales, a self-similar fractal is created. The pattern as a concept also extends to solution space, where this solutions to similar problems are themselves related and define a single template that repeats - with some variation - every time this problem is solved. The underlying idea is to reuse information; whether in repeating a unit to generate a two-dimensional tiling design or three-dimensional original objects.

IIIa. Case study : Geometry of plane and space-Transformations
Theoretical requirements

We describe and illustrate the four features of plane transformation, affine transformations of the plane are composed of scalings, reflections, rotations and translations.
The matrix formulation for the transformation that involve scaling by **r** in the x-direction, by **s** in the y-direction, rotations by **θ** and **φ**, and translations by **a** and **b**; is:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} r\cos\theta & -s\sin\varphi \\ r\sin\theta & s\cos\varphi \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \qquad (1)$$

Scalings: the scaling factor in the x-direction is denoted **r**. The scaling factor in the y-direction is denoted s. Assuming there are no rotations, then if **r=s** the transformation is a similarity. The scaling are always toward the origin consequently the origin is the fixed point.

Reflections: negative **r** reflects across the y-axes, negative **s** reflects across x-axes, reflection across both x- and y-axes is equivalent to rotation by 180 degrees about the origin.

Rotations: **θ** measures rotations of horizontal lines, **φ** measures rotation of vertical lines;
φ = θ gives a rigid rotation about the origin.

In the case of space the transformation matrices can easily be constructed from the plane form. For example the rotation around x-axis means we must rotate all points in the plane YZ (Eq.: 2) The x component of a vector is not affected by a rotation around x-axis. More generally a rotation around a certain axis does not affect the components (x, y or z) from the rotation axis of the vector.

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r\cos\theta & -s\sin\varphi \\ 0 & r\sin\theta & s\cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad (2)$$

III b.Classroom Experiment Form

The students worked varying all parameters involved in the matrix equation applied to different patterns previously constructed, in order to obtain affine transformations of the plane composed of scalings, reflections, rotations and translations (Table I and II).

Table I

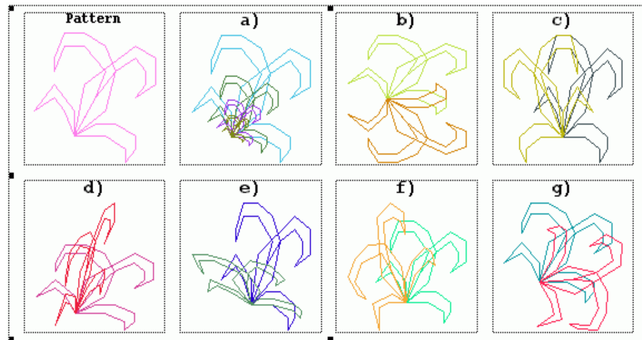
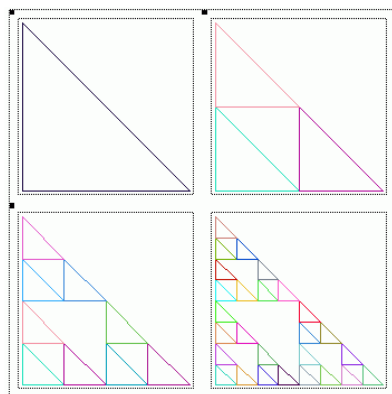
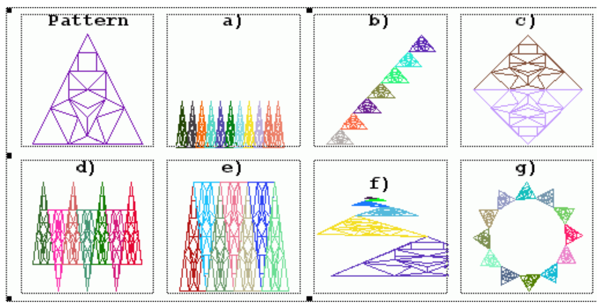
Transformation	Results
a) $r = 0.6; s = 0.6; \theta = 0; \varphi = 0; it = 3$; it= $iterations$ b) $r = 1; s = -1; \theta = 0; \varphi = 0$ c) $r = -1; s = 1; \theta = 0; \varphi = 0$ d) $r = 1; s = 1; \theta = 30°; \varphi = 0$ e) $r = 1; s = 1; \theta = 0; \varphi = 30°$ f) $r = 1; s = 1; \theta = 65°; \varphi = 65°$ g) $r = 1; s = 1; \theta = 30°; \varphi = 30°$	

Table II

Transformation	Results
$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ $\begin{pmatrix} x^n \\ y^n \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} x^{n-1} \\ y^{n-1} \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ $.i) \quad a = 1/2, \quad b = 0, \quad or$ $ii) \quad a = 0, \quad b = 1/2$	

A set of graphics objects are shown, as the result of a creative students' session (Table III), obtained applying random functions to choose colors and parameter values, and changing the number of iterations looking forward original tilings.

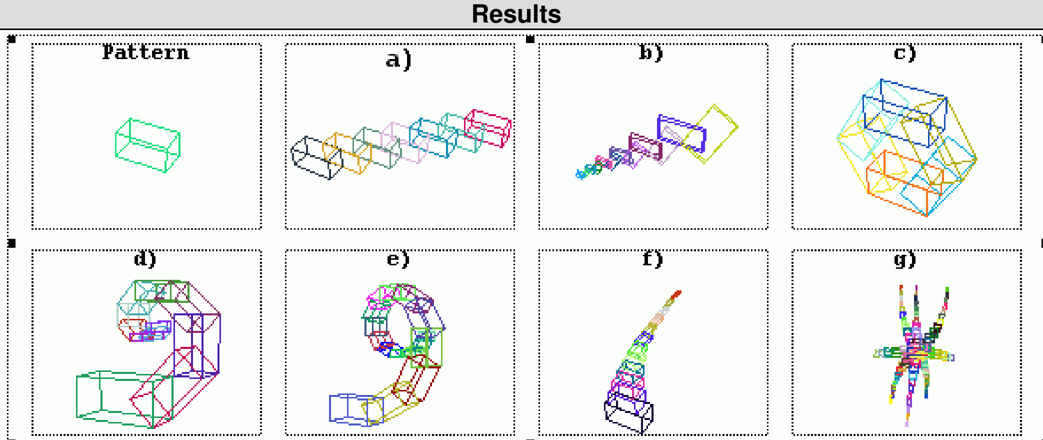
Table III

Transformation	Results
a) $r=1; s=1; \theta=0; \varphi=0; a=10; b=0; it=10$ b) $r=1; s=1; \theta=0; \varphi=0; a=6; b=12; it=6$ c) $r=1; s=-1; \theta=0; \varphi=0$ d) $r=1; s=-1; \theta=0; \varphi=0; a=8; b=8; it=6$ e) $r=1; s=-1; \theta=0; \varphi=0; a=6; b=12; it=6$ $\theta=2\pi.\mathrm{Random}(0,1);$ f) $\varphi=2\pi.\mathrm{Random}(0,1);$ $a=6; b=12; it=5; r=Random(0,1); s=Random(0,1)$ g) $r=1; s=1; \theta=\pi/6; \varphi=\pi/6; a=6; b=12; it=11$ *it= iterations Random=Random Function*	

The students developed spacial vision capacity and dinamyc view forms working with a constitutive geometrical element, for example a poligon, applying an iterative process, matrix transformations, and trying to reach artistic representations in which different movements exist (Table IV).

The level of complexity of the geometrical objects is limited only by the time involved or the hardware capability, thus the important outcome of this process is that the students begin to discover an attractive work and lead them to ask how they can accomplish these goals.

Table IV

Transformations
a) $r=1; s=1; \theta=0; \varphi=0; a=b=c=0; it=6$ b) $r=s=1.2; \theta=\varphi=\pi/3; a=b=1; c=0; it=11$
c) $r=s=1; \theta=\varphi=\pi/3; a=2; b=c=0; it=5$ d) $r=s=0.8; \theta=\varphi=\pi/4; a=2; b=c=0; it=10$
e) $r=s=0.9; \theta=\varphi=\pi/6; a=2; b=0.2; c=0; it=15$ f)
$r=s=0.8; \theta=\varphi=0; a=-0.5; b=1; c=0; it=11$
g) *Set of objects type f) varying a,b,c.* *it= iterations*
Results


IV. Fractals objects

To generate all but the simplest fractals we need to understand the geometry of plane transformations. The idea behind fractals is that complex patterns and shapes may be described by repeated placement of some basic *starter* pattern. This results in self-similarity, that is similarity in the form of the pattern at all levels, looking at small parts of the whole reveals similar symmetric structures.

Having accomplished the shown transformations, the fractal concept was introduced: The word *fractal* is used to describe figures with infinite repetitions of the same form. Students could observe that several fractal objects were drawing graphics obtained by iterations of certain related transformations. They have also discovered that not all the iterations produce fractal images.

V.- Concluding Remarks

The methodology has developed:

- Better understanding of abstract mathematical concepts. Through practice in applications the students developed the principal ideas of movements in the plane and space for better understanding about the role of it concept in design.

- Develop solving skills.The students gained experience in mathematical us a tool, finding by themselves its interpretation.

- Study technological disciplines. Because of the workshop, the students had a better understanding of statics concepts and recognized the role of mathematics as a design tool.

- Acquire computer-designing skills. Knowledge of mathematical methods in contour design is an important prerequisite for CAD studies.

Summing up this new way of teaching mathematics to students make them analyze the role of the discipline in design applications and enhance their creativity. Working the relationship between linear transformations with computer´s graphics and fractals it may be expected to achieve a description, in every representation, of a new architectural space.

Bibliography

Abell M., Braselton J.,*Mathematica by Example* Ed. 1994

Bahder T., *Mathematica for Scientists and Engineers*. Ed. 1995

Gareth W. , *Algebra Lineal con Aplicaciones* Ed. 2002

Maeder R. ,*The Mathematica Programmer II* Ed. 1996

Mandelbrot B., and Frame M., *Fractals, graphics and mathematics education.*Ed. 2004

Mandelbrot B., *The fractal geometry of nature*. Ed. 2003

Nakos G., Joyner D., *Algebra Lineal con Aplicaciones* Ed. 1999