Automatic procedure for the dimensioning and arrangement of space units of an architectural organism

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***Abstract**:*

The application of a Mathematical Programming (M.P.) technique, typical of Operational Research (O. R.), is proposed as a means to cope with the decisional problem of layout dimensioning and arrangement. Within the ambit of O.R., Mathematical Programming. deals with decisional problems of simplest structure: only one decision factor, only one preference function, complete (deterministic) knowledge of the environment in which one operates.

Such a problem, in standard form, presents an objective function $Z=f(\underline{x})$*, of n variables* $\underline{x}$*, to be minimized and a system of linear equations and/or inequalities, on the same variables, which represent the constraints and which define an admissible area for the solution.*

The architectural organism is modelled as an assembly of parallelepiped shaped space entities or units, provided with a certain number of "holes" that permit functional corresponding connection. The pursued intent being optimal assembly.

The model, in its mathematical form, fits a standard Non-Linear M.P. (N.L.P.) problem, since the objective function Z is non-linear and the constraints are represented by inequalities. In its graphic form it reproduces an image of all the space units constituting the organism; moreover it is able to represent these units, in their logical and physical individuality, and their mutual relationship, as well as the ones with the external environment.

In fact, from dimensional prerequisites, for each unit constraints can be generated on its surface or sides, as non-linear inequalities on the dimensional variables; constraint inequalities on the variables representing the 'holes' can be generated from the adjacency prerequisites, that define the functional connections between the units; further constraints, equations or inequalities, can be generated from other architectural prerequisites, such as exposures, alignments, symmetries.

Some limitations inherent in the proposed model are discussed together with the "device" to overcome them, such as linearization of the quadratic constraints on the surfaces, or elimination of unit penetration or detachment between units by minimizing the square of their value. With regard to the objective function consideration was given to minimizing the units surfaces (multiplied by given cost coefficient), or the external perimeter, etc.. The possibility for the introduction of diverse and more complex quantities is not excluded.

Such a formulation allows the application of an iterative resolutive technique known as Gradient Method (G.M.): given an initial point, consistent with the area of admissibility, a solution is automatically generated corresponding to a point of relative minimum for the objective function. Such

a resolution is not unique and many solutions can be generated by changing the initial point.

In a project of great dimensions, in which the spatial arrangement of the units includes the management of many data that are interconnected in a complex way, the proposed method can constitute an especially useful tool. It can be an effective numeric control of the dimensional and relational quantities at stake and then an useful tool to relieve the designer from the burden of purely mechanical operations freeing a lot of time to the exclusive human capacities for creativity.

INTRODUCTION

Methods for building design is a theme of some importance of the research in architecture.

Anyone that is interested in design, is aware of the difficulties in pursuing a solution, in a context where the complexity of "system building" more and more increases and involves, within the design process, different professional figures, highly specialized, which tell mutually their own requests by a precise and codified language.

The solution, therefore, has to be adequate to general objective, and to global limitations (sociological, psychological, formal, technological, economic), and to the peculiar requests of each room or even of each detail component.

Given the large number of parameters to keep in mind, the solution, that must adapt to all these needs, is frequently obtained by a process of issue and feed-back repeated, for successive approximations.

Hardly ever one solution only exists: many a time, several can be found, all possible.

Authoritative scholars, showing the "permeability of architecture" to the scientific method, referring also to structural and functional complexity of used materials, are led to think of architecture as a "system of systems".

More and more urgent is, therefore, the demand of tools for finding solutions to problems concerning "complex systems" (as the architectural ones), where by complex system we can mean, in short, a set of many variables, of different type, mutually connected by relations of even more different type (spatial, structural, formal, technological, economic).

The different nature of variables requires the contribution of various doctrines, that can supply it only if a precise, "formal", representation of the system exists, in short, an efficient "model", for all of them.

Some attempts have been made according to different approaches, but research has not been concentrated yet on a reduced set of general procedures, which can constitute the steady portion of a formal and systematic method for architectural design.

Particularly - we recall - many tools of spatial arrangement that minimize the distance, as cost of moving, have dealt with the problem of optimal location of rooms#1. Another technique can be constituted by a refining of the method of the permutations, originally supplied by Armour and Buffa#2. Some studies have considered the possibilities of the representation of layouts by the "poli-omino" theory#3 or, some others, by the LPAOR of U. Flemming#4, that has constructed a kind of mathematical theory of rectangles.

Among the systematic methods for transforming the adjacency graph in an architectural plan, the most general results were obtained by the method due to J. Roth, R. Hashimshony and A. Wachman#5, that is proceeding through various steps: orientation and separation of a planar graph into two sub-graphs (according to X and Y axis) by a graphic technique called "coloration"; transformation of them in dimensioned sub-graphs (with determination of the critic distances by using the PERT method); generation of a concrete plan; verification *ex post* of the respect of the adjacency requests; selection of plan alternatives.

In this lode, the present work intends to tackle, therefore, a typical problem of architectural design: the dimensioning and arrangement, within a constrained ambit, of layouts already selected and optimized for the relational prerequisites.

The proposed tool can take a place inside the procedures of CAAD.

1. FROM THE PROBLEM TO THE MODEL

In the ambit of the tools of aided design the present method is proposed as an automatic procedure which provides the design of organism with dimensions and distribution in accordance with the dimensional and relational constraints; the first ones consist in fixed intervals of dimensions for each room, the latter consist in the layout and other constraints of typically architectural kind. The procedure realizes the optimization of a pre-set objective.

This happens through the application of a Mathematical Programming (M.P.) technique, typical of Operational Research (O. R.), to a mathematical "model" of the architectural organism#6.

This is possible since the general characteristics that the problem presents, if we introduce some simplifications, match well with what the mathematical tool adopted needs.

In fact, within the ambit of Operational Research, M.P. deals with decisional problems of very simple structure: only one decision factor, only one preference function, complete (deterministic) knowledge of the environment. These characteristics seem to fit well in architectural problems: the greatest limitation is, perhaps, the singleness of the preference function, since, sometimes, more than one (reciprocally contrasting) could be necessary. However the hypothesis can be accepted as first approach.

A problem of M.P., in standard mathematical form, presents an objective function Z=$f(\underline{x})$, of n variables $\underline{x} = (x_1, x_2, ..., x_n)$ to minimize, and a system of linear equations and/or inequalities, on the same variables, which represent the constraints and which define an admissible area A into which the solution has to be found.

min Z = $f(\underline{x})$ objective function

$$\begin{cases} a_{11}x_1 & + a_{12}x_2 & + \dots & + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 & + a_{22}x_2 & + \dots & + a_{2n}x_n & \geq b_2 \\ \dots & \dots & \dots & \dots & \dots \\ a_{p1}x_1 & + a_{p2}x_2 & + \dots & + a_{pn}x_n & \geq b_p \end{cases}$$

linear constraints which define A

$x_i \geq 0$ possible constraints of non-negativeness

The type of objective function $f(\underline{x})$ (o.f.) determines the category of the programming problem in relation to the resolutory technique:

- if the o.f. is linear it is a matter of Linear Programming (L.P.), that can be tackled by rather simple resolutive techniques (for instance by the Simplex Method) ;
- if the o.f. is quadratic and the constraints are exclusively of equality it is matter of Quadratic Convex Programming referable to the above mentioned L.P.)

- otherwise we have Non-Linear Programming (N.L.P.), that must be tackled by not very easy classic methods (for example the application according to Kuhn-Tucker of the technique of the Lagrange-Multipliers), or else by numeric methods of iterative-evolutive type, as the Gradient Method (G.M.).

The crucial point, in this type of problems, lies in managing to make a *"model"*, one way corresponding, in its mathematical form, to the requested structure for the application of resolutory method, another way able to well represent the problem and the whole of constraints to be imposed.

The formulation of the model (of mathematical type, in this case) implies the passage from a description of words, of the object studied and of the problem to solve, to one expressed by a set of symbols, operations and relations that define the connections between the variables and the data of the problem. In the construction of the model, to facilitate the finding of the characteristic quantities that represent the problem with an acceptable percentage, it is convenient to distinguish goals, components, variables, parameters, relationships, events or activities.

In our case, in pursuing the intent of optimal assembly, the architectural organism, reduced to the essential, can be modelled as an assembly of N *Components* constituted by parallelepiped shaped space entities or units, provided with a maximum of one "hole" for each side, that permit functional corresponding connection.

Without lessening the generality of dealing in 3D, for simplicity sake, we refer in the following, to a problem in 2D (Fig. n. 1):

the *Variables* will be represented, for instance, by the 2*N coordinates, indicating the position of each unit, or rather of a characteristic prefixed point (for instance the lower left corner), in the frame of reference; by the 2*N dimensions of the units sides parallel to the Cartesian axes; by the 4*N coordinates indicating the position of each hole, or rather of its characteristic point (for instance the left corner), in respect to the origin of the side on which they are situated;

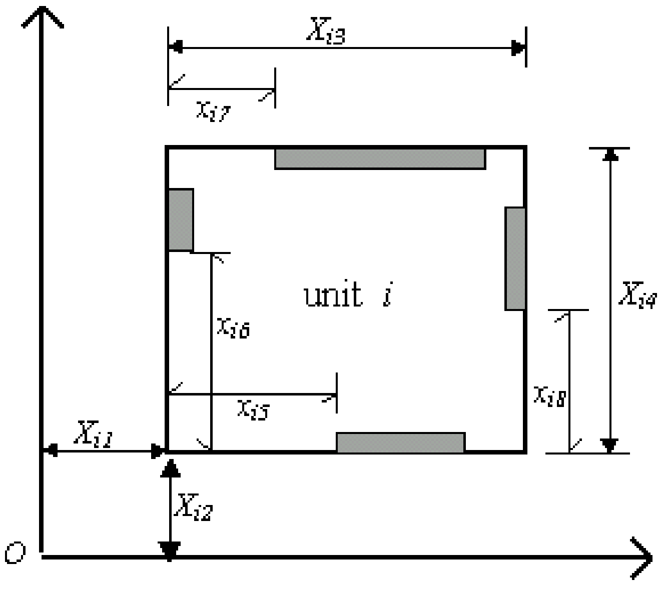

Fig. n. 1. *Basic component of the model: the space unit.*

the *Parameters* will be, for instance, the 4*N values indicating the dimension of each hole, the 2*N values indicating the minimum and maximum surface admissible for each unit, the 2*N values indicating the minimum and maximum dimension admissible for the sides of the unit, the P cost coefficient relative to the surface of the unit, to the perimeter, etc., and others that can be thought out;

the *Relations*, that represent the constraints of the problem, can be expressed as equations or inequalities on the variables and the parameters indicated above. In fact, from dimensional prerequisites, for each unit constraints on its surface or sides can be generated as non-linear inequalities on the dimensional variables; constraint inequalities on the variables representing the ‘holes’ (position and dimension) can be generated from the adjacency prerequisites (layout), that define the functional connection of the units; further constraints equations or inequalities can be generated from other architectural prerequisites, such as exposures, alignments, symmetries.

It is easily verifiable that an aggregation of such elements is able to represent effectively any given layout of an architectural organism.

In fact, the apparent restriction of only one adjacency for each side can be overcome dividing the units which require a greater number of adjacencies in as many further units as many are holes on one side, and making the hole corresponding to the division such that consents the carrying out of the activities expected for that room.

The presence of a high number of adjacencies may depend, perhaps, on the presence of different locations of activities in the unit; therefore the division can happen so as not to interfere with these.

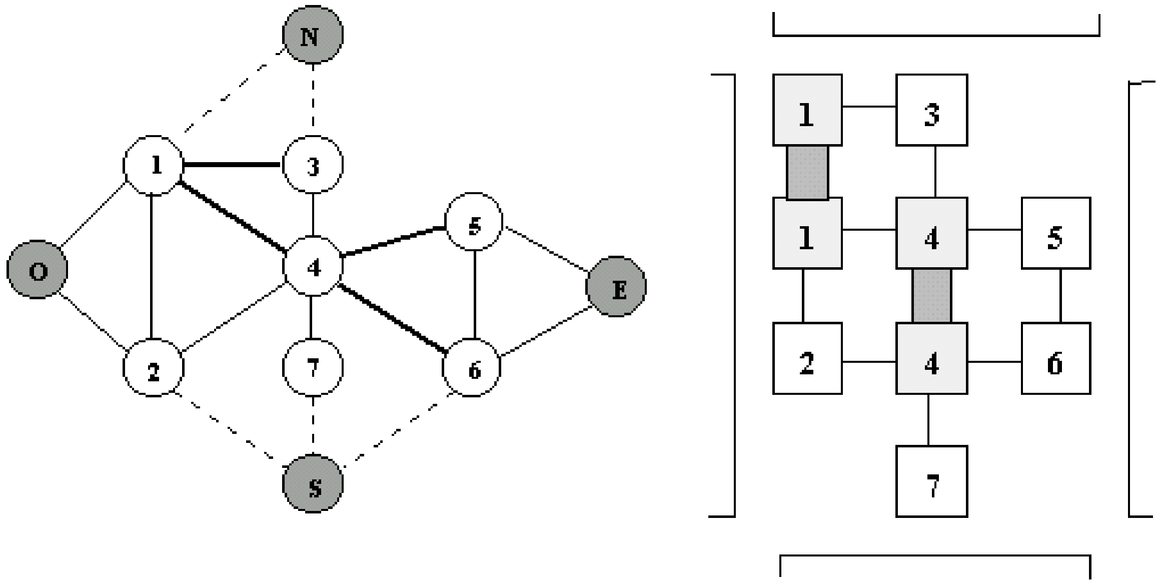

Fig. n. 2. *Transformation of a graph to fit the model.*

Moreover the measure of that special kind of hole can be fixed equal to the same dimension of unit in that direction.

Considering this rule of division of one unit in some ones, any graph can be represented by a set of the above-stated units, each connected, at most, by only one adjacency for every side (Fig. n. 2).

Such an organization is the model on which the optimal dimensioning is operated.

Others limitations are inherent in the proposed model: however some "device" to overcome them can be found.

As one can easily see, for uncontrolled adjacencies, varying the units dimensions, undesirable penetrations or detachments between units (Fig. n. 3) can occur.

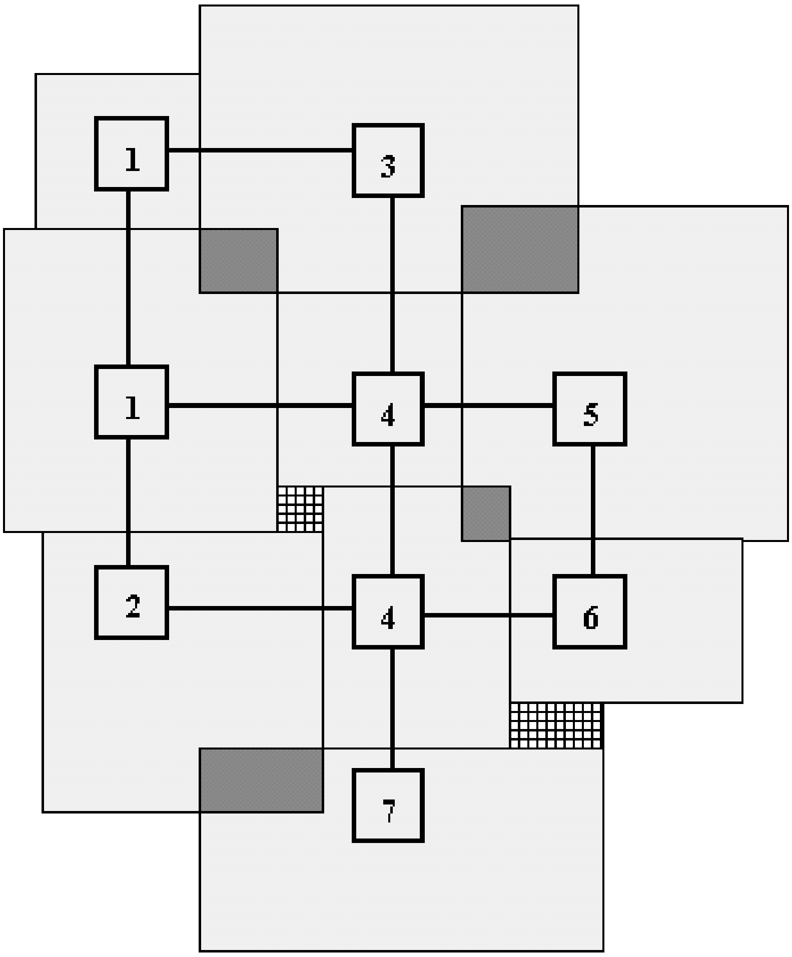

Fig. n. 3. *The penetrations and the detachments to eliminate.*

The resolution of the drawback, as shown in Fig. n. 4, leads up to different kinds of solution according to the way to realize the uncontrolled adjacencies.

In this can be found a decisional mechanism: in fact it is necessary to decide the shape of the units before writing the dimensional relations between them; but this is also a consequence of the the dimensions that the units will be taking after the solution of the dimensional problem. A decision in a state of uncertainty would be taken.

On the contrary, considering that no adjacency is required for the penetrated-detached area, both the lack of adjacencies and the possibility of having different types of them, correspond, in the same way, to a relational prerequisite for the units.

So, if no relation is fixed on the adjacencies relative to the penetrations_detachments, the problem, well stated in accordance to the relations, remains to be solved only for dimensional questions: in short, it will be sufficient to reduce the size of the penetrated or detached areas to the minimum (in absolute value), i.e. to zero.

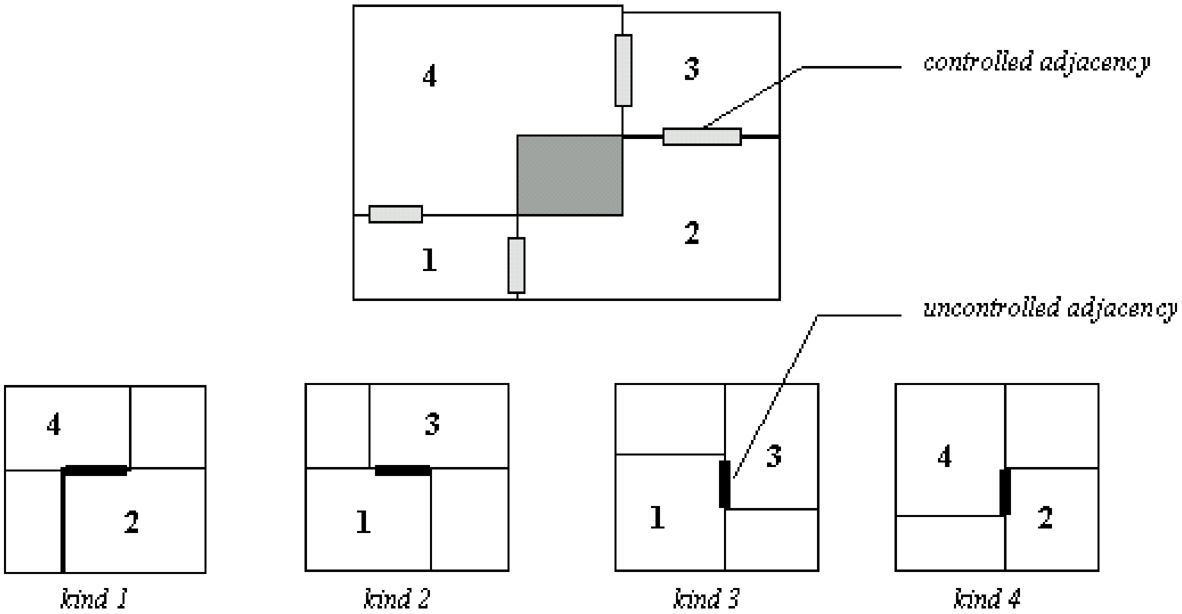

Fig. n. 4. *The overlap implies a decision.*

The approach that is interesting consists in allowing, at the level of the model, the penetration_detachment between units, without worrying to operate before any choice to avoid it, and then in reducing the effects to minimum by optimization (minimizing also the square of their value).

With regard to this, it must be considered that the objective of minimizing the penetrations-detachments could be in conflict with other objectives, as minimizing the area of each unit (eventually multiplied by a cost coefficient) or the perimeter of organism, or with any other objective. To search for the solution a problem of strategy arises that can be tackled by the introduction of an adequate parameter of cost (M) also for the penetrations-detachments; the "character" M, to award to the behaviour of the model, can be defined by analysing the results of various tests with different values of M.

Another difficulty can be the fact that, sometimes, owing to the structure of the area of admissibility, it could be impossible to reach the zero value for each penetration-detachment: in these cases some constraints have to be relaxed or rather some penetrations-detachments have to be admitted and in a measure tolerated.

Therefore the choice of the objectives, and precisely of the objective function (o.f.) in the case of M.P., is an intrinsic part of the model, since o.f. represents the link among Variables, Parameters, Relations (constraints) and Objectives, that leads the search of the solution: a lot of accuracy is needed to set it and to re-define it (in reference to form, variables, parameters and coefficients involved) on the basis of the analysis of the results of various tests. It does not exclude the possibility for the introduction of diverse and more complex quantities.

At this point, if we provide for the further "device" of the linearization of the quadratic constraints on the surfaces, the proposed model, in its mathematical form, fits a standard Non-Linear M.P. (N.L.P.) problem, since the o.f. is, likely non-linear and the constraints are represented by linear equations or inequalities. In its graphic form it reproduces an image of all the space units constituting the organism; moreover it is able to represent these units, in their logical and physical individuality, and their mutual relationships, as well as the ones with the external environment.

Of course the operation of modelling isn't direct and univocal, so a mending (or a total conceptual reformulation) can be required in a process of feed-back on a large number of tests and trials.

2. THE RESOLUTIVE ALGORITHM: CONSIDERATIONS FOR THE APPLICATION OF THE GRADIENT METHOD.

The approach that is considered here refers to the direct use of the o.f. and of the equations of constraint by numeric methods that realize the optimization by evolving from an arbitrary point to a point of minimum for the o.f. with consecutive approximations.

The mathematical formulation of the model as a N.L.P. problem allows the application of one of these iterative techniques known as Gradient Method (G.M.): given an initial point, consistent with the area of admissibility, a solution is automatically generated corresponding to point of relative minimum for the o.f.

I omit to describe it in detail as there is an extensive literature on its workings: its graphic representation, in the simple case of two variables, is shown in Fig. n. 5.

However it can be useful to report a few considerations on the application of the G.M..

1.1 The linearity of the constraint equations.

The presence, in the model, of constraints on the surfaces implies the generation of non_linear equations; on the contrary the G.M. requests the area of admissibility for the variables (A) be defined by a set of linear constraint equations on those variables. The question has been settled by a linearization of those constraints that, for the way as it has been made and for the presence of constraints on the sides of the units, has produced an acceptable approximation.

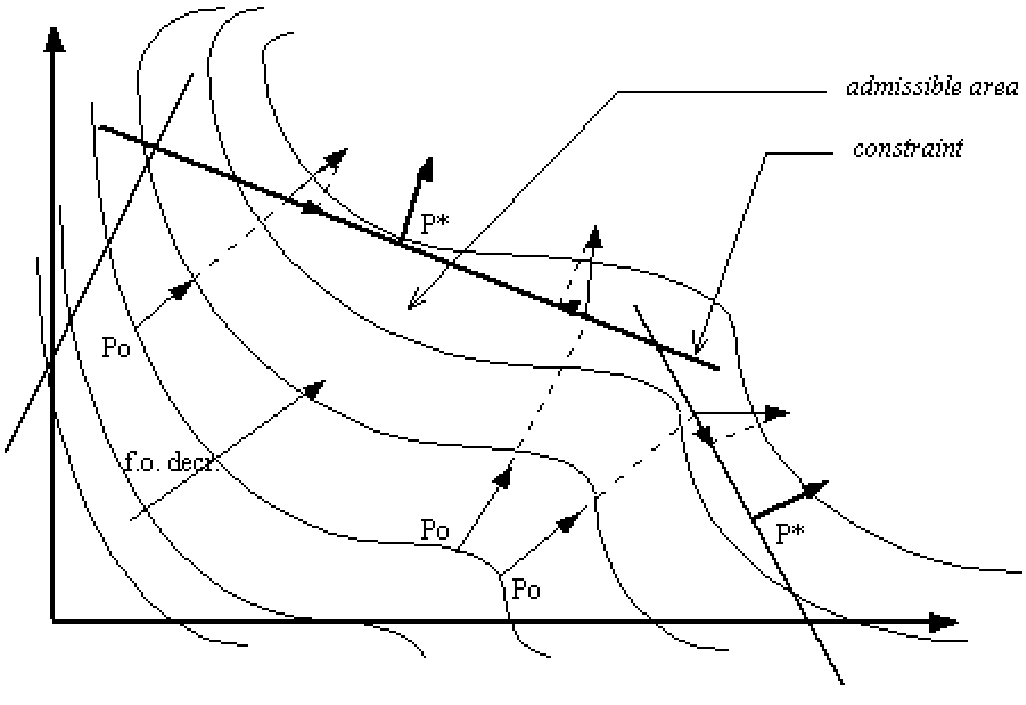

Fig. n. 5. *Local and global convergence of G.M.*

1.2 The convergence of the method.

The theorem on gradient projection assures that the method converges to a global minimum if and only if the o.f. is convex in the admissible area and with continuous second partial derivatives.

This condition for the above-mentioned o.f. has not been specifically investigated, but, in general, could be unverified: in this case the method will converge to a local minimum. Such a resolution is not unique and several solutions can be generated by changing the initial point. Among them, likely, one can be found corresponding to global minimum.

<u>1.3 The request of an initial point.</u>

The necessary condition for beginning the iteration is the initial point (solution) being inside the admissible area by constraints defined. Now, in practice, the possibility to know an initial solution constitutes an extremely complex question, since such a point has to satisfy all the imposed constraint equations, and they are so many.

I' m studying theoretical methods to solve such a problem, but the structure itself of the equations makes each approach difficult. Consequently, for the applications, an automatic routine has been provisionally made that provides a starting-point by a simple arrangement of logical type according to the constraints.

<u>1.4 The implementation.</u>

In conclusion it has to be specified that the implementation that I've set of the proposed method, by the coding of a fitting program, constitutes a first test of verification in practice on the running of the model and on its congruence.

The program, at present stage, requires further improvement and development; however it has shown the effective workings of the method expounded and its capabilities.

An example of arrangement and dimensioning of units supplied by the program is reported in Fig. n. 6.

3. CONCLUSIONS

In a project of great dimensions, in which the spatial arrangement of the units includes the management of many data that are interconnected in a complex way, the proposed method can constitute a useful tool.

The ambit proper of a Computer Aided Design must be to supply the adequate tools for exerting a stringent, mathematical, "control" on the relational and dimensional quantities at stake, a control that doesn't limit at all the freedom of the ideation, but instead increases it: it can facilitate the exclusive human capacities for creativity.

Thus, the present work, far from to constituting a method of design, must be considered as a further occasion to discuss about the permeability of operating in architecture to the scientific method and about the relationships between the questions put to the actors of the design process and the available resources (logic, mathematical tools, modelling, systematization) to tackle them and to make automatic tools of aid and control for the design process.

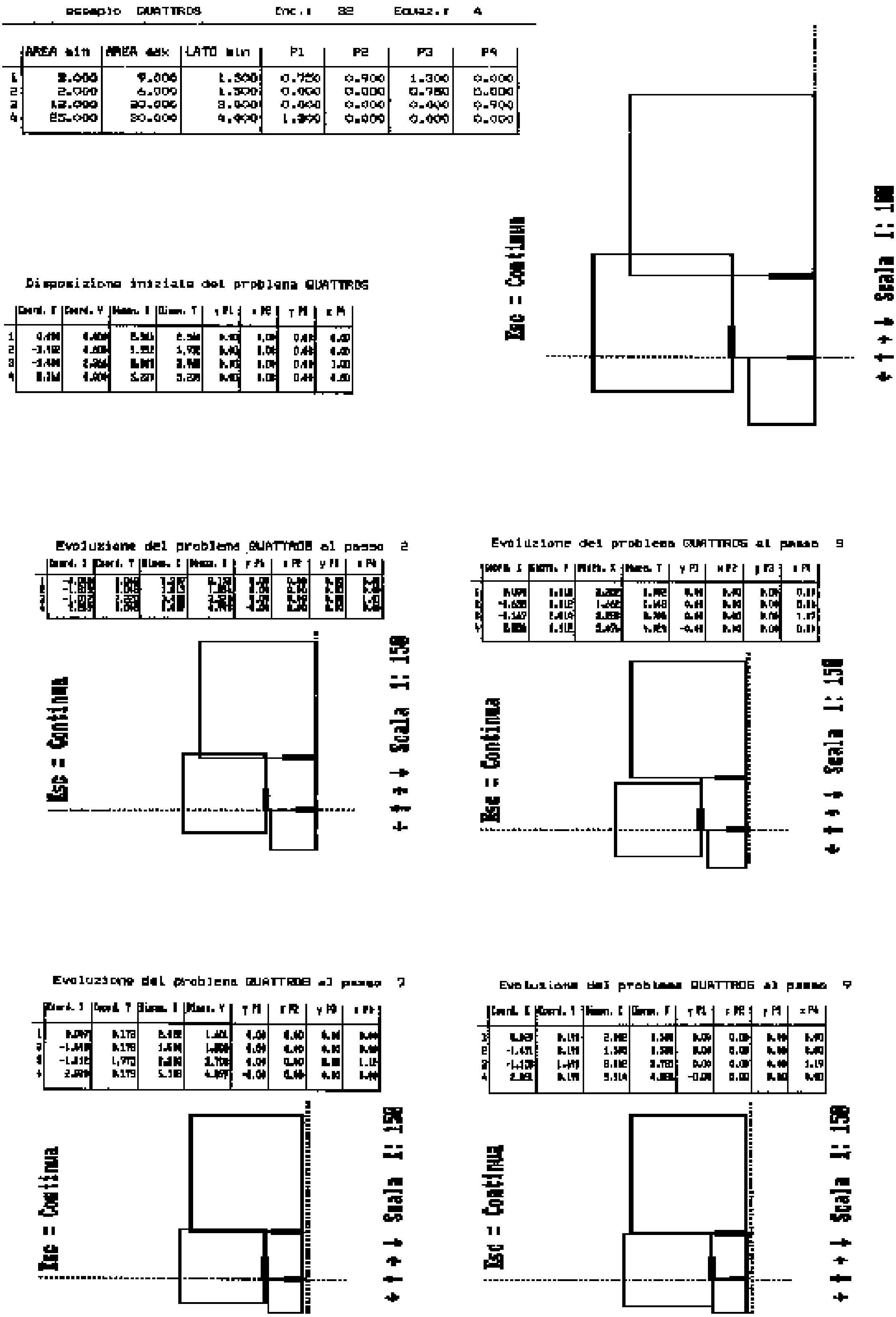

Fig. n. 6. *An example of dimensioning supplied by the program.*

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