Surface Structures: digital design and fabrication

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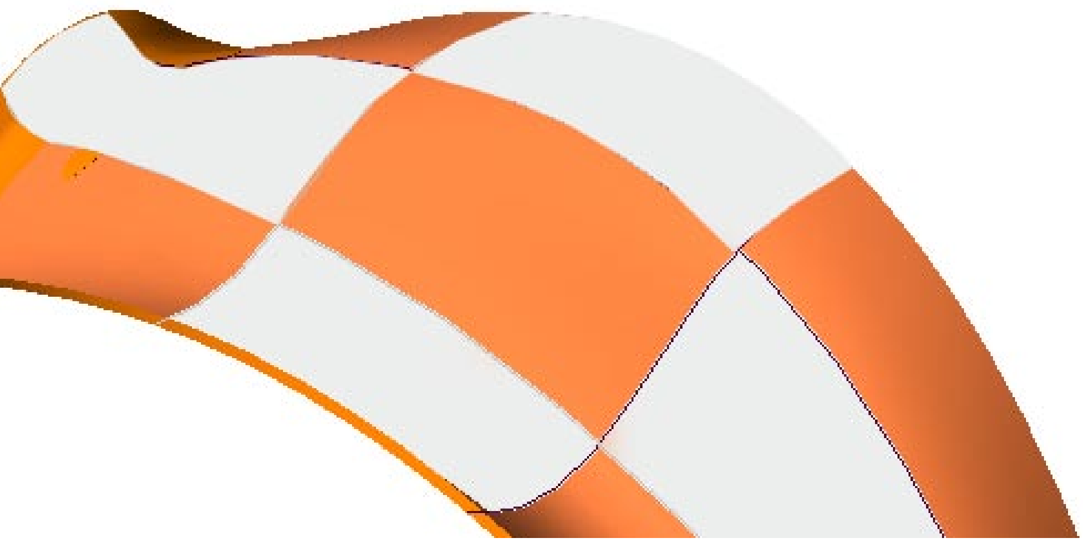

Abstract

This paper presents a study in digital design and manufacturing of shells, which are material-efficient systems that generate their load-bearing capacity through curvature. Their complex shapes are challenging to build, and the few current shell projects employ the same shape repetitively in order to reduce the cost of concrete formwork. Can digital design and manufacturing technology make these systems suitable for the needs of the 21st century?

The research developed new digitally-driven fabrication processes for Wood-Foam Sandwich Shells and Ferrocement-Concrete Sandwich Shells. These are partially pre-fabricated in order to allow for the application of Computer-Numerically Controlled (CNC) technology. Sandwich systems offer advantages for the digitally-enabled construction of shells, while at the same time improving their structural and thermal performance. The research defines design and manufacturing processes that reduce the need for repetition in order to save costs. Wood-Foam Sandwich shells are made by laminating wood-strips over a CNC-milled foam mold that eventually becomes the structural sandwich core. For

Ferrocement-Concrete sandwich shells, a two-stage process is presented: pre-fabricated ferrocement panels become the permanent formwork for a cast-in-place concrete shell.

The design and engineering process is facilitated through the use of parametric solid modeling environments. Modeling macros and integrated Finite-Element Analysis tools streamline the design process. Accuracy in fabrication is maintained by using CNC techniques for the majority of the shaping processes. The digital design and manufacturing parameters for each process are verified through design and fabrication studies that include prototypes, mockups and physical scale models.

Introduction

This paper presents digital design and manufacturing processes for shells, curved structural surfaces that carry loads through material-efficient membrane stresses. Their design, structure, and fabrication need to be addressed throughout the design process, starting during conceptual design. The integration of these aspects is facilitated by digital design environments that integrate design, structural finite-element analysis and support for Computer-Numerically Controlled (CNC) fabrication technology.

Simply lending curvature to a surface does not automatically generate any structural capacity; instead, the shapes of shells need to be carefully designed to well-established structural principles. Shells either follow certain simple geometric shapes such as sphere segments, hyperbolic paraboloids or hyperboloids, or their shapes need to be based on equilibrium figures. Such equilibrium shapes are generated through the balance of external loads and internal forces. For each set of loads, internal forces, and support conditions a single equilibrium shape exists. The shape can be derived from accurate physical model experiments such as those using hanging fabrics (Fig. 1), pressurized membranes as documented in Heinz Isler's work, or through other model experiments. Alternatively, they can be generated through various computational form finding procedures, all of which are essentially devised to reproduce the shaping of physical models. A well-known computational method for formfinding is the force-density method, originally designed to find the equilibrium shape of cable networks but since expanded to include rigid shells (Linkwitz and Schek 1971).

The shaping of shells is well understood, but the challenge of constructing complexly shaped thin surfaces remains. In the past shells have been built mostly in reinforced concrete, with few exceptions in timber, fiberglass, or post-tensioned masonry. The major challenge for concrete shells today is the cost involved in the building of the formwork. Traditional timber formwork is labor-intense, and with high labor costs relative to the cost of concrete and steel, it typically represents a major portion of the cost of a shell. Timber shells, on the other hand, are restricted to ruled surface shapes. This research set out to devise new ways to approach shell construction, addressing the dilemma of formwork cost and shape restrictions. For the design of the shell parametric models are presented that facilitate an integrated approach to shaping, structural analysis and preparation of data for digitally supported manufacturing processes.

First an overview of traditional construction techniques is presented, followed by a summary of the general principles of digital design and fabrication for shells. Wood-Foam Sandwich shell and Ferrocement-Concrete Sandwich shells are then described in more detail, followed by a comparative analysis and conclusions.

Curved Structural Surfaces: the Challenge of Complex Shapes

1. Concrete Shells

Existing construction techniques for concrete shells typically employ rigid timber formwork. A network of columns carries an assembly of curved and straight timber beams. Thin wooden boards are bent over the beams, cut to shape and fastened. The boards form the surface onto which the reinforcement bars are placed. Stiff concrete mixes are then placed in a single pour onto the formwork. This time-consuming construction technique was feasible as long as labor costs were relatively low compared to material cost. During the past decades this cost division has been inverted, because labor costs have increased approximately twice as fast as material costs (U.S. Department of Labor 2003).

Figure 1. Conceptual physical formfinding model: a hanging fabric, supported continuously along its edges, is coated with plaster. As the fabric has to carry the weight of the material, it automatically takes on a funicular equilibrium shape. Each change in loads or support conditions would yield a different shape.

Figure 2. Subdividing a parametric shell model into segments

Several alternative construction techniques evolved late in the 1940s. Pier Luigi Nervi began to use pre-fabricated formwork elements in his 1949 Turin Exhibition Building. The same technique was used in several of his dome shells, such as the Palazetto dello Sport in Rome in 1957. The prefabricated formwork elements were made from a fine, mesh reinforced cement mortar, ferrocement. After assembly of these elements on temporary scaffolding, additional reinforcement was placed, and a site-poured layer of concrete ensured the structural connection between the ferrocement elements. The prefabricated formwork thus became permanently embedded into the finished shell. This construction method reduced the construction time and the formwork cost, because it eliminated the need to construct a complete rigid timber formwork surface for the shell. But the molds used to fabricate the ferrocement formwork had to be used repetitively for the sake of cost cutting, thus resulting in restrictions to symmetrical, regular shell shapes based on simple geometry such as cylinders or spheres.

The 1940s also saw the development of pneumatic formwork in the effort to reduce the construction time and costs of concrete shells. Here membranes are pressurized to temporarily support the reinforcement and the concrete until it is cured. The concrete may be placed from the outside or from the inside. Pneumatic formwork is still used, especially in the United States, but the shells constructed with this method are typically restricted to synclastic shapes. Currently, only regular domes are built with inflatable formwork. Deflections of the membrane during the curing process and the associated weakening of the concrete can be problematic when using pneumatic formwork.

A different approach is taken by H. Isler in Switzerland. His highly irregular forms are built on a set of scaffolding columns and curved, glue-laminated timber beams. Over these a network of thin boards is laid, onto which rigid insulation boards describe the shell shape. The reinforcement is laid onto the insulation board, and once the concrete has been poured these boards become permanently bonded to the concrete shell, thus creating the interior surface finish as well as thermally insulating the roof. The high formwork costs are offset by reusing identical formwork elements several times. Isler's shell designs often feature linear arrays of identical shells that are built with the same formwork elements.

In summary, the alternatives to conventional wooden formwork are either limiting in terms of geometry (pneumatic systems) or are costly and require repetitive use in order to be economically feasible.

2. Timber Shells

Timber is an alternative material in shell construction. It has a favorable strength-to-weight ratio, can be easily manipulated with simple hand tools, and is generally considered aesthetically pleasing. Due to the difficulty of bending wood in two directions, timber shells have generally followed ruled surfaces such as hyperbolic paraboloid (HP) shapes. These shells can be built from boards that are bent in a single direction, usually the one following the set of perpendicular parabolas that describe the HP shape. The boards are applied in several layers that are mechanically fastened to each other. The temporary support structure needed for the shaping of the board layers consists of column-supported beams that follow the ruling lines of the HP shape.

Designers are severely restricted in the shapes available for timber shells. Some recent grid shells, for example the roofs over the spas in Bad Dürrheim or Bad Neuenahr, Germany, have been built to geometrically more complex shapes. However, these were exceptional projects only possible with extremely qualified craftsmen that are increasingly hard to find.

3. Pre-Fabrication

The challenge of fabricating complexly shaped structural surfaces is perfectly suited to exploiting digital design and fabrication techniques. CNC fabrication technologies can facilitate the generation of complexly shaped shell surfaces, but they are currently not available on site. Therefore the new processes presented here are based on pre-fabrication, which at the same time limits weather dependability and potentially reduces construction tolerances.

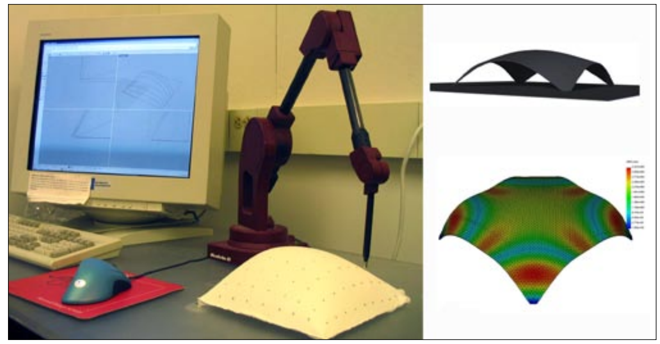

Figure 3. Left: Digitizing a conceptual physical shell model. Laser scanning techniques (not shown) are commonly used for final, accurate measurements. Right: results of computational form-finding techniques using the general finite-element method.

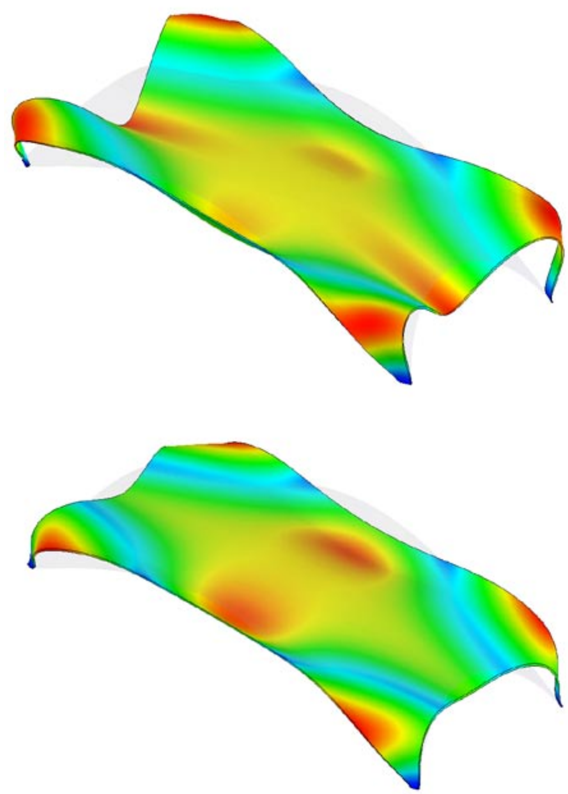

Figure 4. Comparative FEA deflection study of a shell: Model 1 on the left has increased thickness near the supports; Model 2 on the right has constant thickness. The deflections are shown magnified by a factor of 50. Model 2 has much larger deflections, especially near the supports.

Large shells are unlikely to be transportable in one piece. Subdivision into segments, and the assembly and joining of these segments on site are therefore essential components of any digitally supported pre-fabrication process. The allowable sizes of these shell segments are restricted by the maximum size permissible for road transport. Machining processes during pre-fabrication, structural and aesthetic considerations might also impact the joint layout. Segment layouts are likely to be the result of an iterative process – one well supported by parametric modeling that allows for the testing of multiple layouts based on the same shell design (Fig. 2).

4. Digital Design Techniques

Accuracy in both design and fabrication is essential especially for shells that follow equilibrium shapes, since deviations from the ideal shape result in undesirably large deflections. Geometry and structure are so closely connected that an integrated design environment that permits both design development as well as structural analysis using Finite-Element Analysis (FEA) is highly advantageous. The use of high-end parametric design environments such as CATIA or SolidWorks allows for integrated design approaches. These environments also support a variety of CNC fabrication processes that enable the translation of an accurate digital model into a physical reality.

The form-finding process either employs physical or computational modeling techniques. Data from computational processes is directly useable during design development, while the results of physical model experiments are digitized and then imported into a parametric digital modeling environment (Fig. 3). After correcting measuring errors a preliminary FEA can clarify the structural behavior. Some further correction of the shape may be necessary. The edges and support conditions usually need special attention. Once the shape is finalized the shell needs to be modeled as a solid prior to subdivision into segments for fabrication.

Most shells do not feature a constant thickness throughout the surface. Near the supports or at the edges the shell thickness often needs to be increased. The varying thickness of the shell should be incorporated into the model because the digital model is directly used for manufacture. The structural analysis also needs to take variations in thickness into account (Fig. 4). Common techniques for thickening surfaces do not usually allow for variations in thickness to be easily accommodated. Instead of using a single surface as a basis for a thickened solid, two separate surfaces need to be modeled. They follow slightly different geometry, merging where the surface thickness is minimal and diverging where the surface thickness is larger. On thickening both surfaces the resulting solids can be merged into a single body.

There are various ways of subdividing a complexly shaped shell into segments. It is desirable to retain an association to the master shape once the subdivision is completed, so that any changes in the master shape that may be necessary late in the design process propagate down to individual shell segments. One way to create linked subdivisions is to generate the shell segments as configurations or object instances of the overall shell. For each segment, the remaining part of the shell needs to be cut, for example by inserting a cut feature operation. These shell segments can then be rearranged into an assembly file.

Both design and fabrication processes presented here are based on a sandwich cross-section that needs to be incorporated into the digital model. In general, it is advisable to create the individual layers of the shell as separate part models that then can be arranged in an assembly model. This procedure allows for individual material properties to be assigned to the different layers of the shell in order to determine the overall mass, reactions, and, most importantly, to perform a realistic FEA that takes the sandwich build-up into account.

Wood-Foam Sandwich Shells

In order to free wood shells from the restriction to ruled surfaces, it is necessary to recompose the load-bearing surface from smaller wood elements. Commonly used boards are too stiff to be bent into a double curvature. While these boards are normally mechanically connected, adhesive bonding is the only feasible way to generate a rigid surface from thinner wood elements such as rotary cut veneer strips or strands. By relying on these thin strips, wood can be treated as a modern composite: aligning the wood fibers with the principal stresses in the shell maximizes the material efficiency because wood is strongest along the direction of the fibers. Adhesive bonding of several layers of thin wood strips to the desired curved shape can create a strong and visually pleasing structural surface.

Molding techniques in wood are traditionally a craft-based activity. Due to lack of data, craftspeople empirically determine the possible width and thickness of wood strips when molding doubly-curved shapes such as boat hulls. Good parametric modeling environments allow for a curvature analysis of any selected shell segments. Once the direction of the various layers of strips has been determined by the Finite-Element Analysis, the curvature analysis – carried out in the same environment – yields the required radii of curvature both along and perpendicular to the strip orientation for each layer. Algorithms developed by the author then enable the approximate prediction of strip width and thickness for a given wood species, based on the maximum permissible strain during the bending operation. For more tightly curved areas, thinner, narrower strips will be chosen, whereas fairly flat areas can be fabricated with thicker, wider strips. (Fig. 5) Species and grade can be varied according to the stresses present in the shell.

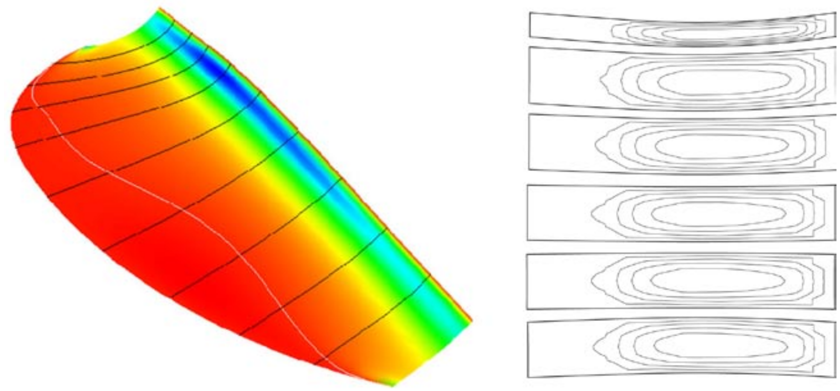

Figure 5. Left: The width and depth of wood strips are derived by determining the internal strains based on the curvature analysis. Right: Digitally flattened strips

Requirements for thermal insulation of the external envelope are significantly higher today than they were when shells were first built in the early 20th century. Instead of adding the insulation as a layer external to the structural surface, the insulation becomes an integral part of the shell as the core of a structural sandwich. Stiff synthetic foams such as high-density polyurethanes or polystyrenes improve the structural behavior of the shell by reducing deflections and improving the resistance against buckling.

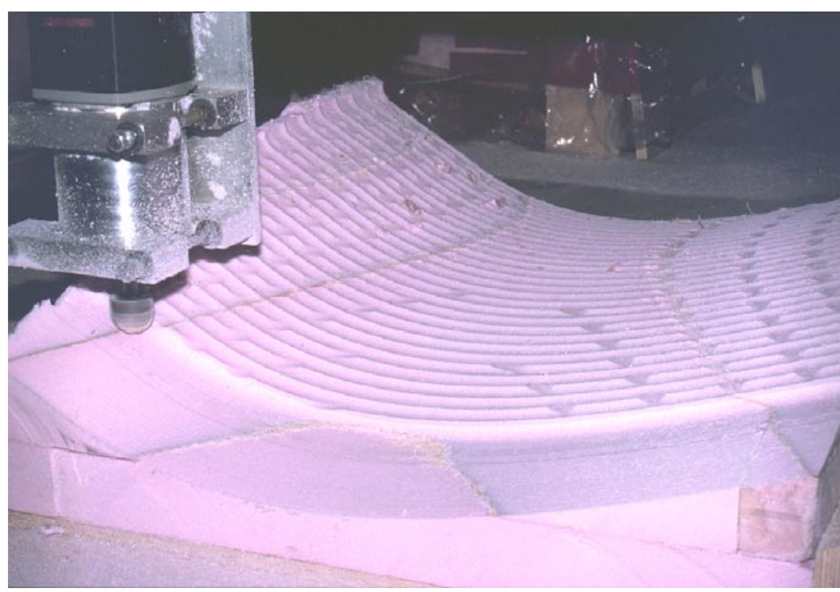

Figure 6. CNC-milling of a foam mold

1. Process Characteristics

A key factor in molding thin wood strips is the need for a mold – a rigid surface over which strips can be bent and to which they are temporarily fastened until the adhesive has set. These molds are usually discarded after the surface is finished. For Wood-Foam Sandwich shells, the complexly shaped molds consist of high-density foam, milled on a CNC-milling machine (Fig. 6). The shape of the molds is directly

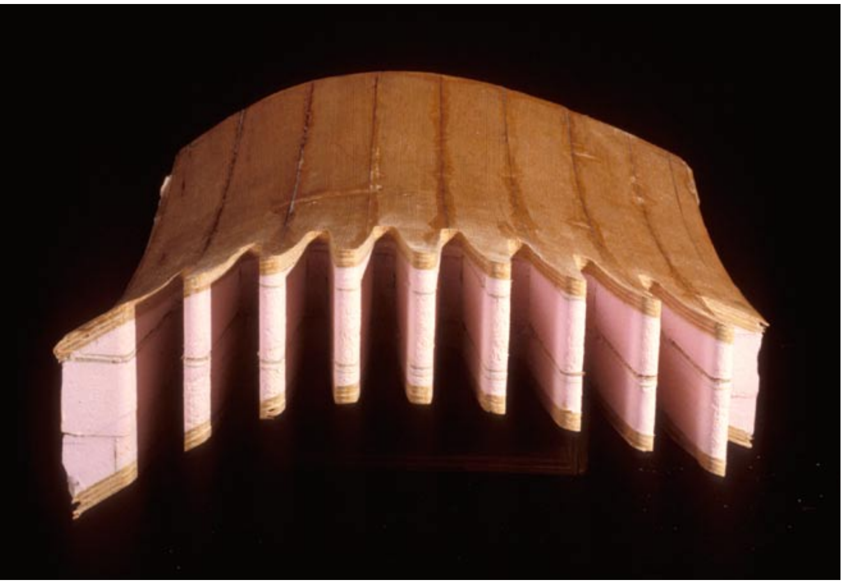

Figure 7. Prototypical sandwich shell panel with a milled fingerjoint

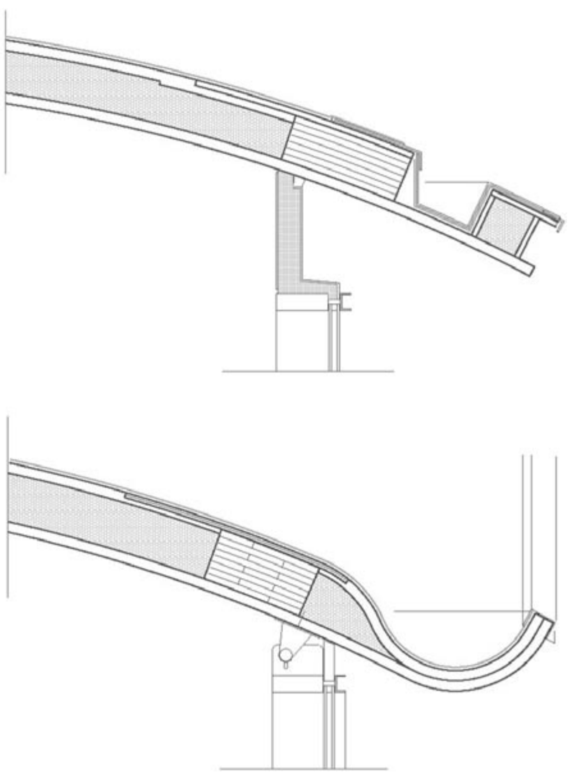

Figure 8. Possible edge details of a Wood-Foam Sandwich shell

derived from the parametric design model. This model allows for the generation of the toolpaths that, once post-processed, guide the CNC machine. Even though milled foam can be easily recycled it is still wasteful to discard this high-quality material. Instead, the mold itself becomes the sandwich core.

Depending on the type of foam used, the approximate shape of each segment mold can be quickly roughed out from a foam block with a CNC hot wire cutter. The final shape is milled on a CNC-milling machine using ball end mills. The strips are laid over the foam and temporarily bonded using an instantly curing adhesive while the structural adhesive, a synthetic, thermosetting resin, is curing. The following layers can be stapled on in order for the strips to remain in place during the curing. With one facing complete, the half-finished shell segment can be turned over for the machining of the other side. A simple egg-crate system is sufficient for the support during the second CNC-milling operation. The first facing provides enough stiffness through its curvature in combination with the rigid foam. The second facing can then be vacuum molded onto the smoothly milled surface, and a rigid, complexly shaped sandwich panel has been created (Figure 7). While still on the egg-crate support, the sides of the panel can be trimmed to the correct perimeter shape.

The shell segments need to be connected to adjacent segments in a structurally sound manner, transferring compressive, tensile, as well as bending stresses. A study of adhesive joint types that included physical load tests of large structural fingerjoints was carried out. The premise was that joints would be machined on the same CNC milling machine used for the shaping of the foam core. The wood-adhesive composite, as fabricated with the vacuum molding process, achieved the strength of commercially produced plywood while only requiring 5% of the lamination pressures that are typical in an industrial production setting. The CNC-milled joints, however, only achieved approximately 40% of the tensile strength of the solid composite. Comparable testing by NASA (Spera et al.) showed a maximum tension transfer of 94% for the same material and joint geometry. Since the NASA joints were cut using a large circular blade, it can be assumed that the milling operation did not generate a surface quality suitable for adhesive bonding. A CNC-machining center with a cutting blade would be the preferred way to prepare the joints. Alternatively, lap joints can be used with a combination of mechanical and adhesive connection. Such joints would also facilitate alignment during assembly.

Once all shell panels have been fabricated, they are transported to site where they have to be mounted on temporary supports, while the segments are structurally joined. The waterproofing and possible roofing material should then be applied as soon as possible.

2. Design Implications

Wood-Foam Sandwich shells are suitable to cover small to medium spans. Edge beams, if required, can be easily integrated into the depth of the sandwich. At the supports, larger areas of solid timber can take into account the force concentrations commonly present here. Steel inserts that connect the shell to the supporting structure are detailed, as would normally be the case for the supports of larger solid timber elements.

Edge features such as gutters can be incorporated into the shell design. They are formed by a reversal of curvature along the edge as is often seen in shells derived from equilibrium figures. Alternatively, gutters can be embedded into the depth of the structural sandwich (Fig. 8).

The shapes of Wood-Foam Sandwich shells should be generated with similar rigor as is recommended for any shell. The sandwich buildup can resist some bending moments, but it is not meant to compensate for a 'bad' shell shape with large bending moments and deflections. Since the sandwich depth, as well as the thickness of the facings and the strength grade of the materials can be varied, the system can accept some deviations from ideal shapes. Due to the relative complexity of the fabrication process, one might not choose to apply the technology to ruled surfaces, which are easily fabricated using traditional techniques.

CNC-Fabricated Formwork for Ferrocement-Concrete Sandwich Shells

For architectural and aesthetic reasons, and in cases where fire performance does not allow for a wood-foam shell, concrete may remain the material of choice. The process presented here is based on Nervi's use of pre-fabricated ferrocement formwork elements that become permanently embedded into the shell.

Ferrocement is a cementitious composite that is reinforced with a wire mesh. Traditional ferrocement techniques are very labor intensive, and the material is predominantly used in developing countries with low labor-to-material cost ratios. Ferrocement components are usually thin – between ¼ inch and 2 inches – and lend themselves well for use in structural surfaces. The fabrication process for concrete sandwich shells proposes to prefabricate the lower sandwich facing in ferrocement using CNC technology, and use this layer as formwork for the pouring of the concrete for the upper, principle layer on-site. This process is suitable for all types of shells – those based on simple geometry as well as those following equilibrium figures.

1. Process Characteristics

Following the subdivision of the shell into segments, a triangulated system of ribs needs to be modeled in a parametric digital design environment. These ribs accurately describe the segment shape; they serve as guides for the fabrication of the ferrocement formwork. All ribs need to be planar section cuts of the complexly curved surface, since they are to be fabricated from flat steel plate stock using a CNC-plasma cutter or other CNC tool. Modeling macros can facilitate the modeling of the ribs, as well as generate the slot and tab system that is devised to facilitate connections between ribs. The triangulation ensures that, once assembled and welded together, the rib network accurately describes the shape of the shell segment. Edge ribs are generally deeper than the intermediate ribs (Fig. 9).

Onto the underside of the ribs thin, bendable steel rods are applied in CNC-cut slots. These rods serve as supports for an assembly of fiberglass cloth and steel mesh. The cement matrix can be applied directly onto this assembly, using either mechanized

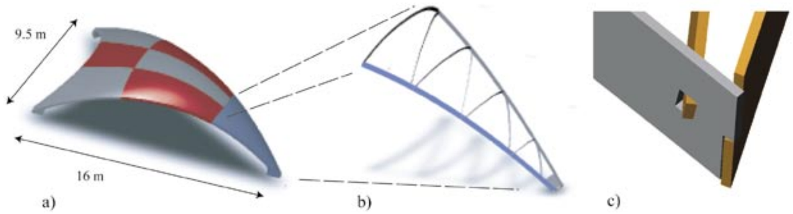

Figure 9. The shell is subdivided into segments (a). A triangulated network of steel ribs is parametrically modeled (b). The slotted connection, modeled with a custom macro (c).

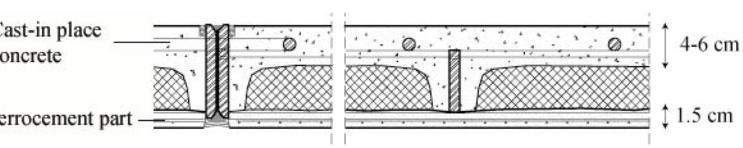

Figure 10. Cross-section through a Ferrocement-Concrete Sandwich shell: a segment joint shown on the left

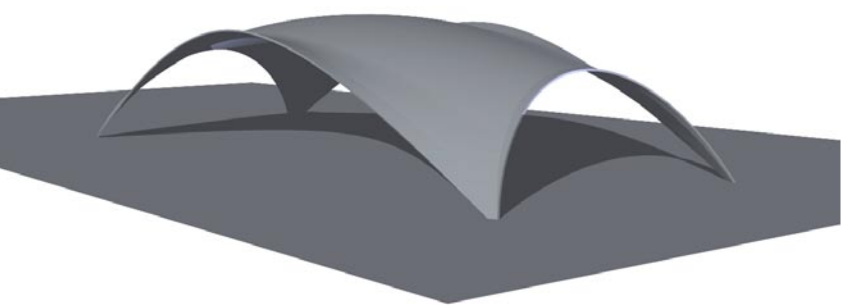

Figure 11. Parametric model of Isler's shell

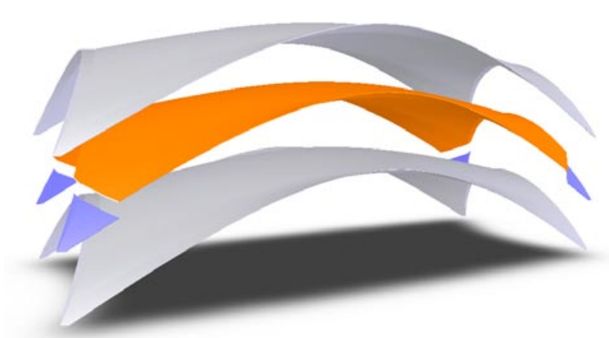

Figure 12. The components of the concrete sandwich shell in exploded view

applicators (shotcreting) or hand-plastering techniques. Once cured, the interstitial space between the ribs is filled with insulating foam. The zone immediately adjacent to the ribs is left free and is later filled with concrete, to ensure sufficient shear stiffness between the outer concrete layer and the inner ferrocement layer.

The prefabricated ferrocement formwork is assembled on-site while supported on temporary scaffolding. The segments are welded together along the edge ribs, into a rigid structure that is only missing its upper concrete layer. The connections of the ribs to the supports need to be made. Slots on the upper side of the ribs guide the placing of the principal reinforcement on-site, greatly facilitating the otherwise complicated measuring and placement process of the bars. With the reinforcement in place, the upper concrete layer can be poured in a conventional manner. Once fully cured the concrete and the ferrocement layers form a rigid structural sandwich. The shear-resistant connection is achieved through the concrete-embedded steel ribs. The foam does not serve any structural role, but improves the thermal resistance of the shell (Fig. 10).

2. Design Implications

Ferrocement-Concrete Sandwich shells are suitable for a wide range of shapes and sizes. In a conservative scenario these shells can be built even within current codes for concrete shells. In that case the inner ferrocement layer would simply not be considered as structural, and all necessary load-carrying activity would be assumed to have taken in the upper concrete layer.

Using the CNC-fabricated ferrocement formwork liberates designers from restrictions to regular geometry or repetitive use of identical shells for the sake of economy. The process is designed such that there are no disadvantages to uniquely shaped shells and shell segments. A maximum complexity is embedded in, and maximum value derived from the rib network that is directly generated from the digital model. Minor design changes can be incorporated later in the design process. Critical design activities such as preparation for fabrication and final structural design can be accomplished in parallel as long as the fabrication models remain associated with the master geometry.

5. Comparative Finite Element Analysis: Solid Concrete, Wood-Foam Sandwich, Ferrocement-Concrete Sandwich

A well-proven shell shape was studied to compare the structural behavior of a solid concrete shell with both types of sandwich shells. An equilibrium shell designed and built by Heinz Isler was chosen. This shell was built in 1982 as an array of 6 identical shells for a sports center in Solothurn, Switzerland. Isler derived the shape from a physical model using a hanging fabric. The shell spans a plan area of 18.4 × 48 m (60.4 × 157.5 ft), with a peak height of 9.9 m (32.5 ft) and a thickness of 90 mm (3.5 in). The geometry is highly complex and contains curvature reversals along all edges. It could not be built with pneumatic formwork systems, and in the case of Isler's project was economically feasible because the same formwork system could be used repetitively. It exemplifies the current problem of shell construction (Fig. 11).

The shell geometry was derived as accurately as possible from the literature (Ramm and Schunck 2002). Shell A was modeled as a solid concrete shell with a thickness of 90 mm (3.5 in), increasing up to 160 mm (6.3 in) at the supports. Shell B was modeled as a Ferrocement-Concrete Sandwich shell, with a ferrocement layer of 20 mm (0.8 in) thickness, 60 mm (2.4 in) interstitial foam layer and 50 mm (1.9 in) upper concrete layer. Shell C represented a Wood-Foam Sandwich shell, with a thickness of each wood facing of 25 mm (1 in) and a 100 mm (4 in) foam core. Both sandwich shells featured an area of solid facing material near the supports (Figure 12).

The study focused on comparing deflections for a snow load of 0.75 KPa (15.7 lb/ft2) and self-weight. The following Moduli of Elasticity were assumed in the Finite Element Analysis:

- Concrete 28 GPa (4 × 106 psi)
- Ferrocement 27 GPa (3.9 × 106 psi)
- Wood 10 GPa (1.4 × 106 psi)

The effect of the shear stiffness of the steel ribs of the Ferrocement-Concrete Sandwich shell was approximated by assigning the foam layer a stiffness of 80 MPa (11 × 106 psi). The deflections as derived from the Finite Element Analysis are shown in Table 1.

Even though deflections are clearly larger for the tested sandwich shell configurations, they always remain within the allowable limits. Even for the Wood-Foam Sandwich shell, the maximum deflection expressed as a fraction of the span L is L/600, approximately half of the allowable deflection. Both sandwich shells use less rigid material than the solid concrete shell. Stresses are generally low, as is to be expected for an equilibrium figure. This study suggests that both types of sandwich shells are structurally feasible alternatives to conventional concrete shells.

Conclusion

The past decade has seen a surging interest in complex shapes, partially generated by new and powerful computational tools that facilitated the modeling of complex shapes. Many of these shapes superficially resemble shells, but their geometry is not usually suitable as a structural surface. Instead, conventional systems of curvilinear frames, beams and columns have to be devised to support these digitally-generated shapes. These structural solutions are often clumsy compared to the elegance of the material-efficient structural surfaces. Amidst this interest in complexly shaped envelopes, there might yet be room for shells, provided the challenges of their construction are overcome. The potential of digital design and production environments for shell construction is only beginning to unfold.

This paper has presented a brief summary of CNC-fabrication processes for sandwich shells that avoid the pitfalls of traditional shell construction. The challenge of translating a large, complex surface geometry into a physical artifact is a good application for advanced parametric modeling techniques that can directly link to CNC fabrication technology. The generation of the complex shape is transferred to a CNC environment: milling machines for the shaping of foam molds or plasma cutters for the fabrication of the shape-defining rib network. The design technique for both types of sandwich shells employs parametric, feature-based modeling that fully supports all steps from conceptual design to design development and design for construction. Integrated FEA tools allow for the seamless integration of structural analysis into the design process.

As digitally-supported manufacturing techniques further penetrate architectural construction, these and other digitally-driven processes are bound to deeply impact practice and design as we know it today. The work on roof shells demonstrates that new fabrication processes need to be studied very carefully to avoid the need for repetition in the quest for cost control and scheduling feasibility.The research suggests that shells can be reintegrated into the vocabulary of architects today through the application of digitally supported design and manufacturing techniques.

Shell Model	Description	Deflection (mm/*inch*) at long edge	Deflection (mm/*inch*) at mid-span
Shell A	Solid concrete shell d = 90 mm	35 / *1.4*	59 / *2.3*
Shell B	Ferrocement-Concrete Sandwich (20 – 60 – 50 mm)	40 / *1.6*	68 / *2.7*
Shell C	Wood-Foam Sandwich (25 – 100 – 25 mm)	46 / *1.8*	77 / *3*

Table 1: Comparative Shell Study: Deflections

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Bechthold's research is dealing with Computer-Aided-Design and Manufacturing applications in architecture, with a particular focus on surface structures. For his work on a new process of designing and manufacturing wooden roof shells he won several awards, among them the Tsuboi Award by the International Association of Shells and Spatial Structures.