# Static Eiegenvalue Analysis as an Aid in Furniture Design

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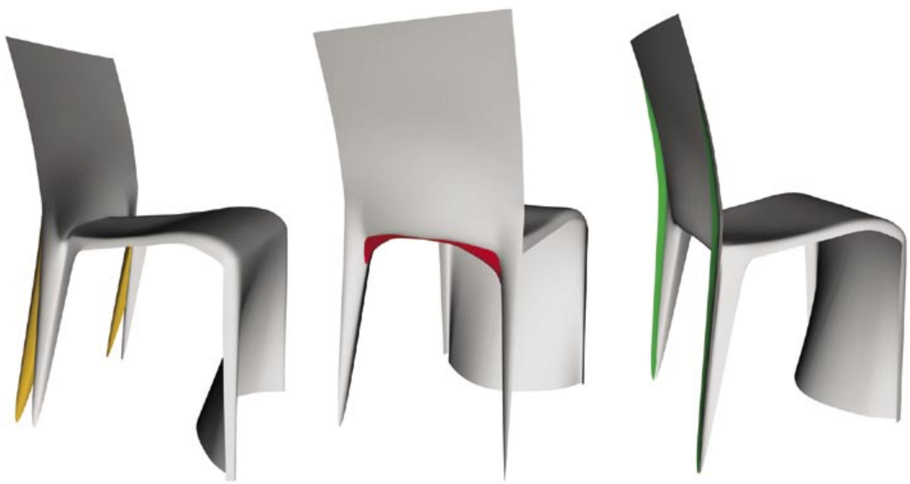


### Abstract

In the design process, knowledge of structural mechanics is often reduced to its being used to determine whether the object that has been designed is sufficiently strong. Strength testing indicates this directly on a yes or no basis, whereas computations are able to compare the level of stress with the strength of the material. Understanding the interplay between load, form, and material which structural mechanics is able to provide can be of considerable and far-reaching importance, both at an early conceptual design stage and while developing parts and details. The aim of this paper is to show how structural mechanics (in particular, static eigenvalue analysis) can be used to create work methods that provide a common language between the designer and the engineer during the design process. A case study is presented in which the Finite Element Method (FEM) was used to perform static eigenvalue analyses aimed at facilitating a collaborative furniture design process in the creation of a shell-shaped chair. Analysis of this sort was chosen because it can be used in a sketch-like manner. The designer found it easy to incorporate the results of the analysis into his own sketching work. It also enabled him to see how different design changes affected the overall structural behaviour of the chair without him having to create a full-scale prototype for physical testing.

## Introduction

The testing of furniture is usually a low-tech procedure: the piece of furniture involved is subjected to a number of predetermined loadcases according to certain standards, such as ISO-standards. In order to perform such testing a prototype or a finished version of the piece of furniture is needed. Prototypes of this sort are often expensive to manufacture, which radically limits the number of design ideas that designers and producers of furniture can test. A prototype should ideally contain all the design ideas the designer wants to test, and also be developed to a sufficient extent in all other respects that the need for further prototypes can be avoided. Since this is very difficult to attain, however, a faster, less limited and cheaper way of testing furniture at the early stage of the design process is needed.

Many of the problems discovered in the testing of furniture are related to structural integrity, material, and strength. Knowledge of these matters can be obtained within structural mechanics, employed primarily by engineers. If an engineer is involved in the furniture design process, however, it is usually in the production phase rather than in the designing process. Collaboration between designers and engineers at the early stages of design should be able to reduce problems of structural integrity during the design phase itself. Efforts should be directed at facilitating such a collaborative design process.

The most common way of using structural mechanics in an industrial context is by use of the Finite Element Method (FEM). This is a numerical method for solving differential equations. FEM is frequently employed in the automotive and aerospace industries. Generally, the object of design has already left the conceptual stage of the design process when FEM is utilised. This leaves little or no room for large changes to be made. This is due primarily to the fact that most FEM calculations require a complete description of the object which is to be analysed, of the load applied, and of the surrounding environment. New methods for using FEM as a design aid in the early sketching stages of the design process are thus needed. Static eigenvalue analysis is one such method. An analysis of this type ranks different deformation patterns of an object in terms of the strain energy (eigenvalue) needed to achieve a particular pattern. No predefined loadcases are required in using this method, the object itself providing the deformation patterns it is most sensitive to.

Few references to research on the use of FEM in furniture design are to be found. The first research published by Eckelman was concerned with calculation methods (Eckelman 1966, 1969, 1970). Gustafsson, who used FEM to analyse the overall structural behaviour of a two-dimensional chair frame, can also be noted (Gustafsson 1995, 1996a, 1996b, 1997). A similar but more detailed investigation was published soon thereafter by Smardzewski (1998). A common element in these studies is that they focus on the later stages of the design process and do so from the standpoint of the engineer rather than the designer. A recent contribution to this research that uses both standpoints is concerned with the use of applied visualisation of structural behaviour in the conceptual stage of the furniture design process (Olsson and Olsson; submitted for publication).

The aim of this paper is to show how structural mechanics (in particular, static eigenvalue analysis) can be used to create work methods that provide a common language for use in the design process by both the designer and the engineer. The focus here is on facilitating the design process at the early sketching stage, since the chances of influencing the outcome of the finished product are much larger then. To explore this approach a case study was carried out concerning the early stages in the designing of a chair.

## Collaborative designing

The case study involved concerns collaboration between designers and structural engineers in the designing of furniture, investigating how this collaboration can be made to function as effectively as possible. Two members of the research group were selected as participants. One, acting as the designer, was an architect with experience in rapid prototyping, which involves Computer Aided Design (CAD), virtual prototypes, 3D scanning, and plotting. The other, acting as the engineer, was a civil engineer experienced in the use of Computer Aided Engineering (CAE) and FEM.

Traditionally, designers and structural engineers, when confronted with a complex design task, tend to differ in the methods and the tools they employ. Engineers attempt to divide a problem into parts and to make predictions for each, analysing the different parts separately. This is often done with CAE-software. The designer, in contrast, attempts to grasp a problem in its entirety, viewing matters of structure, function, and form as pertaining to the entire system. This difference in perspective has led to considerable confusion in discussions between members of the two professions. An innovative use of CAE software for facilitating communication enables both perspectives to be handled at the same time. Simulations of an object's overall structural behaviour, both when deformed or strained and when subjected to stress, can be performed virtually in real time, providing a useful basis for a creative discussion of alternatives. The aim of the case study was to investigate how this could be done in a practical situation.

In this study, the designer made use of a 3D-software package called Rhinoceros, which is based on NURBS (Robert McNeel & Associates 2003). The engineer, in turn, used a CAE software based on FEM, called I-deas (EDS 2003). To be able to perform computations and simulations pertaining to various conceptions developed in the study, it was necessary to use the computer programs in ways not planned by those who developed them in their original form.

The designer and the engineer worked on the case together. Their primary aim was to develop a design approach they felt would be useful, rather than to achieve a perfect design as such. The case involved the creation of a chair, beginning with sketching and continuing on to what in a real case would be the creation of the first prototype. Different 3D sketches were first created in the CAD-software package and then exported to a neutral model file format, such as IGES. They were then imported into the CAE software, where the sketch was analysed and evaluated. All geometry was created in the CAD software. Since the case study was of explorative character, little was determined beforehand. The designer and the engineer were left to use their respective tools and knowledge in such a way that together they could succeed as well as possible at the overall task.

### Static eigenvalue analysis

Since the study concerns the early stages of the design process, the attempt was made to find ways of using FEM that resemble the sketching work of the designer. Static eigenvalue analysis is suitable for such calculations. Static eigenvalue analysis has been used earlier in the context of developing new finite elements (Cook, Malkus and Plesha 1989). In 2003, Olsson and Thelin introduced static eigenvalue analysis as a tool for investigating structural behaviour (Olsson and Thelin 2003). In this paper the method will be tried out as an aid in a preliminary design process.

Consider a finite element model of a structure composed of linear elastic material and having a linear loading response. Assume a load $\{f\}$, proportional to a corresponding nodal displacement $\{a\}$.

$$\{f\} = \lambda\{a\} \qquad (1)$$

For the structure as a whole, the loading response can be written as:

$$[K]\{a\} = \{f\} = \lambda\{a\} \qquad (2)$$

or

$$([K] - \lambda[I])\{a\} = \{0\} \qquad (3)$$

where $[K]$ is the structural stiffness matrix and $[I]$ is a unit matrix. From Eq. (3) a set of eigenvalues $\lambda i$ and eigenvectors $\{ai\}$ can be calculated. An eigenvector shows a specific deformation pattern, a so-called eigenmode (Figure 1). From Eq. (1) it then follows that each deformation mode scaled by $\lambda i\{ai\}$ can also be regarded and interpreted as a load case, such that

$$\{fi\} = \lambda i\{ai\}.$$

Thus far, a set of load cases connected with a structure has been established on the basis of the stiffness properties of the structure alone, without any actual loading of the structure taking place. It is also possible to order these load cases so as to determine the one to which the structure is most sensitive. This is the same as asking which load case would require the greatest amount of strain energy in order for a given state of deformation to be achieved or, correspondingly, which load case would result in the least deformation for a predefined value of the stored strain energy.

Premultiplication of Eq. (2) by {aiT} yields
{ai}T[K]{ai} = {ai}Tλi{ai} or 2Ui = {ai}Tλi{ai} (4)

Ui is the strain energy stored in the structure when displaced by the eigenvector {ai}. Thus, for each load case {fi} = λi{ai} there is an amount of stored strain energy Ui = {ai}T(λi/2){ai} corresponding to it.

To be able to order load cases {fi}, the respective deformation modes need to be made comparable by normalising them in some way. Different proposals for the normalisation of static eigenmodes are provided in Some Numerical Aspects on the use of Static Eigenvalue Analysis (Olsson, Ottosen, and Thelin; to be published). A normalisation of the eigenvectors requires that

{ai}T{ai} = k (5)

Value k is a mesh-dependent scale factor computed by use of some condition, for example, that of assigning the stored strain energy a reference value:

Ui = {λi}T[D] {λi}/2 = 1 (6)

By use of Eq. (6), a scale factor for the eigenvector {ai} can be obtained making it possible to compare different structures in terms of the deformation which occurs when the same amount of strain energy is applied to each.

Analysing an object's "*mechanical nature*" in this way provides a good starting point for a dialogue between the designer and the structural engineer. A difficulty involved is that the theoretical framework for the eigenmode approach is still under development, there being no commercial codes that explicitly support it.

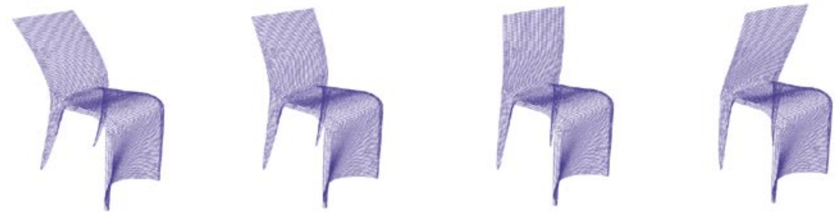

*Figure 1. An animation showing a single eigenmode of a chair (Awuthor).*

**Case Study**

Using collaborative design at an early sketching stage in the case study meant shifting between working with a design presented as a 3D-model and making visualisations of different deformation modes and states of stress. In efforts to close the gap between the know-how of the engineer and that of the designer, a continuous process of dialogue was pursued as modelling and simulation were being carried out.

**1. Design 0**

The task the case involved was to create an easily stackable chair with an organic form for use in a public environment. Emphasis was placed on the chair's overall shape rather than on details or the choice of materials. The goal was to modify the chair digitally to a point past the stage at which one would ordinarily create a full-sized prototype for testing.

An initial design was created and was designated as design 0 (Figure 2). The next step was to make a static eigenvalue analysis of this chair so as to provide the basis for the initial discussion between the designer and the engineer.

**2. Simulation**

By use of simulation, two types of mechanical behavior could be made visible. First, account was taken of the deformation behavior, its being decided that the chair should feel stable but should also have a flexible backrest for comfort. Second, questions of strain on the material were considered, its being asked whether, through geometrical shaping, stress concentrations could be avoided so as to make more efficient use of the material, resulting in a thinner shell and a lighter chair.

*Figure 2. Design 0 (Author)*

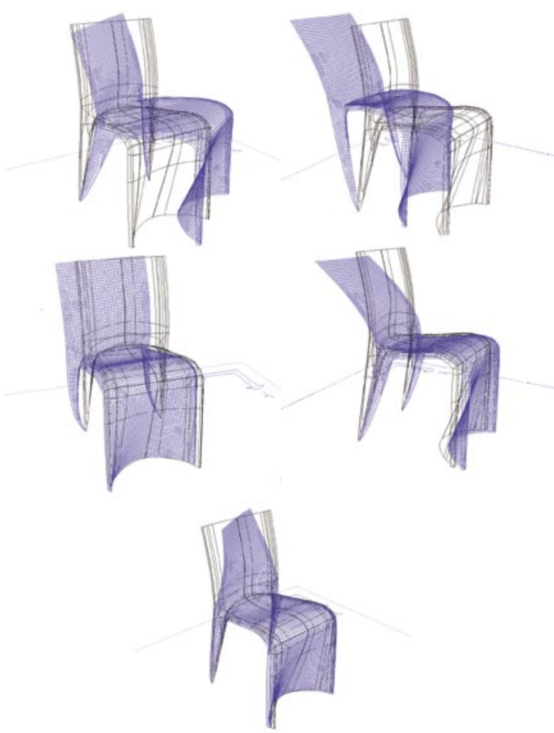

*Figure 3. The first five eigenmodes in design 0 (Author)*

At this stage, the question was to find a set of appropriate load cases that could guide us through different behaviours of the chair, such as when a person is leaning backwards, turning around and so forth. By use of standards for testing of furniture we could have been provided with a set of predefined load cases to simulate and evaluate.

An alternative process, the one proposed and used in this paper, was to compute the static eigenvalues of the chair and allow them tell us which load cases the chair is sensitive to; or, with a structural vocabulary, where it is weak. The static eigenvalue analysis provided us with a set of possible load cases and arranged them in order of weakness.

By use of the eigenmode concept the load cases shown in Figure 3 were calculated. It was decided to examine the first five eigenmodes, since the size of the eigenvalues increased considerably after the fifth eigenmode (i.e., the chair required much more energy to be deformed). These five eigenmodes could be interpreted as load cases appearing when a person is balancing on the back legs, turning around, or leaning on the backrest.

Being spared a boring examination of a dozen of possible load cases, the eigenmode concept provided us not only with relevant load cases, but also with measures of their relative order of weakness.

In the present case, I-deas was used to perform a static eigenvalue analysis of the chair. Since the eigenvalue solver in I-deas is only intended for use in performing dynamic analyses, it had to be used in a different manner in performing static analyses. Dynamic eigenvalue analysis can be modified so as to carry out a static analysis by giving some of the quantities used in the calculations a different meaning. Specifically, this involved replacing the mass matrix by a unit matrix. However, due to limitations in the current version of I-deas the normalisation mentioned earlier could not be achieved. Generally, this would mean that the eigenvalues obtained from the FEM models based on different designs could not be compared. The collection of finite elements, called the FEM mesh, is created as a representation of the design that is to be calculated. The more similar these FEM meshes are, the more trustworthy a comparison can be made between their eigenvalues in the absence of normalisation. The order of the eigenmodes is not dependent upon the choice of normalisation, which in this study means that it can be used without reservation in comparing different designs.

Von Mises stresses were computed to investigate the stress distribution. A von Mises stress is a scalar compound stress value indicating the level of material stress in an isotropic material in which compression and tension are of equal strength, as in the case of steel (Desai and Siriwardane 1984). Since at this point no specific material had been chosen for the chair, an isotropic material was used in the initial calculations. If an anisotropic material had been chosen, the von Mises stresses would not have provided an adequate picture of the stresses involved.

The von Mises stresses were mapped onto the surface of the chair using a colour scale that made it easy to identify the stress concentrations that could prove troublesome (Fig. 4). Since the first eigenmode was judged to be the most relevant deformation pattern for the chair, it was decided to consider primarily the von Mises stresses for this eigenmode.

### 3. Dialogue concerning design 0

The first three eigenmodes revealed deformation patterns associated with instability of the chair, whereas the fourth eigenmode possessed the desired deformation pattern. To increase the stiffness (i.e., the strain energy required as expressed by the eigenvalue) of the first three modes, the shape of the chair had to be changed. The von Mises stresses obtained on the basis of the eigenmode analysis revealed internal stress patterns of alarming character in the area where the rear legs, the seat, and the backrest joined. One way of solving these internal stress problems would be to add more material to that area. Although commonly used in the engineering design of a structure, this approach is not the only way of reducing the stress concentrations involved. The stresses in a problematical area can also be reduced by changing the shapes of other parts of the structure.

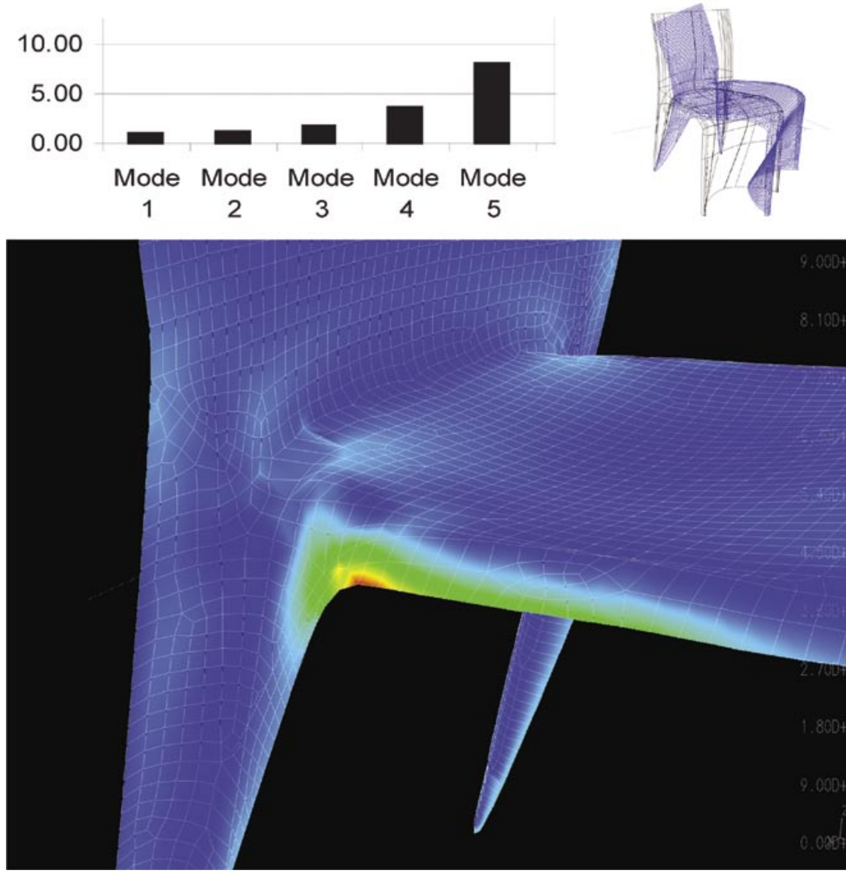


*Figure 4. The first five eigenvalues, the first eigenmode and the von Mises stresses associated with the first eigenmode in design 0 (Author)*

*Figure 5. The design changes shown in comparison with Design 0. Design 1:1 (yellow) has the rear legs moved back, 1:2 (red) has a more vault-like shape between the rear legs and the back rest and in 1:3 (green) the sides of the chair are bent back changing the profile of the rear legs and back rest. (source: author)*

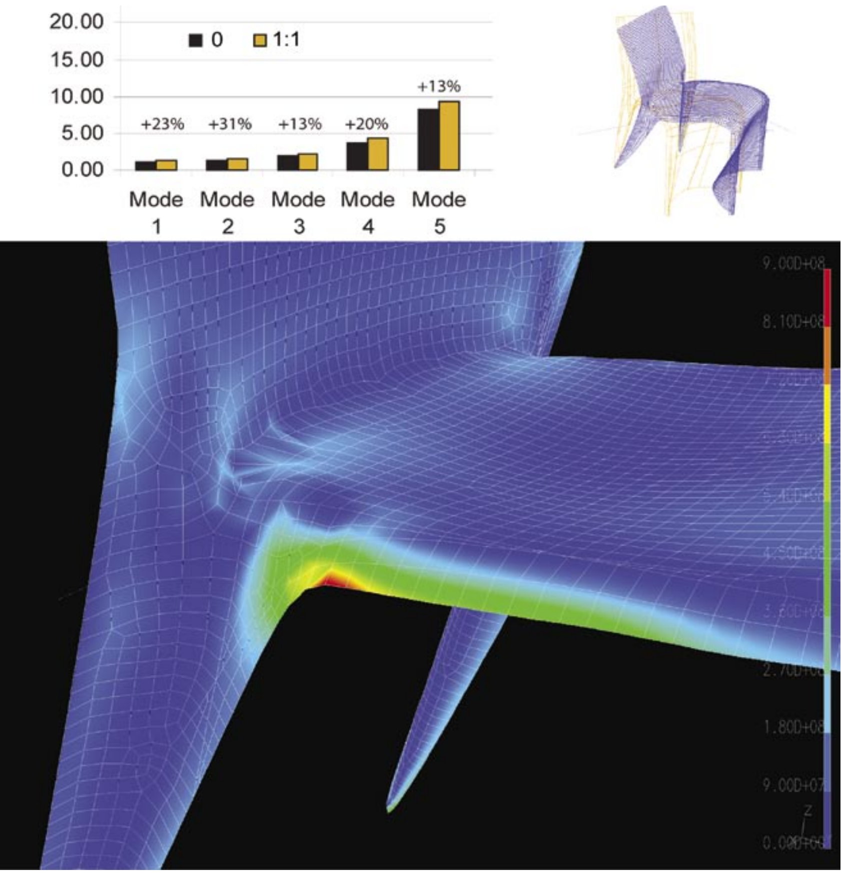


*Figure 6. The first five eigenvalues, the first eigenmode and the von Mises stresses associated with the first eigenmode in design 1:1 (Author).*

The designer tried to reshape the geometry of the chair so that the seat would be stable and the back be flexible and be movable as an element of steady shape. Three possible ways of reducing undesired movements and of smoothing out the stress concentrations were discussed:

- moving the rear legs farther back
- decreasing the curvature in the corners between the rear legs and the backrest
- changing the profile of the rear legs and the backrest

**4. Designs 1:1, 1:2 and 1:3**

A model of each of the three alternatives was produced using the 3D modelling software. The major concern at this point was to see in what ways and to what extent changes in shape could improve the structural behaviour of the chair rather than to explore the visual effects of the differences between the three versions. For this reason, the designer changed only one parameter at a time. The 3D models for the three alternatives were created by modifying the 3D model from Design 0. Only the discussed design changes were different in the three 3D models, ensuring an accurate comparison with the earlier design concerning structural behavior.

Design 1:1 is a modification of design 0. For this design the rear legs are placed further back in an attempt to reduce the stress concentrations and undesired deformations. In design 1:2, the curvature in the corners between the rear legs and the backrest has been decreased with the aim of improving the stability of the chair without adding to its weight. In design 1:3 there is a change in the profile of the leg and the backrest as compared with design 0, this being regarded as a way to increase bending and torsion of stiffness where the rear leg is connected to the seat (Fig. 5).

Comparisons of Design 0 and 1 in terms of eigenmode values, as well as of the deformation mode belonging to the first eigenvalue and of the corresponding von Mises stresses, are shown for the three suggested improvements in design in Figures 6, 7, and 8.

### 5. Dialogue concerning Design 1

Each of the three alternatives was found to improve the stiffness of the chair, the strain energy present having been increased in each case. The percentage of increase in eigenvalues as given in the figures cannot be regarded as exact due to the change of FEM mesh that affects the normalisation of the eigenvector. However, the FEM meshes in these designs are quite similar to that of design 0, which means that the introduced error can be assumed to be small. For example, it is fairly certain that in all three designs the eigenvalue for eigenmode 3 has been increased by 10-20% as compared with design 0.

On the basis of improvements of this sort, design 1:1 was found to be best, followed by 1:3 and then 1:2. These results led to the proposal of a new design, in which the ideas contained in design 1:1 predominated and some of the ideas from designs 1:3 and 1:2 were incorporated. Such design decisions must also take account of material optimisation and shaping. The von Mises plots indicated the second version of the design to be the most effective at reducing the high level of stress found for the first eigenmode in the connection between the backrest, the seat, and the rear legs. Although the second version of the design did not reduce the eigenvalues as much as the other design alternatives did, the von Mises plot showed its characteristics to have a positive effect of a different type. It was felt that this should likewise be taken into account in creating a new design.

### 6. Design 2

The overall shape of this design was the result of what the designer who participated had learned from the earlier discussions. This design is quite different in appearance from the others. Here, the designer used sharp, well-defined angles instead of smooth curved surfaces, wanting to see how this would affect the chair from an aesthetic point of view. He also wanted to see how it would affect the chair in constructive terms. Although the shape of this chair was quite different, it still had the properties the designer had sought in the original chair (Fig. 9, 10).

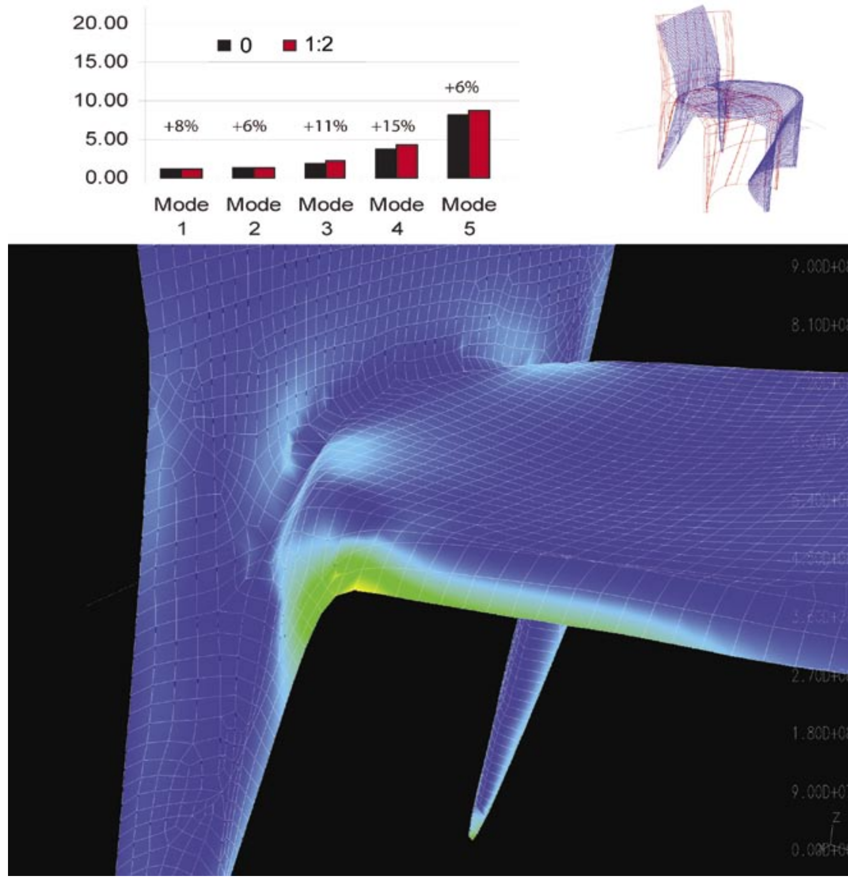


*Figure 7. The first five eigenvalues, the first eigenmode and the von Mises stresses associated with the first eigenmode in design 1:2 (Author)*

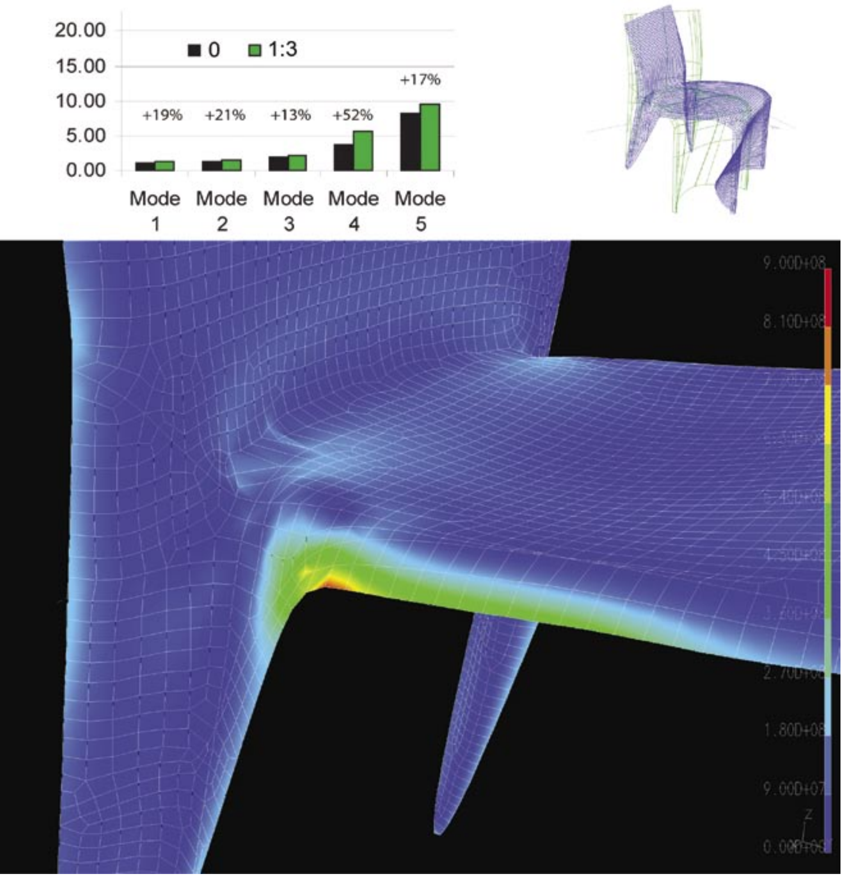


*Figure 8. The first five eigenvalues, the first eigenmode and the von Mises stresses associated with the first eigenmode in design 1:3 (source: author)*

*Figure 9. Design 2. Note that although the shape and the detailing are quite different here, the chair still possesses the inner properties the designer was aiming at in the original chair.*

### 7. Dialogue concerning Design 2

In terms of the eigenvalues obtained on the basis of the calculations, this new design was found to be stiffer, as was intended. This is especially true of eigenmode 3, for which more than twice as much strain energy is needed for a given deformation behaviour to occur (Table 1).

| Design | Mode 1 (%) | Mode 2 (%) | Mode 3 (%) | Mode 4 (%) | Mode 5 (%) |
|---|---|---|---|---|---|
| 1:1 | 23 | 31 | 13 | 20 | 13 |
| 1:2 | 8 | 6 | 11 | 15 | 6 |
| 1:3 | 19 | 21 | 13 | 52 | 17 |
| 2 | 22 | 44 | 104 | 23 | 19 |

*Table 1: Increase of eigenvalue for the first five eigenmodes.*

The von Mises plot of this design shows that the high level of stress is smoothed out by being distributed over a large area. Nevertheless, the stress levels are lower than the maximum stresses found in the earlier designs. This is generally preferable since it reduces the amount of material needed. Not only were the eigenvalues increased, but also the von Mises plot showed fewer stress values of a high level. Thus, this design appeared to be an improvement over the earlier ones.

The information made available to the designer at this early stage of conception can act as an effective reference for design. The influence on design is not simply mechanical or linear in nature. As in any design project, a change in design can represent a leap forward, a step backwards, or do no more that simply contribute to an understanding of the problem.

### Conclusions

The use of eigenmodes in early phases of the designing process provides the designer indications of the overall effects of a particular shape, a particular combination of materials, or a particular set of support conditions. After working with these methods for a period of time, the designer judged the static eigenvalues analysis to be especially useful since the sketch-like way in which it could be employed was similar to his own way of working at early stages of the design process. He also felt that it could help determine whether one was designing the right shape for obtaining the structural behaviour that was desired, as in the case of the flexible backrest. Analysing the internal forces and identifying areas of high stress level provides information useful for early decisions that need to be made. Information regarding structural matters available at this early stage of sketching can both be a guide and an inspiration to the designer, who can transform it into concrete ideas regarding shape.

In the case study presented here, the designer had extensive opportunities to try out and evaluate a number of different design ideas without having to construct a full-scale prototype. The designer learned not only how different ideas affected the design visually but also how they affected its structural behaviour. This approach gave quick and specific answers to the "*what if*"-questions that could be asked.

The case study emphasizes the need for adequate dialogue between the different professions involved. Since calculations alone are of little or no use if they are not correctly interpreted, it is important that the designer have a proper understanding of the engineer's use of simulations and calculations, so that 3D models produced in the CAD programs can be used directly in the CAE program without modification. One difficulty encountered was that of finding a common approach to geometrical modelling that would work well in both CAD and CAE.

Although the chair considered here is in the area of furniture design, the results regarding work methods and collaboration in the design process are of generic value in many areas of design. Two aspects of chair design that make it of particular interest are that it represents a complex yet limited designing task and that, since up to now there has been scarcely any use made of CAE-tools in the designing of furniture, the potential for developments in this field are very great indeed.

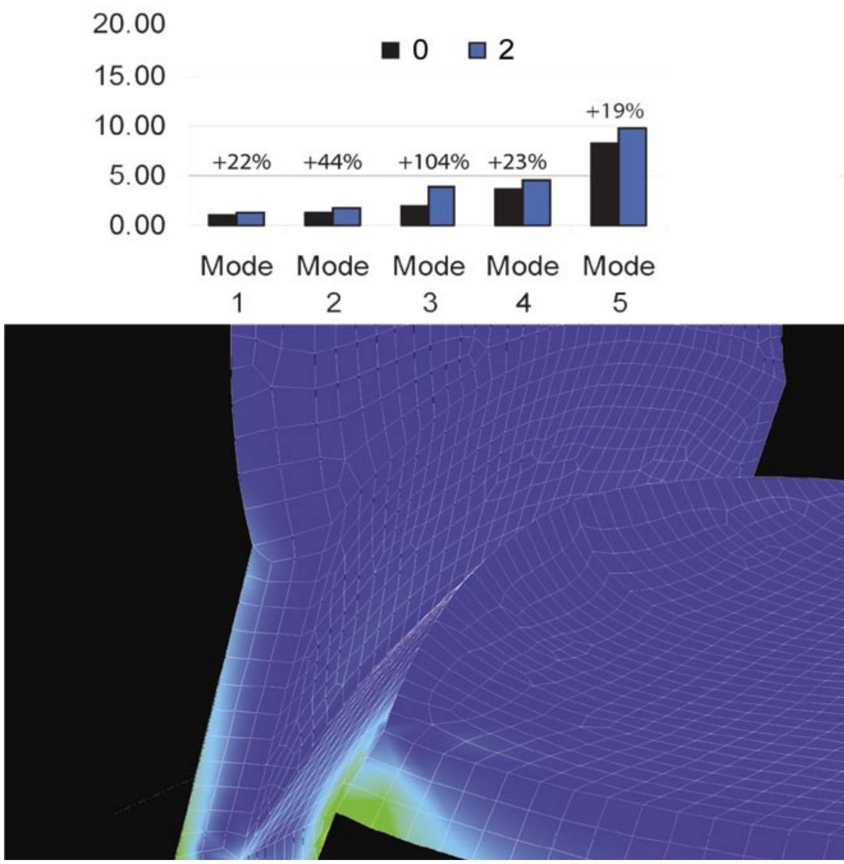


*Figure 10. The first five eigenvalues, the first eigenmode and the von Mises stresses associated with the first eigenmode in Design 2 (source: author)*

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**Pierre Olsson**, first author. My work concerns the exchange of information and dialogue between designers/architects and engineers in design processes, with special attention to structural behaviour. Well-balanced graphical user interfaces can allow these professions to interact and get visual feedback from calculations of structural behaviour whose results can otherwise be both static and hard to interpret.

My most recent project concerns a computer-based graphical tool that will allow architects and engineers to freely sketch different trusses and load cases. The calculation engine behind the graphical interface will at any time during the sketching provide feedback to the user concerning the structural behaviour of the truss. The aim of this project is to provide a possibility to get a more refined understanding of the structural behaviour of trusses through free and unbiased experimentation by the user.