Algorithm
Analysis

Computation

Design

Environment

A Computational Analysis of Fractal Dimensions in the Architecture of Eileen Gray

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THIS PAPER IS THE FIRST INVESTIGATION OF THE FRACTAL DIMENSIONS OF FIVE OF THE HOUSE DESIGNS OF EILEEN GRAY; A PROMINENT ARCHITECT WORKING MAINLY IN FRANCE BETWEEN 1922 AND 1956. In this paper, a computational variation of the "box-counting approach" (used to determine fractal dimension) is applied to a multi-dimensional review of the houses of Gray. As a contemporary of Le Corbusier, Gray is a significant architect for such an analysis. This research is important because it expands the set of examples of early Twentieth Century architects who have been analyzed using the method. This paper provides a computer-assisted mathematical analysis of characteristic visual complexity in five houses designs by Eileen Gray.

1 Introduction

In 1996, Carl Bovill produced an approximate fractal dimension calculation for a façade of Le Corbusier's Villa Savoye and another for a façade of Frank Lloyd Wright's Robie House. Bovill's analysis showed that the façade of the Robie house had a higher fractal dimension than the façade of the Villa Savoye (a variation in the order of 10%). Bovill's method has also been described and extrapolated by a number of researchers and it has been used to determine the fractal dimension of selected views of ancient as well as modern architecture (Bovill 1996, 1997; Bechhoefer and Appleby 1997; Makhzoumi and Pungetti 1999; Burkle-Elizondo, Sala and Valdez-Cepeda 2004). Despite this, Bovill's initial results and method remained largely untested for nine years.

When Lorenz (2003) repeated Bovill's analysis, he was able to partially replicate his results but he also suggested that the approach should be revised to employ computational methods. Ostwald, Vaughan and Tucker (2008) combined the programs *Archimage* (vers. 2.1) and *Benoit* (vers. 1.3.1) to produce a more balanced result for the fractal dimension of an architectural drawing. In the same work, they tested this method on five of Le Corbusier's houses and five of Frank Lloyd Wright's houses. When this larger sample was analyzed the difference in characteristic visual complexity between the two architects' houses was found to be in the order of 4%. Despite expanding the set of cases being analyzed with this method, the results were still insufficient to offer a clear determination of the usefulness of computational fractal analysis in architecture. To improve the statistical validity of the result, additional data sets are required. The present paper expands the set of cases analyzed using this method for determining the characteristic visual complexity of domestic architecture. Five houses designed by Eileen Gray, a contemporary of Le Corbusier, are the focus of the present paper. The interest in these houses is precisely that, if the method is valid, then there should be clear numerical similarities between the results for Gray and for Le Corbusier.

But why is the *box-counting method* in general, and the expansion of its results to include other examples in particular, of interest? First, this is because there are relatively few methods available for quantitatively analyzing architectural elevations. Moreover, of the methods that have been proposed, few, if any, have ever been consistently applied (Stiny 1975; Krampen 1979; Elsheshtawy 1997; Stamps III 1999, 2003). The box-counting method is a rare quantitative method that has been repeatedly used in architectural analysis, even if it is poorly understood. The very limited set of results that have been published to date using this method have been inconclusive, yet this has not stopped researchers using the results to argue that; natural objects have higher fractal dimensions than synthetic objects and therefore architecture which has a higher fractal dimension will be more conducive to human inhabitation than architecture with a lower fractal dimension. This proposition remains unconvincing for a number of philosophical and mathematical reasons (Ostwald 2001, 2003; Ostwald and Wassell 2002) but the limited number of examples of consistently determined fractal dimensions for architectural elevations complicates the situation.

2 Fractal Geometry

Fractal geometry came to prominence in mathematics during the late 1970's and early 1980's (Mandelbrot 1982). However, it was not until the late 1980s and the early 1990s that fractal dimensions were applied to the analysis of the built environment (Ostwald 2001). For example, Batty and Longley (1994) and Hillier (1996) have each developed methods for using fractal geometry to understand the visual qualities of urban space. Oku (1990) and Cooper (2003, 2005) have used fractal geometry to provide a comparative basis for the analysis of urban skylines. Yamagishi, Uchida and Kuga (1988) have sought to determine geometric complexity in street vistas and others groups have applied fractal geometry to the analysis of historic street plans (Hidekazu and Mizuno 1990). Eglash (1999) has compared the geometric patterns and formal relationships found in indigenous architecture with fractal geometry.

The box-counting method is the most common mathematical approach for determining the approximate fractal dimension of an object. In its architectural variant, the method commences with a drawing of an elevation of a house. A large grid is then placed over the drawing and each square in the grid is checked to determine if any lines from the façade are present in the square. Those grid boxes that have some detail in them are recorded. Next, a grid of smaller scale is placed over the same façade and the same determination is made of whether detail is present in the boxes of the grid (see figures 1 – 3). A comparison is then constructed between the number of boxes with detail in the first grid and the number of boxes with detail in the second grid. This comparison is made by plotting a log-log diagram for each grid size (Bovill 1996; Ostwald and Tucker 2007). By repeating this process over multiple grids of different scales, an estimate of the fractal dimension of the façade is produced (see figure 4). While this process can be undertaken by hand, as Bovill does, the programs *Benoit* and *Archimage* automate this operation.

There are several variations of the box-counting approach that respond to known deficiencies in the method. The four common variations are associated with balancing "white space" and "starting image" proportion, line width, scaling coefficient and moderating statistically divergent results. The solutions to these issues that have been proposed by Foroutan-Pour, Dutilleul and Smith (1999) and Ostwald, Vaughan and Tucker (2008) are adopted in the present analysis.

3 Method

Five of Gray's house designs were selected for the present analysis: Small House for an Engineer (1926), E.1027 (1929), Four Storey House (c.1934), Tempe à Pailla (1934) and House for Two Sculptors (1934). The drawings used for the analysis have been adapted from reconstructions of Gray's original work produced separately by Constant (2000), Hecker (1993) and Wang and Constant (1996). Only three elevations are analyzed for the Small House for an Engineer because Gray's archives only contain those three. The standard method for the fractal analysis of visual complexity in houses is as follows.

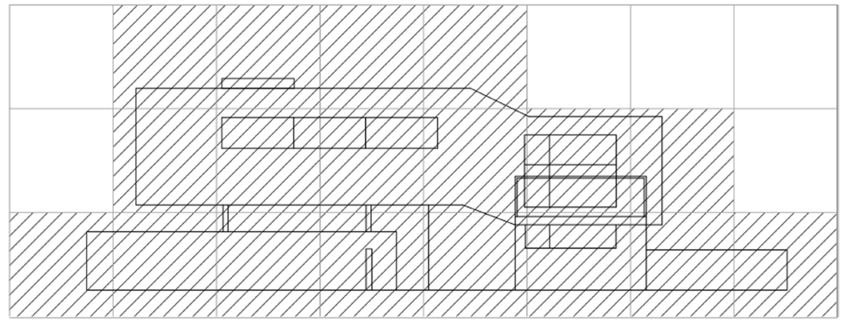
1. The drawings or views of each individual house are separately grouped together and considered as a set.
2. Each view of the house is analyzed using *Archimage* and *Benoit* programs producing, respectively, a $D_{(Archi)}$ and a $D_{(Benoit)}$ outcome. The settings for *Archimage* and *Benoit*, including scaling coefficient and scaling limit are preset to be consistent between the programs. The starting image size ($IS_{(Pixels)}$), largest grid size ($LB_{(Pixels)}$), and number of reductions of the analytical grid ($G_{(\#)}$), are recorded so that the results can be tested or verified. *Archimage* results are typically slightly higher than those produced by *Benoit* although the variation is consistent.
3. The $D_{(Archi)}$ and $D_{(Benoit)}$ results for the elevation views are averaged together to produce a separate $D_{(Elev)}$ result for each program for the house. These results are a measure of the average fractal dimension of the exterior façades of the house.
4. The $D_{(Elev)}$ results produced by *Archimage* and *Benoit* are averaged together to produce a composite result, $D_{(Comp)}$, for the house. The composite result is a single D value that best approximates the characteristic visual complexity of the house.
5. This process (steps 2 to 5) is repeated for each house producing a set of five $D_{(Comp)}$ values. These values are averaged together to create an aggregate result, $D_{(Agg)}$, which is a reflection of the typical, characteristic visual complexity of the set of the architect's works.

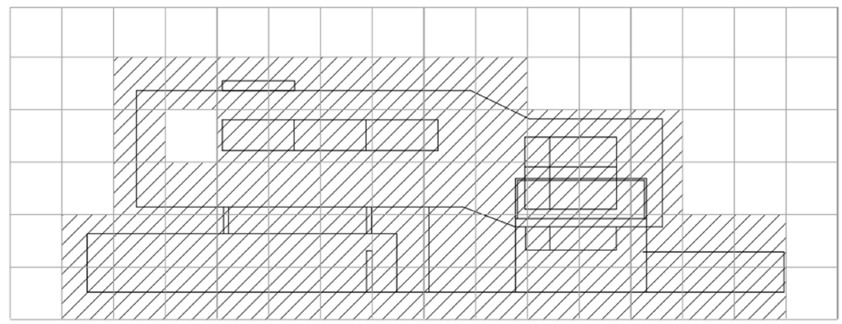
Importantly, this method does not produce a D result for the three-dimensional form of the house rather, it generates a series of average D results for the two-dimensional visual qualities of a structure.

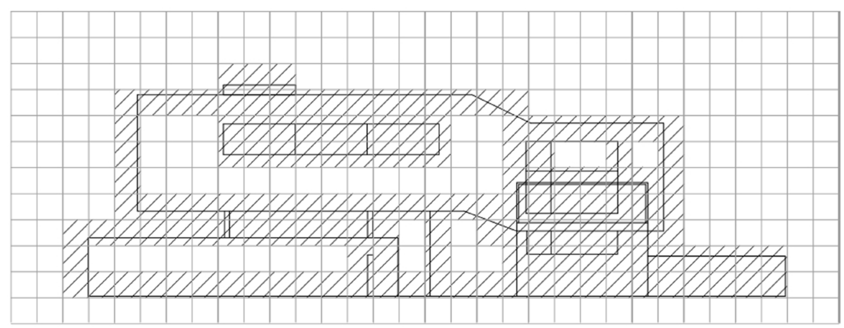
Table 1 contains a summary of the abbreviations. Tables 2 to 6 contain the $D_{(Archi)}$ and $D_{(Benoit)}$ results that are combined to produce $D_{(Comp)}$ values for each of the five houses by Gray (steps 2 to 4 above) and table 7 records the $D_{(Agg)}$ result for all of Gray's houses (step 5 in the method above).

4 Discussion and Results

Born in County Wexford, Ireland, in 1879, Eileen Gray studied at the Slade School of Art in







FIGURES 1 AND 2. FIRST GRID (TOP) AND SECOND GRID (MIDDLE) PLACED OVER ELEVATION 1 OF THE HOUSE FOR AN ENGINEER.

FIGURES 3. THIRD GRID (BOTTOM) PLACED OVER ELEVATION 1 OF THE HOUSE FOR AN ENGINEER.

London and the Académie Colarossi and Académie Julien in Paris. Most of Gray's architectural designs remain unrealized, although many are sufficiently well documented so that, according to Murphy, they display a "full and original understanding of the language" of the Modern movement (1980:306). Gray's built works are constructed from concrete, stone, steel and glass. They feature intersecting white, concrete planes, glazed bands in fine, steel frames and tubular steel balustrades. Her houses are set on local stone bases, a detail which seems to connect each house indelibly to the site.

The House for an Engineer has the lowest composite D result of all of the five houses analyzed ($D_{(Comp)}$ = 1.2895). Only three elevations for The House for an Engineer were found in Gray's records. Of these, Elevation 1, records the highest result for the house ($D_{(Archi)}$ =1.421) with Elevation 2 ($D_{(Archi)}$ =1.290) giving the lowest result for all of Gray's five houses. Elevation 3 ($D_{(Archi)}$=1.361) has a D result in line with the overall average fractal dimension of all the elevations for this house ($D_{(Elev, Archi)}$ = 1.357).

E1027 has the highest composite D result of 1.464, which is generated from the elevations' average result of $D_{(Elev, Archi)}$ = 1.512 and $D_{(Elev, Benoit)}$ = 1.416. The East and West elevations for E1027 produce D results close to these average results; respectively $D_{(Archi)}$ =1.516 and 1.502 and $D_{(Benoit)}$=1.416 and 1.409. In contrast, the North elevation is considerably lower in value and the South higher; respectively, $D_{(Archi)}$ =1.447 and 1.353 and $D_{(Benoit)}$=1.583 and 1.486. The Southern elevation produces the highest D result for all of the five houses analyzed.

The elevations for the Four Storey Villa (1.350 < $D_{(Archi)}$ < 1.470 and 1.214 < $D_{(Benoit)}$ <1.318) produced D values generally lower that all the other five houses, except House for

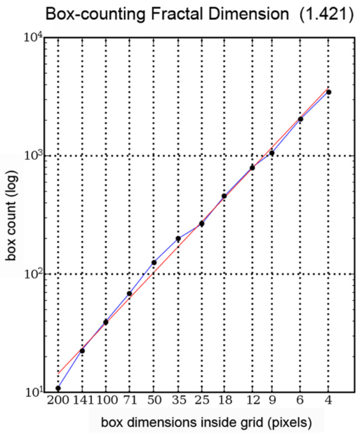


FIGURE 4. LOG-LOG DIAGRAM (RIGHT) FOR ELEVATION 1 OF THE HOUSE FOR AN ENGINEER.

an Engineer. With an overall $D_{(Comp)}$ of 1.337, the Four Storey Villa has the second lowest composite D result.

Tempe à Pailla has a D result for its North elevation ($D_{(Archi)}$ =1.340) (see figure 5) which is significantly lower than all other elevations of this house. The other three elevations for Tempe à Pailla all fall into a tight range (1.457 < $D_{(Archi)}$ < 1.468). This result occurs because the northern elevation is only one storey high; the rest of the northern part of the building is set into the hillside of the site. The other three elevations each display more than one storey as the building descends the hillside and have higher D values. The low result for the North elevation brings the composite D result for the house down to 1.378, which is the same as the average composite result for all five houses.

The House for Two Sculptors (see figure 6) has consistently high D results for all four elevations (1.441 < $D_{(Archi)}$ < 1.519). The resulting average composite result ($D_{(comp)}$ =1.425) is the second highest of the five houses.

The final composite D results for the five houses by Eileen Gray range between the lowest result, for the House For an Engineer ($D_{(comp)}$ = 1.289) Gray's first architectural design, and the highest for E1027 ($D_{(comp)}$ =1.464) Gray's first built house. Overall, the aggregated result for all of Gray's five houses was $D_{(agg)}$ =1.379. The composite D values of three particular houses are at a similar level of visual complexity, 1.338 (Four Story Villa) 1.378 (Tempe à Pailla) and 1.425 (House for Two Sculptors). These fractal dimension results are all similar to the aggregated result.

The higher average fractal dimensions were found in Gray's two built houses, E1027 ($D_{(comp)}$ =1.464) and Tempe à Pailla ($D_{(comp)}$ = 1.378) (see figures 5 and 7) and the unbuilt House for Two Sculptors ($D_{(comp)}$ =1.425) (see figure 6). The visual complexity found in the House for Two Sculptors is derived from its complex geometric form, the curved walls indicated in the images by dashed lines in the elevations. This increase in detail is reflected in the results.

It is known that Gray completed the detailed work on her designs as they were in the process of being constructed and her built works reflect this with their higher D values. The final drawings published and analyzed for E1027 and Tempe à Pailla are more complete, in terms of visual detail, than Gray's unbuilt works. It is possible that if Gray had realized the other three houses in this study, their fractal dimension would have marginally increased.

5 Conclusion

Past research using this method has recorded results for:

- 5 houses by Frank Lloyd Wright built between 1901-1910: $D_{(agg)}$ =1.543
- 5 houses by Le Corbusier built between 1922-1928: $D_{(agg)}$ = 1.481

At the beginning of the present research project it was anticipated that the results for Gray would be close to the results for Le Corbusier. Yet, Gray's aggregate result is lower by 0.102 or approximately 7.4% than Le Corbusier ($D_{(agg)}$ =1.481) and lower than Wright ($D_{(agg)}$ =1.543) by 0.165 or 10.6%. This is a significant difference in a field wherein houses by the same designer and from the same era are often clustered within a 1% D range. What then does this mean?

An intuitive or qualitative assessment of Gray's architecture would suggest that it has slightly less visual complexity than the architecture of Le Corbusier, but not in the magnitude identified in the present research. One explanation for this unexpected result is found in the difference between Gray's completed works and her unbuilt ones. The completed works have $D_{(Comp)}$ results that are much closer in value to those of Le Corbusier. The unbuilt works are conspicuously lacking the same level of detail development that the built projects have, and, as a result, they have much lower $D_{(Comp)}$ values. An alternative explanation is that the computational analysis may be producing a more attenuated reading of the difference between these architects' works than the human eye can usually detect. Ultimately, while the former explanation seems more likely, as the set of examples tested using this method is expanded in future research, different explanations may be uncovered that will assist future researchers attempting to use quantitative measures for analyzing architectural elevations.

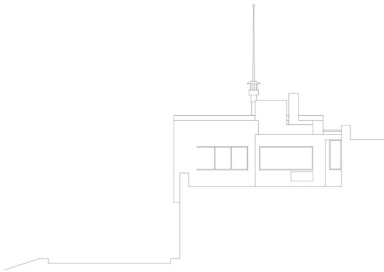


FIGURE 5. TEMPE À PAILLA NORTH ELEVATION: $D_{(ARCHI)}$ = 1.340

6 Appendix

TABLE 1. ABBREVIATIONS AND DEFINITIONS.

Abbreviation	Meaning
D	Approximate Fractal Dimension determined using the box-counting method.
$D_{(Archi)}$	D calculated using *Archimage* software.
$D_{(Benoit)}$	D calculated using *Benoit* software.
$D_{(Elev)}$	Average D for a set of elevation views of a house using a specified program.
$D_{(Comp)}$	Composite D result averaged from both *Archimage* and *Benoit* outcomes for the elevations of a house.
$D_{(Agg)}$	Aggregated result of five composite values used for producing an overall D for a set of architects' works.
$IS_{(Pix)}$	The size of the starting image measured in pixels.
$LB_{(Pix)}$	The size of the largest box or grid that the analysis commences with, measured in pixels.
$G_{(\#)}$	The number of scaled grids that the software overlays on the image to produce its comparative analysis.

TABLE 2. SMALL HOUSE FOR AN ENGINEER DATA AND RESULTS.

Views	$IS_{(Pix)}$	$LB_{(Pix)}$	$G_{(\#)}$	$D_{(Archi)}$	$D_{(Benoit)}$	$D_{(Comp)}$
Plan	1200 x 715	400	14	1.342	1.231	
Elev 1	1200 x 400	200	12	1.421	1.272	
Elev 2	1200 x 557	150	11	1.290	1.171	
Elev 3	1200 x 572	150	11	1.361	1.222	
$D_{(Elev.)}$				1.357	1.221	1.289

TABLE 3. E.1027 DATA AND RESULTS.

Views	$IS_{(Pix)}$	$LB_{(Pix)}$	$G_{(\#)}$	$D_{(Archi)}$	$D_{(Benoit)}$	$D_{(Comp)}$
Plan	1200 x 848	400	14	1.522	1.415	
Elev E	1200 x 848	400	14	1.516	1.416	
Elev N	1200 x 572	150	11	1.447	1.353	
Elev S	1200 x 672	300	13	1.583	1.486	
Elev W	1200 x 925	300	13	1.502	1.409	
$D_{(Elev.)}$				1.512	1.416	1.464

TABLE 4. FOUR STOREY HOUSE DATA AND RESULTS.

Views	IS(Pix)	LB(Pix)	G(#)	D(Archi)	D(Benoit)	D(Comp)
Plan	729 x 1200	365	14	1.445	1.318	
Elev 1	1200 x 486	150	11	1.356	1.214	
Elev 2	1200 x 472	150	11	1.470	1.311	
Elev 3	1200 x 457	150	11	1.350	1.252	
Elev 4	1200 x 472	150	11	1.435	1.310	
D(Elev.)				1.403	1.271	1.337

TABLE 5. TEMPE À PAILLA DATA AND RESULTS.

Views	IS(Pix)	LB(Pix)	G(#)	D(Archi)	D(Benoit)	D(Comp)
Plan	1200 x 500	150	11	1.465	1.359	
Elev E	1200 x 486	150	11	1.461	1.341	
Elev N	1200 x 848	400	14	1.340	1.239	
Elev S	1200 x 1079	300	13	1.468	1.390	
Elev W	1200 x 543	150	11	1.457	1.328	
D(Elev.)				1.431	1.324	1.377

TABLE 6. HOUSE FOR TWO SCULPTORS DATA AND RESULTS.

Views	IS(Pix)	LB(Pix)	G(#)	D(Archi)	D(Benoit)	D(Comp)
Plan	1200 x 913	300	13	1.384	1.306	
Elev 1	1200 x 515	150	11	1.441	1.312	
Elev 2	1200 x 629	300	13	1.476	1.387	
Elev 3	1200 x 629	300	13	1.519	1.421	
Elev 4	1200 x 515	150	11	1.493	1.352	
D(Elev.)				1.482	1.368	1.425

TABLE 7. COMPOSITE AND AGGREGATE RESULTS FOR GRAY'S HOUSES.

House	D(Comp)
Small House for an Engineer	1.289
E.1027	1.464
Four Storey House	1.337
Tempe à Pailla	1.377
House for Two Sculptors	1.425
D(Agg) for Gray	1.378

FIGURE 6. HOUSE FOR TWO SCULPTORS ELEVATION 3: $D_{(ARCHI)}$ = 1.519

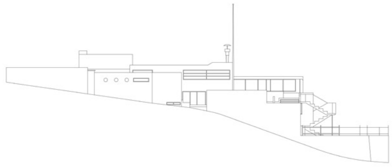


FIGURE 7. TEMPE À PAILLA WEST ELEVATION: $D_{(ARCH)} = 1.457$

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