THE NOVEL STONES OF VENICE:
THE MARCHING CUBE ALGORITHM AS
A STRATEGY FOR MANAGING
MASS-CUSTOMISATION

ABSTRACT
The Marching Cube (MC) algorithm is a simple procedural routine for the surface representation of three-dimensional scalar fields. While much has been written of the algorithm’s efficiencies and adaptive nature within the domain of computer graphics and imaging, little has been explored within the context of architectural geometry and fabrication. This paper posits a novel implementation of the MC algorithm coupled with robotic fabrication (RF) techniques, to realise an open-ended design method that approaches mass-customisation as the unique geometric distortion of a finite set of topologically consistent families of tectonic elements.

The disciplinary consequences of this and similar methods that intimately couple algorithmic design techniques with robotic fabrication are discussed. These include the re-affirmation or expansion of the role of the architect as master builder that is enabled by challenging Leon Battista Alberti’s 15th Century division between design concept and building.

The method and its disciplinary potentials are illustrated through the description of an installation built by the authors for the Australian Pavilion at the Venice Biennale. Clouds of Venice serves as a case study for a new integrated mode of production, one that increases the quality and number of feedback relations between design, matter and making.
1 INTRODUCTION

1.1 DIFFERENCE AND REPETITION

Difference and repetition are recurring themes within architectural discourse. Through the championing of digital techniques (algorithmic, associative or other) and numerically controlled fabrication methods, contemporary practice seeks an expansion of the linear and highly standardised protocols of industrial production. Algorithmic design methodologies, when coupled to robotic fabrication, enable an explicit and bidirectional traversal of the modern division between design and making. This paper describes one such method that modifies the familiar marching cube algorithm to take advantage of its latent possibilities for fabrication efficiency.

1.2 LABOUR DIVIDE

The contemporary exploration of the architectural potentials of robotic fabrication and algorithmic design techniques perhaps most significantly permits a re-affirmation or expansion of the role of the architect as master builder by challenging the 15th Century division between design and building established during the Renaissance most notably by Leon Battista Alberti. As Mario Carpo surmises:

“Alberti’s entire architectural theory is predicated on the notational sameness between design and building, implying that drawings can, and must, be identically translated into three-dimensional objects [and that] the design of a building is the original, and the building is the copy” (Carpo 2011: 26).

Thus through Alberti begins a historical and temporal divide between design and realisation through the fundamental displacement of architectural enquiry away from material praxis towards the creation of new techniques of representation (drawings) that leaves no space for material influence nor artisanal (builder) interpretation. Alberti’s “preference for the purely theoretical aspect of his art…[and to]…always withdr[aw] into the background when the moment came to carry out a project” (Borsi 1977: 10) is relayed and contextualised within Franco Borsi expansive opus of Alberti’s career. Critically, this retreat has fundamental consequences for architectural production as it restricts the number, nature and quality of influences able to permeate design. It does not allow bi-directional communication channels, or feedback loops, to exist between the processes of design and the materials and systems engaged in its materialisation. Design becomes an incredibly insular operation within the narrowest phase-space of possibility, tantamount to Ludwig Wittgenstein’s famous dictum that, “the limits of my language mean the limits of my world” (Wittgenstein 1972: 56). For architecture this means that representational constraints are at least as significant as the more recognised limitations of structure, time, and budget (Benjamin 2004: 348). A disciplinary conundrum best observed by William Mitchell, “[a]rchitects tend to draw what they can build, and build what they can draw” (Mitchell 2001: 354). Thus the concurrent shifts from shop drawings to the direct generation of instruction code and from highly linearized production chains to flattened systems of intense feedback represent significant factors in one of the contemporary transformations of our discipline.

1.3 CASE STUDY: ‘THE CLOUDS OF VENICE’

A recently completed project, The Clouds of Venice (Figure 1), serves as a case study for a new integrated mode of production, one that increases the quality and number of feedback relations between design, matter and making. Commissioned by the Australian Institute of Architect’s for the 2012 Venice Architecture Biennale, the installation seeks the realisation of a novel spatial experience that is no longer reliant on a limited set of discrete spatial elements—walls, ceilings and floors for its spatial definition. Instead, the project posits a highly diffuse and gradient spatial reading through an ultra-high population assembly of mass-customised, robotically fabricated steel rod elements (Figure 2).

2 MARCHING CUBE [MC] A STRATEGY FOR MASS-CUSTOMISATION AND PART STANDARDISATION

2.1 MC METHOD

The polygonal volume representation (isosurface extraction) of three-dimensional data sets (scalar fields) is a common problem within computer graphics. The MC algorithm, although not the oldest, is the most well-known and implemented approach to this problem. The process combines simplicity with high-speed since it works almost entirely on lookup tables (Bourke 1994). The algorithm has many applications; ranging from the representation of surface-based contours through mathematical space, to the volumetric reconstruction of MRI scan datasets within the field of medical imaging. The most significant attribute of the algorithm within the context of architectural fabrication lies in its capacity to engender an open-ended approach to complex geometric representation that uniquely embeds scalable mechanisms of shape standardisation.
Fifteen (15) isosurface intersections scenarios defined by the MC algorithm

The MC method consists of an ordered set (three-dimensional grid) of cubes located within a spatial lattice of vertices—where each cube comprises eight such vertices. Each vertex within the lattice attains a scalar value via sampling the system’s input data set. The data set is arbitrary but is commonly a set of weighted three-dimensional points. Thus the lattice denotes a scalar field of values either above or below a desired threshold —isolevel—from which the status of any cube can be extracted: entirely inside, entirely outside or partly intersected. The cubes that cross the threshold will contain part of the isosurface that is defined in a procedural way. Since each of the cube’s eight vertices can be either “marked” or “unmarked”, there are 256 ($2^8$) possible conditions. (Newman & Yi 2006: 856) In leveraging aspects of reflective and rotational symmetry, the MC algorithm elegantly reduces that count to fifteen (Figure 3). The first scenario is considered trivial; as all of the cube’s vertices lie above or below the desired isolevel and as such produce no geometry. For each of the remaining fourteen cases, between one and five triangular facets (faces) will be added to the resultant mesh object (isosurface). The final process of the algorithm establishes the actual moments of intersection that define each triangular facet by either linearly interpolating (stepwise or scalar) or more simply by referencing the midpoint of the “cut” edge.

### 2.2 MC Fabrication: Opportunities for Increased Processing Efficiency

The MC is a fast and efficient algorithm owing to the incorporation of lookup tables. Once the vertices of a cube are sampled, the resulting score is used as an index to access not only the MC isosurface scenario, but also to retrieve the intersected edges required to construct the resulting isosurface facets (mesh faces). The efficiency of the MC system is that it is entirely dependant upon explicit conditions or pre-determined topological responses. Thus, the most obvious efficiency gains and those that are easiest to implement, is to capitalise on the shared nature of vertices and edges that exist. Thereby only processing such attributes once.

A second improvement is to establish higher orders of spatial binning or partitioning. Spatial binning is a poly-scalar approach that aims to eliminate trivial calculations by storing elements (cubes, vertices and edges) in spatial groups: octree regions or similar. This enables a pre-pass...
to determine which of these large spatial groups are active and require further evaluation. Thus only the MC cubes that lie within an active region are processed. Given the exponential nature of three-dimensional sets, such improvements offer appreciable rewards that significantly outweigh any added upfront coding complexity, or delays incurred during storage initialisation at runtime.

During the design and development of the case study presented within this paper, the authors implemented the MC algorithm within the architectural modelling environment of Rhinoceros3D via Python script, and also within Processing, an open-source visual sketchpad that utilises the Java language. The performance gains were especially significant in the latter environment, given the continuous mode of computing and OpenGL drawing (at a desired twenty-five frames per second) this context affords.

2.3 MC FABRICATION: AS A MODE OF GEOMETRIC STANDARDISATION

The MC algorithm relies upon two core parameters; the density of the cubic field and the degrees of freedom granted to interpolate along the intersected edges of a given cube. Within the context of computer graphics these key attributes enable trade-offs to be made between the generating of smooth, detailed and seamlessly shading volume representations against acceptable computation times. Considered from the perspective of the constraints of architectural fabrication: economic, material, geometric or other, it is the precise ability to catalogue elements (by MC type) or ability to permit and/or restrict possible outcomes via stepwise edge intersection routines that provides new possibilities for describing and managing geometric variations.

Increasing the density of the cubic field permits a greater level of detail; however such gains are made to the detriment of overall system’s performance owing to the exponential increase in computing tasks. Given the exponential increase in the number of geometric elements, this tactic may ultimately prove counter-productive depending on the ability of the production or assembly line to absorb such quantums.

Stepwise edge interpolation however, remains a powerful and expansive mechanism of geometric variation control. Granting free-reign to each facet to traverse along an intersected edge enables a more efficient method of approximating the true mathematical isosurface. Equally, complete freedom is potentially problematic and at times akin to opening Pandora’s box of infinite variation. The facets may owe their DNA to a finite number of underlying MC marking scenarios, each resulting facet will invariably be unique owing to the scalar effects - geometric transformations and distortions—of the field. Conversely, completely denying a facet’s right to traverse an intersected-edge, the isosurface can only intersect a cube-edge through its edge midpoints. Observation of the fourteen basic MC configurations reveals that only seven unique triangles are implemented. Thus, any volume can be approximated, albeit crudely, using a very limited palette of principle shapes. Thus it can be seen that through the introduction of a stepwise approach to edge interpolation, better surface approximations may be achieved at the rate of 7N. Should we permit three possible intersection points along an edge, our seven triangle set expands to a possible 342 unique facets, while the allowance of four possible locations, rapidly escalates to a possible 2,401 facets. Selecting a workable threshold is aided only by a thorough understanding of what those variations entail, notably from the perspective of fabrication. It is also evident, that engaging such orders of mass-customisation demands an automated approach to fabrication information, namely a file-to-factory protocol to support the generative (algorithmic or explicit) modelling.
2.4 MC FABRICATION: 3D BENT ROD TECTONIC SYSTEM

The tectonic system employed by the Australian Pavilion case study does not mimic the mesh facets of the MC volume rendering. This strategy was quickly dismissed on the grounds that its spatial and architectural character would remain overly captive to the most obvious traits of the MC. Instead, the project operates at a higher-order of abstraction through opportunistically tracing the fourteen unique grids the algorithm provides. The tracery sinuously flirts with the intersecting scenarios and, dependent on situation, either the inside (those being below the isolevel) or outside (those being above the isolevel) of the cube edges themselves. Thus the fourteen scenarios yield twenty-eight possible “families” of geometry with their selection and placement entirely directed by an extended implementation of the MC algorithm (Figure 4). In collapsing possibility to definable ‘families’ the designers were able to tune the visual quality and fabrication viability of each scenario.

Once the generalised patterns were settled upon, opportunities for mass-variation were reopened through global deformations being applied to the MC lattice itself (Figure 5). These deformations were specific to the pavilion installation and in response to: the unusual curved geometry of the exhibition space; improve pre-assembly, packaging and shipping efficiencies; achieve false perspectives and visual displacements, reinforce circulation routes and establish more legible territories, grains and densities within the installation itself. The result is that the installation consists of 1,080 entirely unique parts all organised through the spatial grid of an extended MC approach.

3 ROBOTIC FABRICATION [RF]: ROBOTICALLY TENDERED CNC ROD BENDER

3.1 RF: BACKGROUND

Central to the ongoing exploration of RF conducted by the authors is the introduction of material and fabrication constraints within the digital modelling environment. In a broad sense, this research is deeply engaged with re-establishing a necessary component of design, an explicit feedback between designing and making. We use the word explicit because we believe this is a concept that is often strived for, but seldom reaches a level of fluidity where it can become a truly meaningful part of the design process. Reducing or eliminating the steps of translation associated
with the transition from design to fabrication provides the possibility of creating feedback loops between design intent and fabrication logic, especially where the design process incorporates algorithmic methodologies. The drive towards streamlining these translation steps is not meant to be reductive, merely acting in the service of efficiency; instead it is meant to aid in the development of a reciprocity between designing and making.

This pursuit has been supported by the realisation of new grammars of fabrication capable of incorporating an open-ended set of process-specific variables and calculations. Conceptually, this grammar is software independent; initially the software was developed in RhinoVB, but was recently rewritten in Python, in each case working within McNeel’s Rhinoceros3D software. What is important is the integration of the process constraints from the conceptual phase, which can be generative or explicit, completely through to the machine simulation and final output of machine instruction codes. From the beginning the code has been developed with the intention of creating an open-source framework thus permitting implementation by a broader set of users. This desire is sought primarily for the purposes of feedback and the evolutionary development such a culture permits, and is formally supported by the Fabrication Robotics Network co-founded by the authors in 2011. To date it has been utilised to generate machine code and simulations for Kuka and ABB robots, as well as 5-axis CNC G-Code.

3.2 RF: COLD-FORMING (3D ROD BENDING)

The case study celebrates the constraints of creating structure through bundling, imparting a one-dimensional material with surface- and volume-like qualities through principles of aggregation and variegated densities. Research and development pertaining to tool design, multi-station fabrication processes and sequences continues previous trajectories that saw the realisation of a custom-made, freestanding, robot-tended, CNC bender, controlled as an additional axis of the robotic system. Using this, the robot controls the angle and plane of each bend and the distance between bends, thus creating a series of complex and unique parts (Figure 6). The sequence of operations requires a precise choreography of external clamps, the robotic gripper, and the motion of the arm. The algorithmically generated instruction code makes typical construction drawings redundant and interestingly, also bares little resemblance to the typical point-to-point motion programs of CNC machines.

The process itself is an adaptation of well-established wire and tube bending strategies used in the mass production of formed parts. What is significant about this application is the development of the tool and process coupled with a generic platform, that is robotic manipulator. This allows continued feedback of process parameters into the generative algorithms, which breaks with the traditional workflow of CNC bending and forming. The tight, often
simultaneous development of fabrication hardware (bending dies, hydraulic grippers, integrated servo) and software (KRL code generation from modelled or scripted geometry) means that one can intercede with either aspect equally. For example, the limiting acute angle that can be bent can be made smaller by rebuilding the bender itself in a more compact fashion, or (Figure 7) by replacing any acute angle generated beyond that limit with a double bend. We implement both approaches simultaneously, and as a result, an expanded fabrication dexterity and wider palette of formal possibility develops far faster and with more immediately testable output, than pursuing either avenue alone ever would.

4 CONCLUSION

As demonstrated by the Clouds of Venice installation, the potential offered by the complete and bi-directional integration of robotic fabrication, algorithmically generated form and instruction code allows for a radical transformation of the linear processes associated with industrial production. Algorithmic techniques enable non-hierarchical, non-linear and explicit negotiations between an enlarged set of architectural intentions and the material substrates and fabrication concerns through which they operate. Flattening, reducing or eliminating the steps of translation necessary to the transition from design to fabrication provides the possibility of creating feedback loops between design intent and fabrication logic. Critically, this drive towards streamlining the translation steps is not meant to be reductive, merely acting in the service of efficiency; instead it is meant to aid in the development of a reciprocity between designing and making. It is a novel approach that shifts disciplinary concerns away from object-centric notions of artefacts including how such things are made, towards a deeper concern for the implications of the structures underlying the production processes themselves.

The underlying repetition of the MC algorithm provides an open-ended framework for the fabrication of non-uniform and topologically complex surfaces (and volumes) while maintaining a controlled number of geometrically and topologically consistent elements (or procedures necessary to the fabrication of non-uniform elements). This is especially, and obviously, beneficial for repetitive part production e.g. casting, however, as demonstrated through the case study, even within numerically controlled fabrication scenarios this can often lead to significant production efficiencies. By simplifying workflow, minimising re-jigging, increasing the predictability of motion sequences and/or minimising errors resulting from anomalies the MC algorithm, when strategically implemented relative to fabrication constraints, offers architects a valuable tool in designing and realising freeform assemblies.
WORKS CITED


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