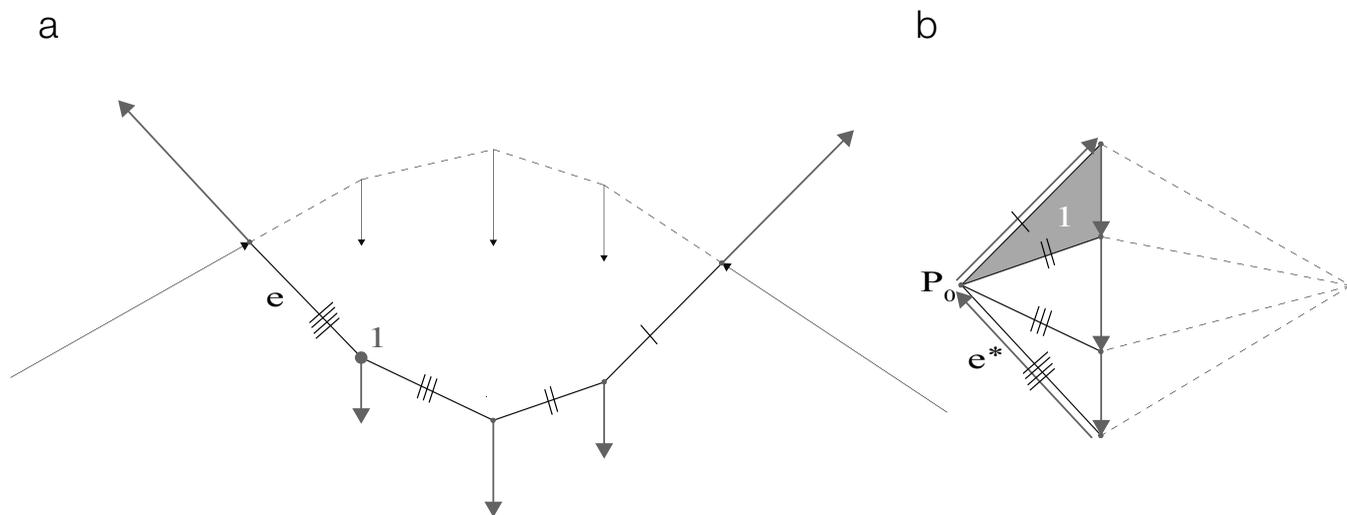


# FUNICULAR SHELL DESIGN EXPLORATION

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1 The form diagram (a) for a tension/hanging funicular and its force diagram (b). The dotted line shows an alternative compression/standing funicular resulting in higher reaction forces.

## ABSTRACT

This paper discusses the design exploration of funicular shell structures based on Thrust Network Analysis (TNA). The presented graphical form finding approach and its interactive, digital-tool implementation target to foster the understanding of the relation between form and force in compression curved surface structures in an intuitive and playful way. Based on this understanding, the designer can fully take advantage of the presented method and digital tools to adapt the efficient structural system to the specific needs of different architectural applications. The paper focuses on simple examples to visualize the graphical concept of various modification techniques used for this form finding approach. Key operations and modifications have been identified and demonstrate the surprisingly flexible and manifold design space of funicular form. This variety of shapes and spatial articulation of funicular form is further investigated by discussing several built prototypes.

## 1. INTRODUCTION

In the last two decades, the rise of computer-aided design and modeling techniques have enabled a new language of doubly curved surfaces in architecture, and structural concepts are being integrated as organizing principle of form, material and structure (Oxman 2010). New digital fabrication methods furthermore made the realization of complex forms technically and economically feasible. To achieve an efficient and elegant design for these non-standard structures, a close collaboration between architects and engineers from early stages in design, based on shared computational tools, gained importance (Tessmann 2008). In order to deal with hard engineering constraints in an intuitive manner in the design process, visual representation (Fergusson 1977) and real-time feedback (Kilian 2006) of structural information became essential. Particularly in funicular structures, form and structure are inherently linked to each other. The designer thus needs to understand this relation to fully take advantage of this efficient structural system in order to adapt it to the specific needs of different architectural applications.

Historically, particularly hanging models and graphic statics have been used to design vaulted structures. In the beginning of the 20th century, Antoni Gaudí used hanging models in the design process of the Crypt of Colònia Güell (Tomlow et. al. 1989); Frei Otto and his team used hanging models to find form for the Mannheim gridshell (Burkhardt & Bächer 1978); and Heinz Isler designed his concrete shells based on hanging cloth models (Chilton, 2000). Around the same time as Gaudí, the Guastavinos were designing large thin-tile vaults for important buildings all over the United States using graphic statics (Ochsendorf 2010). Such form-finding techniques, both physical and graphical, allow the exploration of three-dimensional systems, but the design process is time-consuming and tedious, particularly due to a lack of global control—each local change affects the overall geometry. In the last 15 years, a few three-dimensional computational methods have been developed for the equilibrium design of vaults. Kilian developed a virtual, interactive and real-time hanging string modeling environment, using particle spring systems adopted from the computer graphics industry (Kilian 2006). His approach emphasized the exploration experience, but had challenges to steer the design in a controlled manner. Tools such as Kangaroo or the built-in Maya cloth simulation are based on similar solvers (Kilian & Ochsendorf, 2005). Most recently, several interactive tools allowing for real time exploration of funicular networks have been developed (Piker 2011; Harding & Shepherd 2011).

The Thrust Network Approach (TNA), extending graphic statics to the third dimension for vertical loading, enables the explicit representation and control of all degrees of freedom in funicular networks. TNA has been implemented into an interactive, bidirectional design framework for compression-only vaults (Rippmann et.

al. 2012). This paper provides insights on how to use this graphical approach to extend the known design space usually associated with funicular structures. In the last section, several built prototypes are shown that were designed using the approach discussed in this paper.

## 2. A GRAPHICAL APPROACH TO FORM FINDING

This section describes the concepts of graphic statics and its three-dimensional extension, TNA.

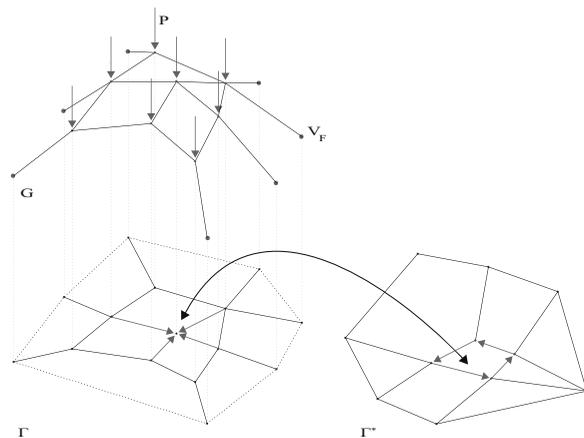
### 2.1 GRAPHIC STATICS

Graphic statics is a method for design and analysis of structures based on geometry and drafting (Culman 1864; Cremona 1890). It uses two diagrams: a form diagram, representing the geometry of the pin-jointed structure (Figure 1a), and a force diagram, also referred to as a (Maxwell-) Cremona diagram, representing the equilibrium of the internal and external forces of the structure (Figure 1b). The power of graphic statics is based on its inherent bidirectional capabilities; one can either use the form diagram to construct the force diagram, or apply the inverse process and construct parts of the form diagram from an intended force diagram, that is either form or force constraints can drive the design exploration (Kilian 2006).

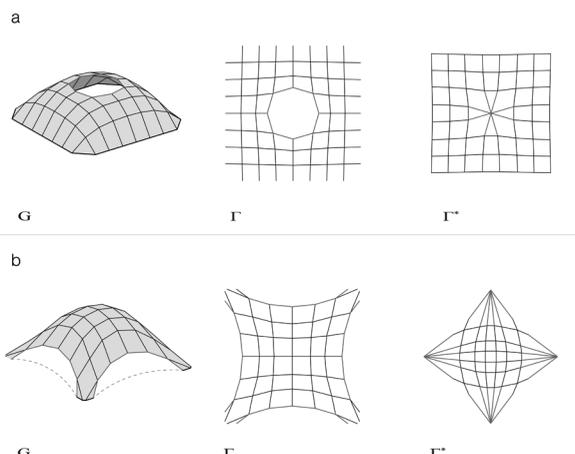
The force diagram is constructed by combining all force vector polygons, graphically expressing the equilibrium of the nodes (local), and structure as a whole (global) of the form diagram. Because the elements of the force diagram represent force vectors, the diagram has as many elements as the form diagram; its elements are parallel to their corresponding elements in the form diagram; and, the lengths of the elements are a measure of the magnitude of axial force in the corresponding elements in the form diagram. Geometrically, the relation between the form and force diagram is called reciprocal (Maxwell 1864).

### 2.2 THRUST NETWORK ANALYSIS

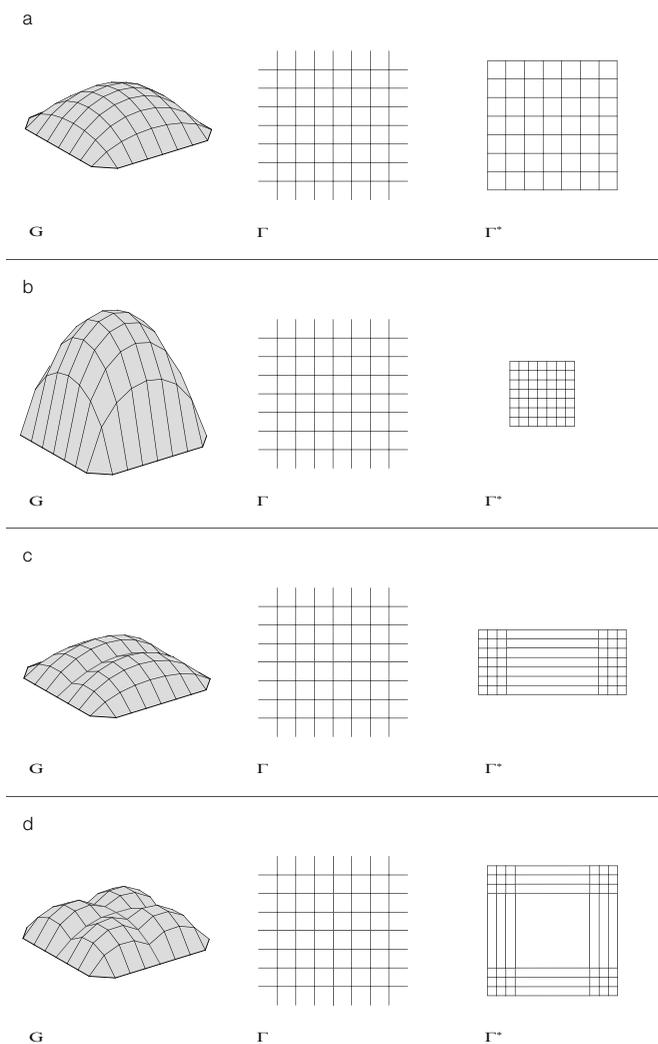
Thrust Network Analysis is a recently developed form-finding method using discrete networks for the design and analysis of funicular structures with complex geometry and vertical loading (Figure 2). These networks are not necessarily actual structures, but rather spatial representations of compression forces in equilibrium with the applied loads. The form diagram  $I$  defines the plan geometry of the structure and the force pattern. Its corresponding reciprocal force diagram  $I^*$  represents and visualizes the distribution of horizontal thrust. Based on this graphical representation of form and force in plan, the funicular thrust network  $G$ , in equilibrium with the given vertical loading, is defined. Because of the vertical loading constraint, the equilibrium problem can be decomposed in two steps: (a) Solving horizontal equilibrium: Since the vertical loads  $P$  vanish in  $I$ , which is defined as the horizontal projection of the thrust network  $G$ , the in-plane equilibrium of  $I$  also represents the horizontal equilibrium of  $G$ , independent of the



2 An overview of the different components used in TNA: form diagram  $\Gamma$ , (reciprocal) force diagram  $\Gamma^*$ , and thrust network G. A detailed description of the method and its implementation is given in the cited papers



4 Creation of convex inner openings (a) and convex open edge arches (b)



3 Global decrease (b) and local increase (c,d) of forces showing the resulting changes in the thrust network.

vertical loads (Figure 2), and is represented by the reciprocal force diagram  $\Gamma^*$  which is drawn to scale. (b) Solving vertical equilibrium: For a given horizontal projection,  $\Gamma$ , and equilibrium of the horizontal force components, given by  $\Gamma^*$  a unique thrust network G, in equilibrium with the given loading P, is then found for each set of boundary vertices,  $V_p$ .

### 3. STEERING FORM AND FORCE

This section gives a detailed overview of the different modifications of form and force using the graphical approach discussed in Section 2.2. The simple examples (Figures 3-7) help to explain the structural logic of funicular shapes, showing the surprising flexibility in design of these structures, as well as their formal, respective structural limitations.

#### 3.1 INTERACTIVE FORM FINDING

To explore the design space of funicular shapes, the TNA method was implemented as an interactive, digital tool, which was developed for in-house research but also released under the name RhinoVAULT (Rippmann et. al. 2012) as a free plug-in for the CAD software Rhinoceros (McNeel 2013). It takes advantage of the inherent, bidirectional interdependency of form and forces represented in visual diagrams, which are essential for a user-driven and controlled exploration in the structural form-finding process. Thus, the implementation and design of the form-finding tool focused on design through exploration, underlining the visual and playful nature of the approach, mainly targeting the early structural design phases. RhinoVAULT emphasizes the inherent simplicity and visibility of the graphical approach to explicitly steer form and forces. This not only fosters the understanding of the form-finding process, but also promotes knowledge of structural design in general. The tool was used for the design of the case studies presented in Section 4.

### 3.2 THE RELATION OF FORM AND FORCE

The TNA method provides the user with a high level of control over the force distributions in a funicular network, in order to accomplish a certain design goal. The following key operations and modifications to shape funicular form and steer the form finding process have been identified: (a) global and local attraction of forces, (b) creation of openings and open edge arches, (c) redirection of the flow of forces, (d) change of support conditions and (e) integration of continuous tension ties.

#### (1) Global and local attraction of forces

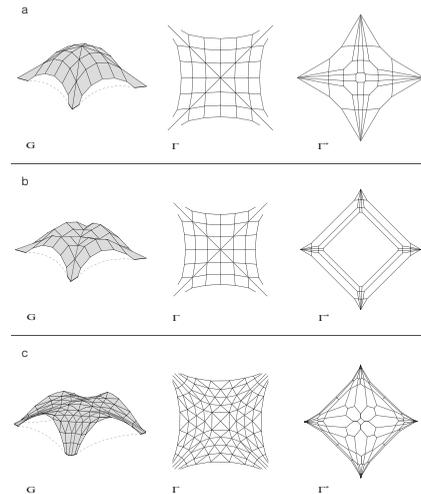
The TNA framework allows controlling the multiple degrees of freedom in statically indeterminate networks. In other words, a statically indeterminate form or force diagram can be geometrically modified while keeping horizontal equilibrium. This means that the length of corresponding elements of the form and force diagram can be modified while guaranteeing their parallel configuration. Consequently, this leads to a local or global increase or decrease of forces since the length of each element in the force diagram represents the horizontal force component of the corresponding element in the structure. The examples in Figure 3 demonstrate this type of global (Figure 3a-b) or local (Figure 3c-d) modification of horizontal thrust and the resulting changes of the thrust network. Figure 3b shows the uniform scaling of the force diagram, globally decreasing the horizontal thrust, which consequently affects the height of the thrust network. Note that this is analogous to move the pole of a funicular polygon in graphic statics (Figure 1) or how reaction forces increase by tensioning a cable, aiming for a nearly straight configuration.

#### (2) Creation of openings and open edge arches

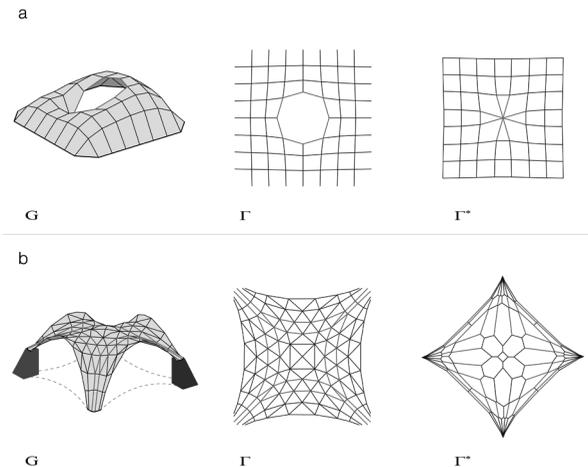
Openings such as an oculus in a dome (Figure 4a) or open edge arches of a shell only supported at the corners (Figure 4b) are typical features of funicular structures. These openings always form a funicular polygon in the form diagram. Note the direct relation of an open edge arch (Figure 4b) and a funicular polygon in graphic statics (Figure 1). Consequently, the inner openings and open edge arches of compression-only structures are by definition convex.

#### (3) The redirection of the flow of forces

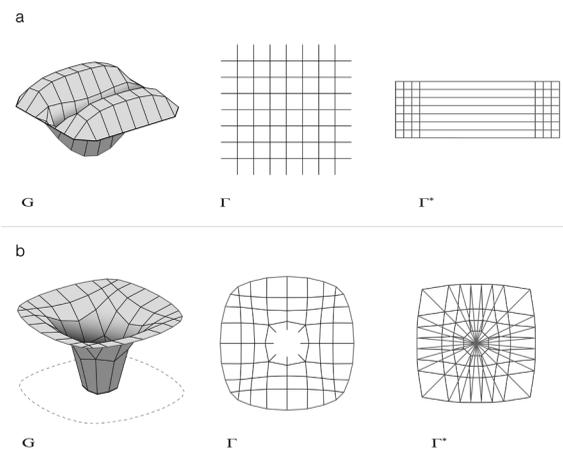
The layout of the form diagram defines the force pattern of the structure in plan. Consequently, forces can only be increased (attracted) or decreased in the directions defined in the form diagram. Therefore, the topology of the form diagram might need to be modified in order to achieve a specific force redistribution to subsequently adjust the shape of the structure. Compared to the form diagram in Figure 4b, additional, diagonal elements were added to the form diagram in Figure 5a, enabling the attraction of forces along the diagonals of the structure, resulting in the cross-vault-like thrust network shown in Figure 5b. A more complex example (Figure 5c) shows the attraction of forces offset to the open edge arches. Due to the lower forces in the corresponding open edge arches, the openings flare up.



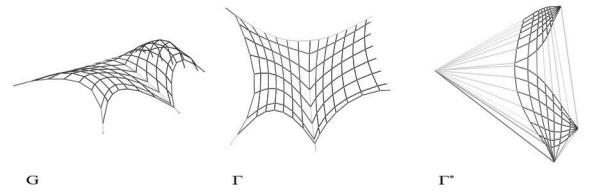
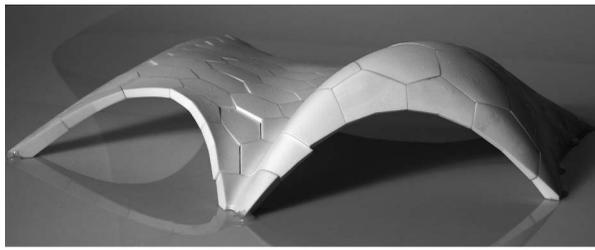
5 Changing the topology of the form diagram (a) in order to redirect the flow of forces by specifically modifying the force diagram (b,c), arches (b)



6 Modifying support conditions by adding new (vertical) supports (a) and changing their vertical position (b)



7 Integration of continuous tension elements in compression structures resulting in a hanging funicular (a) and a continuous tension tie along the open edge of the structure (b)



8 Final structure and TNA form finding result of the Radical Vault – Scale Model.

#### (4) Modification of support conditions

Differentiated support conditions can be simply added to existing solutions (Figure 4a) by fixing additional nodes while solving for vertical equilibrium (Figure 6a). Note that this modification has no effect on the horizontal equilibrium. Consequently, the newly defined supports take only vertical forces. Further, any supports can be modified in height (Figure 6b).

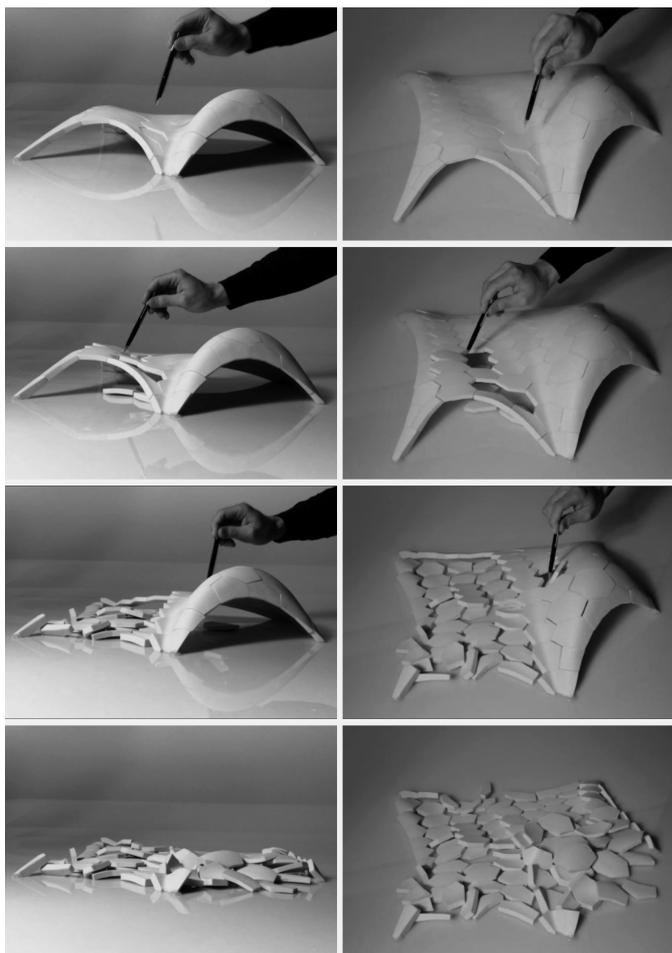
#### (5) Integration of continuous tension ties

An interesting property of graphic statics, and subsequently TNA, is its equivalent use for funicular compression and tension structures as well as for combined compression-tension structures (Van Mele et. al. 2012). This property opens up exciting possibilities for the exploration of new funicular shapes. Whether an element in the thrust network is in compression or tension depends on the orientation of the corresponding elements in the form and force diagram. Note that there is again an analogy to a funicular polygon in graphic statics, which can be in tension or compression according to the position of the pole  $P_0$  (Figure 1). The networks in Figure 7 demonstrate the integration of continuous tension elements or ties in compression structures. Figure 7a highlights the aligned tension elements in the thrust network that form a hanging funicular, which supports the adjacent compression vault caps. The corresponding, flipped tension elements in the force diagram now overlap their neighboring compression elements. The example in Figure 7b shows a ring of continuous tension elements forming an unsupported, cantilevering edge that acts as a tension tie. As for any other openings discussed before (Figure 4), the corresponding elements form a funicular polygon in the form diagram. Note that in contrast to the examples in Figure 4 the funicular polygon is concave due to the corresponding flipped tension elements in the force diagram.

## 4 CASE STUDIES

In the last three years, several built prototypes and scale models have been designed using RhinoVAULT. The presented 3D-printed structural scale models were primarily used as proof of concept studies to verify the structural stability of block configurations of discrete vaults (Van Mele et. al. 2012). In contrast, most full-scale prototypes were built using thin-tile techniques, giving the opportunity to focus on the link between form finding, fabrication and erection (Davis et. al 2012). The thin-tile technique (also called Guastavino or Catalan vaulting) enables efficient erection with minimal guide work and is relatively easy to learn. As a result, several, short student workshops could be organized, starting with an introduction to structural design using the discussed tools and subsequently result in some of the built prototypes shown in this section.

The order of appearance of the following case studies is related to the key modifications of the form and force diagram listed and



9 Collapse study of the Radical Cut-stone Vault – Scale Model.

discussed in Section 3.2. The fact sheet (Table 1) at the end of this section helps comparing the case studies, and summarizes technical details and general information of all structures.

*(1) Radical Cut-stone Vault – 3D-printed Scale Model*

The 3D-printed model shown in Figure 8 was one of the first structural models designed and form found using TNA and its early design tool implementation. It served as a first case study to verify the stability of a discrete, compression-only shape. Despite its free-form appearance, it stands in compression and only partially collapses after several blocks are pushed out of the hexagonal bond (Figure 9).

The asymmetric shape with two high points on varying heights is related to the local attraction of force (horizontal thrust) on the left side of the structure. A shallow open edge arch on the back and the converging fold in the middle of the two bumps cause the highest horizontal thrust. Note that these high local forces affect the local stability of the structure and define certain stable sections, which can be identified during the collapse testing (Figure 9).

*(2) Funicular Brick Shell - 1:1 Thin-tile Prototype*

This full-scale, thin-tile vault prototype has been planned and realized focusing on technical and aesthetic criteria aiming for a light and open form, which included multiple open edge arches, a point support and high degrees of curvature.

The structural fold feature demonstrates the control enabled by the TNA approach: by stretching a section of the force diagram, while maintaining the parallel and directional relationship (this is enforced by RhinoVAULT), forces are locally increased in that region of the vault surface, creating the anticlastic undulation in the compression-only thrust network.

*(3) TU Delft Hyperbody MSc2 Studio Foam Shell - 1:1 Prototype*

During a one-week workshop in collaboration with TU Delft and ROK, Rippmann Oesterle Knauss, the possibilities of combining form finding with a fabrication-based design approach were explored. More than 50 unique foam components were defined using generative design strategies informed by fabrication constraints and construction-aware criteria. All components were later cut from EPS using robotic hot-wire cutting.

The form diagram's topology was directly used to inform the number of components, their size and generative geometry. The integration of multiple open edge arches helped to create a light and open structure while keeping the surface area to a minimum, saving material for this relatively large prototype. The use of foam of course meant that the structure was very lightweight, which thus demanded gluing the discrete foam components to guarantee stability under asymmetric loading. The individual support heights were adapted to the site-specific context.

*(4) ETH Zurich Seminar Week Vault - 1:1 Thin-Tile Prototype*

This thin-tile, vault prototype was constructed by students during a one-week workshop that covered the basics of vault design, from form-finding strategies to hands-on construction work using traditional brick vaulting techniques.

The form finding was driven by the reduction of surface area to allow the students, who are entirely new to the construction method, to construct the shell in only three days, resulting in long-span open edge arches and one central oculus support combination based on an additional vertical load support.

*(5) UT Sydney Ribbed Catalan Vault - 1:1 Thin-Tile Prototype*

This student workshop focused on the form finding and erection of a rib vault structure using thin-tile techniques. After being introduced to tile vaulting and three-dimensional equilibrium design, using RhinoVAULT, the students developed the structural design and an efficient formwork system for the complex 3D rib network. After the erection of the primary rib structure on falsework, the vault webs were filled in using tile vaulting.

The form finding process focused on the integration of an array of smaller openings and open edge arches as well as on the modification of the supports heights.

*(6) Guastavino Staircase – 3D-printed Scale Model*

This discrete and unglued 3D-printed staircase structural scale model serves as one test result of the ongoing research on optimization methods for funicular structures based on TNA (Panozzo et. al. 2013). The staircase structure is inspired by the elegant tile staircases built by the Guastavino Company more than 100 years ago.

The difference lies in the vertical modification of the supports, which rise along the support walls of the staircase.

The compression only structure is based on the same principle as the previously discussed vaults with open edge arches (e.g. Figure 12).

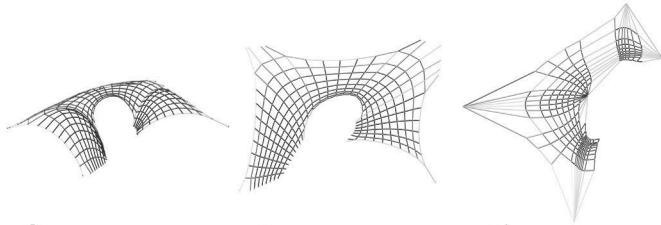
*(7) Stuttgart 21 Vault – 3D-printed Scale Model*

This discrete 3D-printed structural model showcases another test result of the ongoing research on optimization methods for funicular structures based on TNA. The vault structure is inspired by the elegant shell roof of the new Stuttgart main station designed by Ingenhoven Architects together with Frei Otto.

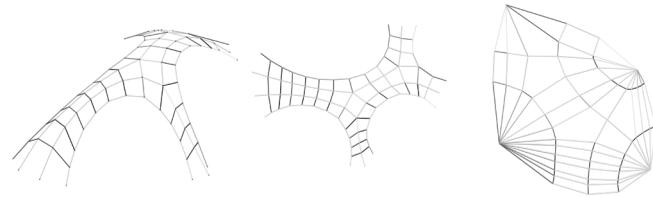
The very flat structure features two central oculi in combination with pulled-down supports, which are achieved by providing vertical reaction forces on one side of each opening.

*(8) MLK Jr. Park Stone Vault – 3D-printed Scale Model*

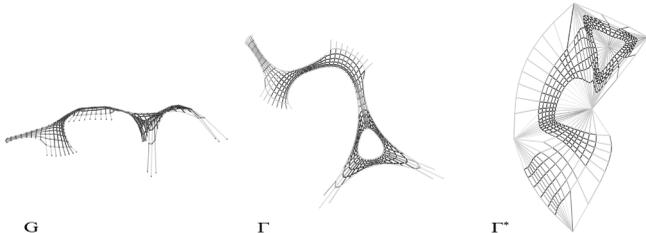
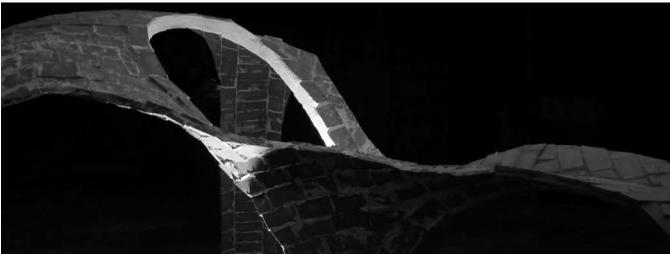
This discrete 3D-printed structural model shows the design for a radical stone structure to be used as a multi-purpose community space in Austin, TX, USA (Rippmann & Block 2013).



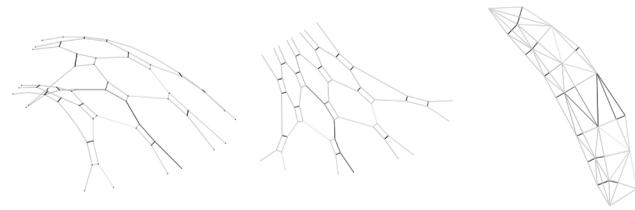
**G** **Γ** **Γ\***  
10 Final structure and TNA form finding result of the Funicular Brick Shell - 1:1 Thin-Tile Prototype.



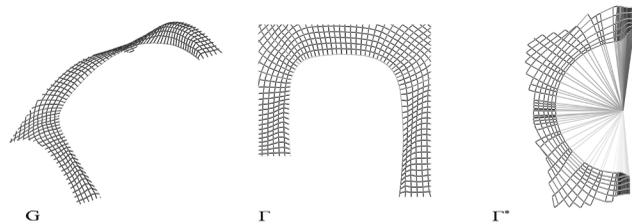
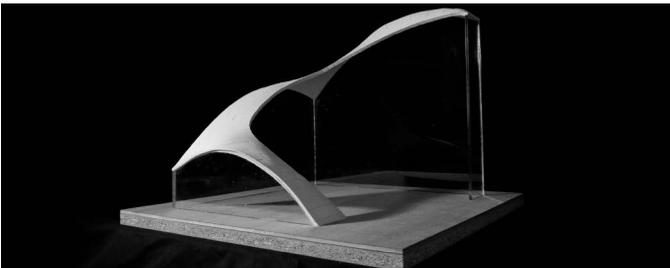
**G** **Γ** **Γ\***  
11 Final structure and TNA form finding result of the TU Delft Hyperbody MSc2 Studio Foam Shell - 1:1 Prototype.



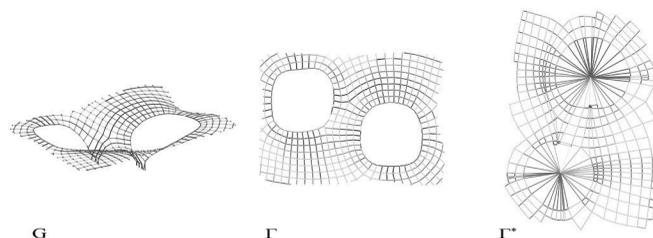
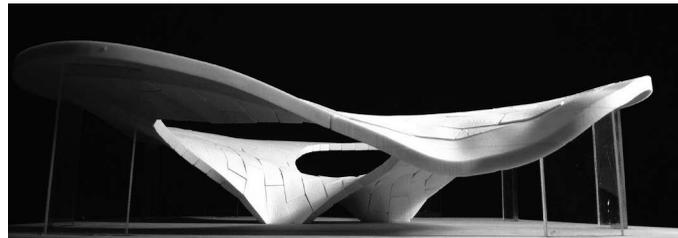
**G** **Γ** **Γ\***  
12 Final structure and TNA form finding result of the ETH Zurich Seminar Week Vault - 1:1 Thin-Tile Prototype.



**G** **Γ** **Γ\***  
13 Final structure and TNA form finding result of the UT Sydney Ribbed Catalan Vault - 1:1 Thin-Tile Prototype.



**G** **Γ** **Γ\***  
14 Final structure and TNA form finding result of the Guastavino Staircase - 3D-printed Scale Model.



**G** **Γ** **Γ\***  
15 Final structure and TNA form finding result of the Stuttgart 21 - Scale Model.

The design combines several features already discussed in previous case studies, such as combined oculus-support combinations and support height modifications. A key feature of the structure is the integration of the flaring-up edges, inspired by Isler's reinforced concrete shells (Chilton, 2000), to open up the covered space. This was possible by carefully adjusting the force flow of the structure in combination with the local attraction of forces.

*(9) Pittet Artisans Vault - 1:1 Thin-Tile Floor System*

This project shows one of the first commercially built structures that use RhinoVAULT for its structural design. The two-layer, thin-tile floor system was installed during extensive renovation work of an historic building.

The structure features three rib-like creases for structural stability and aesthetical reasons. This was possible by carefully adjusting the force flow of the structure in combination with the local attraction of forces.

*(10) Ribbed Cut-Stone Funnel Vault - 3D-printed Scale Model*

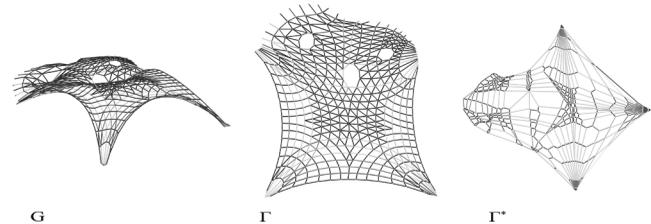
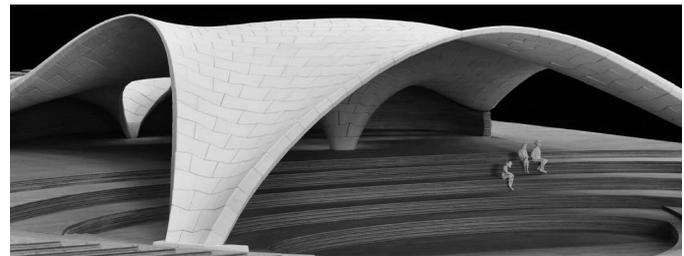
This discrete 3D-printed structural rib model showcases current research on compression structures in combination with continuous tension ties to enable the design of funnel-like shells with free boundaries (Rippmann & Block 2013).

The structural model of this rib structure features a ring of continuous tension elements forming a concave open edge.

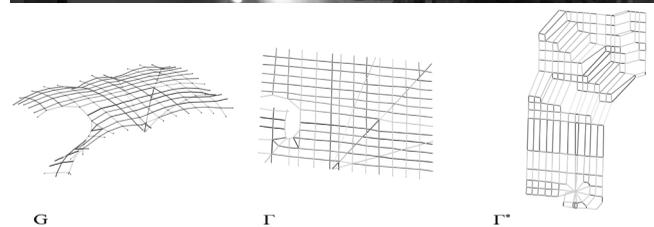
## 5. CONCLUSIONS AND FURTHER DEVELOPMENTS

This paper presented research on the design exploration of funicular shell structures based on Thrust Network Analysis (TNA). It discussed TNA as a form-finding technique for various funicular structures through its interactive, digital-tool implementation. The paper identified various, comprehensive modification techniques based on the relationship between form and force, using simple examples to visualize the underlying graphical concepts. The flexible and manifold design space of funicular form was explored by showcasing several built prototypes and scale models, emphasizing the variety of shapes and spatial articulation of funicular form through TNA.

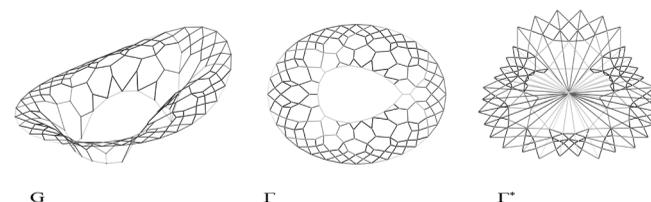
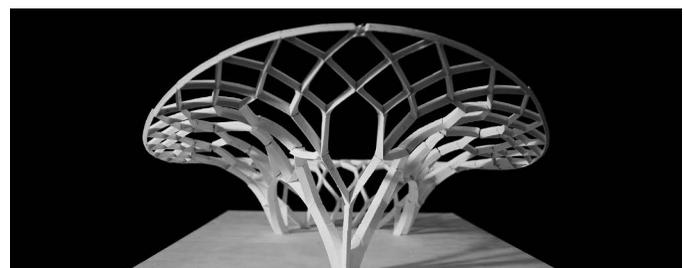
Future research in this area will include the survey of the current design-tool approach in order to further improve the intuitive and educational aspects of the form-finding process. RhinoVAULT was downloaded by more than 3000 people in the year 2012 and the current user-base is constantly growing. The user's knowledge and experience with the software can help to find new user-interface concepts and additional features to attract more designers using this approach to structural form finding. As a result, more architects and designers could intuitively integrate structural considerations in their early design work.



16 Final structure and TNA form finding result of the MLK Jr. Park Stone Vault – 3D-printed Scale Model.



17 Final structure and TNA form finding result of the Pittet Artisans Vault - 1:1 Thin-Tile Floor System.



18 Final structure and TNA form finding result of the Ribbed Cut-stone Funnel Vault – 3D-printed Scale Model.

Overview of 1:1 Prototype Case Studies

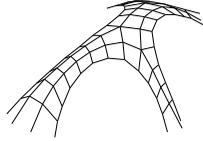
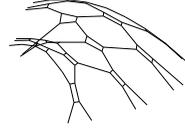
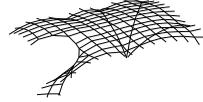
	Funicular Brick Shell	TU Delft Hyperbody MSc2 Studio Foam Shell	ETH Zurich Seminar Week Vault	UT Sydney Ribbed Catalan Vault	Pittet Artisans Vault
					
<b>Credits</b>	Disclosed after blind peer preview	Disclosed after blind peer preview	Disclosed after blind peer preview	Disclosed after blind peer preview	Disclosed after blind peer preview
<b>Year</b>	2011	2012	2012	2012	2012
<b>Location</b>	Zurich, CH	Rotterdam, NL	Zurich, CH	Sydney, AU	Corbeyrier, CH
<b>Material</b>	2/3-layer thin tile	EPS	2-layer thin tile	U-profiles formed with tiles 1-layer thin tile	2-layer thin tile with diaphragm walls
<b>l / w / h (m)</b>	7.7 / 5.7 / 1.6	8.9 / 6.4 / 3.4	7.3 / 4.1 / 1.6	5.5 / 4.9 / 2.4	8.2 / 3.6 / 2.6
<b>Surface Area (m<sup>2</sup>)</b>	28.6	23.8	7.9	9.3	36
<b>Discrete Elements</b>	No (continues tile bond)	50 (glued)	No (continues tile bond)	No (continues tile bond)	No (continues tile bond)
<b>Compression / Tension</b>	Yes / No	Yes / No	Yes / No	Yes / No	Yes / No

Table 1 Case Study Fact Sheet for 1:1 Prototypes.

Overview of Scale Model Case Studies

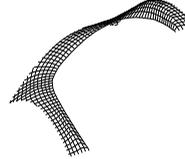
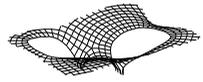
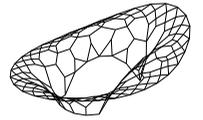
	Radical Cut-stone Vault	Guastavino Staircase	Stuttgart 21 Vault	MLK Jr. Park Stone Vault	Ribbed Cut-Stone Funnel Vault
					
<b>Credits</b>	Disclosed after blind peer preview	Disclosed after blind peer preview	Disclosed after blind peer preview	Disclosed after blind peer preview	Disclosed after blind peer preview
<b>Year</b>	2010	2013	2013	2012	2013
<b>Material</b>	ZCORP 3d-print	ZCORP 3d-print	ZCORP 3d-print	ZCORP 3d-print	ZCORP 3d-print
<b>l / w / h (cm)</b>	55 / 52 / 14	39 / 39 / 39	47 / 33 / 9	94 / 87 / 25	60 / 47 / 25
<b>Surface Area (cm<sup>2</sup>)</b>	1733	712	1562	5885	2605 (continues surface)
<b>Discrete Elements</b>	103	148	242	737	341
<b>Compression / Tension</b>	Yes / No	Yes / No	Yes / No	Yes / No	Yes / Yes

Table 2 Case Study Fact Sheet for Scale Models.

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