ABSTRACT

This paper presents a novel method for the integrated design, optimization and fabrication of space-frame structures in an autonomous, digital process. Comparative numerical studies are presented, demonstrating achievable mass reduction by application of the method by comparison to equivalent to normative space truss designs and dimensions. As such, a principal digital fabrication and assembly scheme is developed, where an architectural design methodology relative to the described process is established, and the proposed process is demonstrated through scaled digital fabrication experiments.
1.0 INTRODUCTION
This paper puts forward a novel method for the coupling of truss-based topology optimization of space-frame structures with digitally controlled assembly processes. The research represents a pilot joining of three preceding investigation trajectories of the authors within the fields of topology optimization, robotic fabrication and architectural design of morphogenetic structures. The presented work establishes the concept of an integrated optimization and fabrication process that fosters the development of novel, material efficient architectural space-frame structures, and identifies key methodological implications to its application within architectural design.

2.0 BACKGROUND
Topology optimization (TO) of continuum structures, as introduced by Bendsoe & Kikuchi (Bendsoe 1988), has seen widespread adoption within the aeronautic, automotive and naval industries for optimization and design of vehicle structures and machine parts (Bendsoe 2004). While studies have indicated potentials for disruptive design innovation and efficiency gains using topology optimization of continuum structures as a constituent design tool (Frattari 2011, Dombernowsky 2012), the coupling of these methods to architectural construction technologies and processes remains but little explored. In a previous research project, Unikabeton, the authors of this paper have investigated topology optimized concrete structures using large scale robotic CNC-milling (Figure 1-2). These investigations allow us to identify a number of methodological challenges of topology optimization of continuum structures in the context of architectural fabrication, serving as a background for the here presented work.1

Topology optimization of continuum structures can be summarized as a process in which material densities are iteratively redistributed within a FE-discretized design domain to meet an objective function, typically the minimization compliance (Figure 3). Such processes allow for the generation of novel, unexpected designs that achieve high structural performance gains through the development of advanced, structural morphologies, transcending typological classification. However, this method is primarily suited to designs that are also constructed through fabrication processes delivering continuum bodies, such as in-situ cast concrete or—extrapolating from current trends in additive manufacturing—3d-printing of concrete (Le 2011, Buswell 2007).

Although in-situ concrete is often used in conventional construction, many architectural structures are assembled from discrete members such as bars, beams, trusses, plates and façade elements, prefabricated in formative processes and assembled on-site. Key advantages of prefabricated construction include low-cost prefabrication of parts, fast on-site assembly, and avoidance of elaborate and complex formwork production. On the other hand, in-situ concrete casting has severe production constraints, especially when erecting outer scaffolding systems to withstand large casting pressures, and is prohibitively complex for the realization of topology-optimized spatial structures.

3.0 TOPOLOGY OPTIMIZATION OF SPACE-FRAME STRUCTURES
In an early paper presenting his studies for “material-economic structures”, Danish engineer and structural theorist Erik Reitzel presented a comparative scheme for the ratio between structural performance and the spatial distribution of material in steel structures (Reitzel 1971) (Figure 4). The comparative study lists incrementally sophisticated structural designs capable of sustaining an equivalent point load at midpoint of an identical span of five meters, finding that a reduction of 95.4% of the mass is achievable through a 13.6 times increase in the allowable structural height.
A framework for negotiation between and investigation of architectural parameters is hereby provided: while the structure increases in height, its layout takes on an increasingly expressive form, inversely proportional to its material consumption. Arguably, it is from here that richness in form can arise from reductionist thinking in resource management, and that through such an operation, the architectural presence of the structure becomes increasingly evident as it introduces geometries of new specificities and volume.

An architectural argument can be given for the spatial exploration of new material-efficient, lightweight structures. Space-truss and space-frame designs can be constructed in a number of materials relevant to the methods presented here. However, since its introduction by Alexander Graham Bell, the space-frame typology has predominantly been produced in steel. For steel specifically, price trends mapping the cost development of steel and concrete respectively (Figure 5) indicate that while the relative cost of concrete has followed a persistent development and remained stable 2009-2012, volatile steel prices at 2012 arrive at 1100 USD/ton, by comparison to 175 USD/ton average for concrete (Portland 2012). Consequently, raw material prices of steel construction processes range from 30-40 percent of the total construction cost compared to 21.8 percent by concrete. Factoring in the fabrication cost of semi-manufacturers of standardized members and profiles, the end-user material cost expense ranges between 60-80 percent of total construction costs, while actual on-site fabrication accounts for 10-15 percent, compared to the approximate 80 percent on-site construction cost for in-situ concrete (Lab 2007) (Figure 6).

Compared to concrete construction, a higher economic incentive is thus given for developing material effecient steel structures with less structural members and smaller dimensions of members. Complementarily, robotic fabrication of bend steel rod designs has demonstrated the feasibility of the production of complex architectural forms (Mcgee 2011, Cortsen 2013).

Continuing these developments, a dual argument is provided for topological optimization of new space-frame structures: a reduction in material consumption and increase in performance is expected, proportional to the allowable spatial volume for optimization operations. This targets spatial structures in particular which, due to the inherent complexity of topological optimization results, become increasingly difficult to construct via in-situ concrete casting. The topological optimization of space-frame structures thus extends the architectural domain achievable through digital construction, while simultaneously allowing for an exploration of new, material efficient designs.
4.0 TRUSS-BASED OPTIMIZATION

The challenges of realizing continuum TO structures using in-situ construction methodologies indicate numerous advantages of applying truss-based topology optimization to discrete architectural structures. As continuum-based TO methods are unsuited for such operation, addressing the challenge of developing discrete structural systems implies a shift in optimization methodology.

While the theoretical development of truss-based optimization procedures preceded continuum-based methods, truss-based optimization, unlike continuum-based optimization, has not seen a widespread adoption in industry. However, for architectural structures, the relative simplicity of truss-based results allow a prescription of the optimization results within a pre-defined range of components, leading to straightforward interpretation and realization processes.

Fundamental investigation of optimum structural topologies dates back to the beginning of the 20th century, in Michell’s classical work on minimum-weight grid-like continua (Michell 1904). This field matured much later into the general layout theory for frames and flexural systems (Hemp 1976, Rozvany 1976). Topology optimization of truss structures began gaining interest with the introduction of numerical methods for automatic optimum design (Dorn 1964).

The majority of established computational procedures follow the so-called “ground structure” approach, where the design domain is discretized using a fixed set of nodal points, which are then connected by a set of potential truss bars. The only requirement in setting the ground structure is that it should be able to transfer the loads to the points of supports without forming a mechanism. Then, the purpose of topology optimization is to determine the optimal cross-section areas of all potential bars, including eliminating unnecessary bars by assigning them a zero cross-section area.

In its most basic form, the topology optimization procedure aims at “finding the stiffest truss.” This can be stated as follows: find the structural topology and cross-section areas, so that stiffness is maximized, subject to an upper limit on the volume of material used, and provided that structural equilibrium can be satisfied. The corresponding mathematical statement is:

\[
\begin{align*}
\text{minimize} \ f & = f_u \\
\text{s.t.} \quad & \sum a_i K_i u = f \\
& \sum a_i l_i = V \\
& a_i \geq 0
\end{align*}
\]

where \( f \) is the external force vector, \( u \) is the displacements, \( a_i \) represents the cross-section area of the \( i \)-th bar, \( K_i \) is the parametric stiffness matrix of the \( i \)-th bar, \( l_i \) is the length of the \( i \)-th bar, and \( V \) is the total allowable volume. The measure \( f_i \) is usually termed compliance and is related to the external work of the forces. In practical implementation, a non-zero lower bound on \( a_i \) is required in order to obtain a non-singular stiffness matrix for which the equilibrium equations can be solved numerically. Then the optimization problem can be solved rather easily using nonlinear programming methods, even for large-scale structures, see discussion for example in (Bendsøe 2003).

In the investigation presented here, MATLAB implementations were developed for the formula shown above, and close variations of it. An important extension, which is not yet included, is the consideration of buckling under compressive forces. Nevertheless, several investigators suggested reformulations that account for either global or local stability (Ben-Tal 2000, Achtziger 1999) so we believe that this does not constitute a significant methodological barrier. Another challenge is related to limiting the design to accept only specific cross-section areas for practical purposes arising from fabrication considerations. The corresponding optimization problem then takes only discrete-valued design variables as opposed to continuous variables in the current investigation. From a mathematical
perspective, discrete-value optimization is much more challenging. State-of-the-art procedures can only deal with relatively small problems, thus limiting the resulting structural complexity and consequently the architectural contribution.

5.0 COMPARATIVE STUDIES

The potential of truss topology optimization as a facilitator of more efficient use of material resources, as well as its ability to integrate into fully digital architectural workflows, are discussed in this section by means of a demonstrative example. We examine the design of a space truss carrying a uniform load of a square roof, with vertical supports located on its perimeter. The span of the roof in both directions is 20 meters; the height of the truss is 1.25 meters; and the total load is 660kN. A typical design can be found in (MacGinley 1981), where the bars of the top chord are chosen as CHS 114.3/3.2, while the bottom chord and webs are constructed from CHS 60.3/4, assuming Grade 50 steel. The layout of this design is presented in figure 7. The total volume of steel utilized in this structure, which will serve as reference for optimized designs, is 0.9923 cubic meters.

Carrying out a structural FEM-analysis of the original design reveals a compliance value of 7.577kNm, a maximum deflection of 0.0333 meters, and extreme stresses of 105MPa in tension and 74MPa in compression. Significant improvement of the weight-compliance trade-off can be achieved even with the same structural layout as a ground structure, but allowing more freedom in sizing of cross-section areas. This corresponds to implementing the minimum compliance formula, with lower and upper bounds on \( a_i \) and the same volume as in the original design. With the upper bound chosen as CHS 114.3/3.2 and the lower bound CHS 48.4/4, more efficient distribution of material is achieved: compliance is reduced to 5.964kNm and the maximum deflection is 0.0263, meaning 21 percent improvement in performance for the same weight.

More dramatic savings in material consumption can be achieved by enabling more freedom in the topology optimization process. This is achieved by defining a more complex ground structure that contains a wide variety of bars, thus facilitating a more efficient transfer of forces to the supports via optimal paths. The simplest examples of such ground structures are three-dimensional boxes of various resolutions, where in principal any node can be directly connected to all other nodes by a bar; for practical purposes, a connectivity index is defined for controlling the maximum distance between two nodes that are connected directly. Examples of such ground structures are presented in figure 8 & 9. Individual architectural expression can be achieved by controlling the grid resolution and the connectivity index, as well as by imposing gradual resolutions. An example of an expressive optimized layout is presented in figure 10.

In order to ensure global stability, one can utilize a certain basic structural layout with minimal cross-section areas, upon which a
more elaborate ground structure is added. Then the main optimized layout results from finding the optimal distribution of bars in the ground structure, but if some bars are completely eliminated this is not a problem because the underlying basic layout provides the necessary stability. The combined ground structure is presented in figure 11, where grey bars represent the basic structural layout, identical to the original design in this case, and the green bars represent the additional ground structure for optimization.

The strategy described above is implemented in conjunction with the minimum compliance formula. We used CHS 21.3/2.6 as minimum throughout the basic layout, CHS 88.9/4 as a maximum throughout the entire structure, and zero cross-section area as minimum in the additional ground structure. We imposed a challenging volume limit of 0.5 cubic meters, which is roughly 50 percent of the original weight is used. Nevertheless, the optimized design exhibits a maximum deflection of 0.0312 meters and a compliance of 6.876kNm, both reduced compared to the original design. The extreme stresses are 117MPa in tension and 109MPa in compression, still well within the allowed stresses in steel. The optimized design is presented in figure 12. It is clear from this result that the optimal load transfer is via a “rotated” grid rather than in the original orthogonal path. In conclusion, the possibility of significant savings in material consumption justifies the use of advanced computational optimization procedures, while realization of the complex optimized design can only be achieved effectively through digital fabrication.

Based on interpretation of the optimized design, a realistic space truss was modelled using only four types of CHS tubes. The volume of the interpreted truss was 0.525 cubic meters, only a minor deviation from the volume used in optimization. A structural analysis carried out in the RSTAB framework analysis software revealed a maximum displacement of 0.0352 meters under the action of self-weight and the uniform load (Figure 13). Therefore it is clear that the optimization procedure can suggest new, efficient layouts that are physically viable.

6.0 ROBOTIC FABRICATION

The fabrication of the presented structures using established methods would require the use of auxiliary structures while establishing a structural welded connection between individual elements. Instead, we investigate a principal robot-assisted process to avoid the excessive use of jigs and additional formwork to stabilize elements during fabrication.

Purely geometric descriptions of building elements do not include all the information needed to allow a digital and automated fabrication workflow. Augmenting the geometry with additional fabrication data and processing it using different software packages is a time consuming and error prone process. In this case study, all the information needed for fabrication is merged into one coherent data
model within one CAD environment. This allows a seamless workflow from the optimization result to its physical fabrication.

A fabrication methodology was developed following the argument structure illustrated in figure 17. From the TO model, line segments representing the member axis are retrieved and organized in a data structure. Within this structure, each member is represented by a start and end point. From this data, the member orientation vectors, member position and the member length are retrieved and stored within the data structure. Subsequently, the member objects are sorted according to a feasible construction sequence, and respective approach trajectories for robotic fabrication are defined. A simple initial solution is computed, but currently manual interaction with the data model through an intuitive interface within the CAD environment is needed to solve about 82 percent of the members. We plan to introduce advanced collision detection and motion planning strategies in a next step in the process.

The data model consecutively outputs URscript that is sent in real-time to a UR controller via socket communication in a three-step fabrication and assembly sequence.

Sequence a) translates the stock feed item (metal rod) along the x-axis to position it for cutting in the dimensions of the member length. Sequence b) defines the end position of the object and generates the movement target. Sequence c) computes an approach trajectory from the cutting bed to the end position target.

To minimize chance of collision with the already built structure in this non-optimized stage, the approach trajectory translates through an intermediate point at a fixed height above the structure. The workflow (Figure 16) was implemented as Rhino.Python scripts within the Rhinoceros 5.0 modeling software.

To assess the principal feasibility of this approach, fabrication experiments were performed on a topology optimized sample structure, which was generated from asymmetrical support conditions, giving rise to a very high level of complexity (Figure 14-15). The experiments were performed using a standard 6-axis Universal Robot manipulator with a range of 850 mm. Based on data derived from the TO results four different wire cross sections were being used. The trimming of the individual wire elements to their precise length is integrated in the robotic fabrication process. A customized gripping tool ensures the precise positioning and orientation of the wire elements within the fabrication space. The connections between the wire elements were subsequently soldered manually. From the input data, an automated production sequence was generated, based on the z-height of element positions. A possibility for manual editing of the sequence, as well as approach trajectory was included (Figure 17-20).

The experiment yielded the following results: the structure could be produced using this method, however, it required extensive manual planning of the construction sequence and editing of
approach trajectories to avoid self-clashing. As TO structures are characterized by a very high degree of non-uniformity, this leads to complex member configurations, in particular with respect to node connectivity, which vary to a degree where systematization of the assembly sequences based on member properties alone are infeasible. Subsequently, alternative measures are needed to avoid elaborate, manual fabrication prone to collision errors. One possibility is deriving an alternative vector for approach trajectory planning, based on a collision detection scan of the angle field of the trajectory. Another is to modularize the structure into minor segments respecting simpler tool path analysis strategies. In both cases, further work would be required to meet demands of time-efficiency of industrial scale construction.

7.0 DESIGN PROCESS

Through the method presented in the preceding chapter, an autonomous, form-generating process is achieved from the onset of optimization until fabrication of the final shape. The radicality of this approach can be examined through a derivative of the tectonics theory that has served as a conceptual apparatus for the development of the presented work.

Tectonics, as characterized by Karl Christiansen, is defined as the relation and interaction between form, matter and technology (Christiansen 2004). The MTF-model (Figure 21) is confronted through developed experiments to arrive at a revisited description, which may operate as an aesthetic decision making model in design processes involving autonomous structural form-generation and fabrication (ASFF).

Historically, structural design has relied on the transfer of knowledge through experience accumulated in typologies. These typologies have been developed, extended and adopted through empirically based experimentation.

In these cases, design can be considered an intellectual activity, which reshapes the unity of parts constituted by the design task into a new configuration. With the advent of CAE form-finding technologies, such as size and shape optimization, dynamic relaxation or membrane simulation, parts of this intellectual design activity is replaced with computation. However, the domain of the architect here remain superior to that of the computer; the layout and composition of the architectural structure is defined entirely through his design activities, and the computational form-finding in essence fine-tunes the already given geometry.

In topology optimization, the most fundamentally constituent part of a geometrical construct, the topology, is subject to change. This translates into a dramatic reconfiguration of the construct, which subsequently bears little or no resemblance to its point of departure.

Hence, the role of computation with TO fundamentally defines the constitution of the design in new directions, and is thus elevated to a level of autonomy relative to the intellect of the designer, such that the execution of main design activities now stems from a purely calculative source. As a consequence, the designer is now detached from any direct control over the process, which, as a function of the embedded complexity of the calculations, arrives at unforeseeable, unexpected morphological results.

While such processes may initially appear as a realization of the ideal of an “objectivized” architecture presented through the proponents of the early modernism, we argue that while the direct design process can here be considered a linear deterministic effect of the optimization setup, the configuration of this setup will inevitably—within the application of optimization to architectural structures—be a product of both cultural and structural considerations, as building

Diagram of principal fabrication methodology
proportions, layout and other factors define the shape of the optimization matrix and supports. The cultural or architectural design sphere with the ASFF optimization process can thus be defined as design of the optimization domain, the design space. While the processes taking place within this domain follow the before given characteristics of autonomy, thus providing no means to direct the design in an immediate, intuitive manner, the configuration of the optimization domain does provide a high degree of indirect control, as the optimization setup can be reconfigured within the conventional limits of architectural design intervention, including the positioning of supports, proportioning of design spaces, and so on. Thus, it is possible to establish an iterative, indirect design process, in which optimization results are architecturally assessed, optimization setups revised, and the process restarted to arrive at revised results (Figures 22-23). These iterations do not provide a platform for the execution of a design intent in a conventional understanding of the term, but rather the possibility for a design exploration, where aesthetic potentials are discovered along the way, and the overall design—of which the optimized structure plays a larger or smaller role—is influenced accordingly.

Having noted these characteristics, we arrive at the following extension of the tectonic MTF model: the formal intent is separated into an intellectual and computational activity that operates on individual premises. The formal intent will be distinguished in the categories of structural formality and architectural formality, of which the latter embodies the numerically non-quantifiable parameters. The relation between structure, material and technology is detached within a closed and autonomous digital unity, operating on the basis of computational logic. Formal intent, detached from direct influence of this unity, operates on an indirect, primary level that initiates the process, and evaluates its results, while utilizing scaled robotic fabrication of optimized space-truss design.
the morphological unpredictability of results as a vehicle of architectural exploration. Thus empowered by the computational autonomy of the structural form-finding procedure, the development of the architectural concept is informed in new directions, unanticipated at the onset of the process (Figures 23-24).

8.0 CONCLUSIONS
A digital process has been established in which data is generated and treated in the following steps (Figure 25). The procedure represents an autonomous structural form finding and fabrication process, which operate independently of human design intervention from the onset of the process. The process was implemented via exchange procedures between the softwares listed.

The established process enables the generation of new, topology-optimized space-frame structures, which exhibit architectural characteristics specific to the ASFF procedure. Comparative numerical studies have been performed indicating that 30-50 percent reductions of material consumptions are achievable by comparative to normative space-truss designs. Robotic fabrication experiments have been performed demonstrating the principal feasibility of digital construction procedures for the realization of the optimized designs.

9.0 OUTLOOK/DISCUSSION
With the increase in availability and decrease of cost in digital manufacturing procedures for bespoke steel structures, such as NC-production of individual member lengths, topological optimization and robotic assembly of space-frame structures addresses an inherent architectural potential of these developments: The application of the ASFF process enables the generation of adaptive designs which can accommodate geometrical irregularities such as topographic variations of the site, skewed position of neighboring buildings or asymmetries of the building program. As such, the process points to a perspective for a contextualization of architectural structures, in which a computational response to the complexities of the project environment results in a structural articulation of contextual specificity.

A significant challenge encountered through the performed fabrication experiments was the high concentration of connections at specific nodes suggested by the optimization procedure, giving rise to steep joining angles and reachability issues in the robotic assembly. This requires the incorporation of fabrication-related constraints on node numbers and number of connections per node. Such constraints may be modeled using integer and mixed-integer optimization methods that have been applied recently in truss optimization (for example Kanno & Guo 2010, Mela & Koski 2013). For further development of an integrated optimization and fabrication procedure, implementation of such constraints represents an important next step.

While a number of development tasks remains, the generic nature of the presented fabrication methodology allows for an adaption to the fabrication of full-scale steel space-frame structures. Full-scale industrial robots featuring a larger working perimeter are available. Robotic welding of repetitive standard connections is a well-known method in the metalworking industry. However, due to the complexity of the construction task, a significant challenge remains for tool path planning with collision avoidance in relation to the build object. Further, the range limitation of robot reach suggests the necessity of constructing with individualized modules. Achieving a modularized design is achievable through control of the connectivity index in the optimization procedure, however at a cost of performance. A possible alternative could be robotic fabrication of nodes and member lengths in combination with manual on-site screw assembly. The research will address these challenges in planned full-scale fabrication activities.
Indirect control of the resulting topologies through reconfiguration of optimization parameters. In the above example, a) top row: connectivity index (the maximum number of diagonals the result is allowed to span) b) middle row: designspace proportions, c) support positions

24
25

ASFF Dataflow

Diagram symbol key

Abbreviations:

Start: Designscape configuration
End: Fabricated prototype
Decision 1: assembly sequence (AS) and approach trajectory (AT) cleared
Action a: Manual revision of A.S. and A.T

Matrix configuration: Rhinoceros 5.0 / Grasshopper

Topology optimization: MATLAB

Member parametrization: Grasshopper

fabrication data model: Rhinoceros 5.0 / Rhino.Python

Robot instructions: UR Script

(assembling sequence and approach trajectory NOT cleared)

(assembling sequence and approach trajectory cleared)
ENDNOTES


WORKS CITED


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