CONSTRUCTING MORPHOGENETIC OPERATORS WITH INVERSIVE GEOMETRY

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Biological membrane deformations are among the most difficult continuous transformations to model algorithmically. The process of epithelial morphogenesis can produce holes, handles, cusps, and intussusceptions—yet change in membrane curvature remains continuous throughout these topological phases.

To model bio-inspired membrane structures in a CAD-like program, we seek computational operations that are continuous, reversible, and parametrically controlled and yet still produce complex topological variations.

We have found that inversive geometry provides a powerful mathematical framework for handling both geometric changes of curvature and topological changes of genus. Our approach to modeling membrane transformations makes use of the bending and “turning-inside-out” capacities of inversive geometry.

For curvature deformations, we construct geometric mechanisms that exploit the correspondences between round and flat elements. For topological changes, we make use of basic incidence relationships contained within geometric algebra to mark topological events.

Radii determined by their distance from the control point to create an invaginating fold.
By examining the geometric and topological properties of canonical transformations under membrane morphogenesis, we have constructed the beginnings of a generative modeling framework using inversive geometry. This could prove to be a powerful and expressive tool in computer-aided design.

Our implementation of inversive geometry is based on conformal geometric algebra (CGA). An introduction to CGA can be found in the references (Dorst 2007: 355-500).

**REFLECTION OF CONTROL SHAPES INTO SURFACES**

In Figure 1 we investigate deformations of a surface created by treating points on the surface as round elements (circles in 2D and spheres in 3D) that can be reflected into. By adjusting the radii of the round element and inverting our control point into it, we can create cusps and folds.

**REFLECTION OF SURFACES INTO CONTROL SHAPES**

In Figure 2 we consider the bulging out of a flat surface by reflecting it into a circle. In Figure 2a a nominal bulge is created as the inverting sphere moves away from the surface. In Figure 2b further translating the inverting sphere upwards increases the bulging. In 2d the bulge pinches off into a second surface when the inverting sphere is translated fully out of the original surface.

**GENERATING ORBITS**

Figure 3 can be seen as a cross-section of a handle formation. These transformations are generated using real and imaginary circles as described by Dorst and Valkenburg (2011). The notion of a real or imaginary element is a component of conformal geometric algebra: they are round elements with negative squared radius. In Figure 3a, we see the start of a hole formation using a real point pair. In 3c we see a topological transition with tangent point pairs. In 3d a hole has formed with an imaginary point pair.

**WORKS CITED**


WESLEY SMITH is a computational artist and researcher. He is currently a developer at Cycling ‘74 in San Francisco, having received his master’s from the Media Art and Technology Program at UC Santa Barbara. His research focuses on the geometric modeling of biological processes using geometric algebra and programming real-time audiovisual composition environments.

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