ABSTRACT

This paper examines the development and usage of a computational drawing practice that emerged from Euclid’s *Elements* (300.BCE). Through an analysis of the mechanics of the drawing practice, it argues that computational drawing is not an outgrowth of digital production, but rather a practice that has been central to the discipline of architecture for centuries. Specifically, it focuses on the drawings ability to develop non-spherical three-dimensional vaults independent of orthography or any form of constructed “view”. Historical examples, drawings by the author, and student work accompany and support the text.
INTRODUCTION

Computational design describes the processing of informational design inputs to produce variable outcomes (Mitchell 1989). These inputs can range from the performance of materials to the variables in surficial geometry. In whatever form computation presents itself in current architecture discourse, it is almost always paired with the instruments that enable these sophisticated processes. However, what is less clear is what actually constitutes computational drawing. Achim Menges has pointed out that materials and biologic systems are capable of computing in their own right, thereby foregrounding the role of computation without digital computers (Menges 2013). Others have focused solely the liberating potential of instrumentally driven design strategies (Witt 2010). Computational design is therefore understood as either something derived from outside the discipline (biology) or as a contemporary method founded on the instrumental practices that have enabled the work in the first place. However, the discipline of architecture and its drawing practices are founded on computational principles. It is not only intrinsic to the discipline, but it could be argued that it constitutive. Computational design and specifically computational drawing is not an outgrowth of digital production, but rather a practice that has been central the discipline of architecture for centuries.

Euclid’s Elements (300 BCE) does not contain a single reference to architectural drawing (Euclid 2010). Regardless of its absence from the text, the architectural practice of graphically representing and geometrically calculating the shape of buildings through orthographic projection is based largely on Euclidean propositions. While orthography is the most obvious architectural derivative of Euclid, other drawing practices also emerged from this text that are less discussed, but perhaps more relevant to contemporary discourse. A clear example of this is the drawing practice that calculated the shape of the stones within vaults construction the sixteenth and seventeenth centuries. Orthography and the use of views and sections is not sufficient to deal with the wide variety of formal conditions that arise in the practice of vault design. The greatest difficulty that orthography faces arises in the construction of non-spherical domes. Spherical domes contain circular sections that are identical through both ninety and one hundred and eighty degrees of rotation. This allows for side and section views to be orthographically constructed from front views without additional information. In contrast, ellipsoid, hyperboloid, and paraboloid vaults do not share this characteristic. The section in each of these figures varies through ninety degrees of rotation (Huerta 2007). Stated differently, an ellipsoid cannot be generated from an ellipse through orthography alone. An additional means of drawing is required: a drawing practice that has much more to do with computing than it does with visualizing.

HISTORICAL INCIDENCES OF COMPUTATION

In Bernard Cache’s recent work he has pointed to several figures within Albrecht Durer’s three treatise that he describes as “parametric” (Cache 2013). One figure that appears in all three of Durer’s treatise, is a triangle that is used alternately to develop variable configurations of curves, lines, and the human body (Figure 1). Cache has specifically pointed to this figure and described it as an example of parametric design because of its ability to produce variable outcomes. Furthermore, while much of Cache’s work focuses on the description and parametric modeling of the actual instruments that Durer developed to create his curved drawings, this is the only example of a drawing capable of calculating form solely through the use of a compass and ruler (Cache 2013). Similar figures also appear in the Juan Caramuel y Lobkowitz Architettura Civile Recta y Obliqua (1678) and in Guarino Guarini’s Architettura civile (1735) (Figures 263).
In Caramuel’s and Durer’s treatise the figure appears in isolation as a means of calculating proportional relationships (Durer1977). However, in Guarini’s Architettura civile the figure is wedged into the workings of an orthographic drawing. Its position within the drawing is not simply convenient, but also generative. Guarini uses this wedge to calculate the curvature of ellipsoid vaults. He later states we know this figure from “Our Euclid” (Guarini1736). Guarini’s statement leads to his Latin treatise on geometry Euclide Adauctus et Mathematicas Universalis (1671), which explains the relationships between parallel sections in ellipsoids (Guarini 1671).

Further examination reveals that although Guarini has used it develop ellipsoid vaults, the figure itself has little if any direct relationship with an ellipse. In fact, the wedge is used solely for the purpose of calculating the distance from the center of the ellipse to points on surface of a smoothly curving ellipsoid. It is not a drawing of any part of the ellipse, instead it merely calculates variable distances from a common point. It is used not as a form of visualization, but rather as form of computation.

Guarini’s reference to the chapter on ellipsoid sections from his mathematical treatise might lead us to believe that this figure has something to do with ellipses, and it does in so far as it can accurately describe the proportional relationship between the changing curvature on the surface of ellipsoid vaults. However, its different use in both Durer’s and Caramuel’s text suggest a another origin. One that is not explicitly linked to ellipses. Durer used the wedge both to develop fortifications as Cache has pointed out, but also to develop measuring lines for the accurate orthographic construction of the human figure (Durer 1594). In these cases it is used to construct variable measurements along curved and straight lines respectively. Guarini’s use of the wedge solely involves the construction of elliptical sections within an orthographic projection. A closer examination of the drawing practice demonstrates the precise computational mechanics at work within Guarini’s drawing. His construction begins by striking of two non-parallel lines that converge at a point. One line is equal to the major axis of the ellipse and one to the minor. A line is drawn that connects the end points of both lines. Additional lines are constructed parallel to this new line from each of the orthographic projectors of points from the original ellipse (Figures 4) and (Figure 5). Ignoring the ellipse for a short time, we can see that the relationship between the two non-parallel lines and the parallel lines is central. The parallel lines divide the intersecting lines into proportional segments. Repeating this process with all of the projectors of the ellipse and transferring them to the lines of that pass through the ellipses center will produce a series of curves. The curves are precise transverse sections through the ellipsoid vault. Although in this case the drawing process has been inserted into an orthographic projection, it is not an orthographic drawing.
Orthographic drawings work by establishing a series of "views" connected by parallel lines. These views can range from single sectional profiles of objects, to oblique projections that visualize the objects full three-dimensional volume. In all cases, the orthographic drawing is always a view of some aspect of the object. It is structured in relationship to a constructed understanding of vision in which parallel lines do not meet (Evans 1989). The wedge deployed by Guarini, which calculates the distance of variable curvature from a center point, does not relate to any view of the object. The wedge calculates proportional relationships between distances and is divorced from any notion of vision. Its sole purpose is to compute and calculate form. It is not a drawing of an object, but rather a generative graphical calculation that outputs smooth three-dimensional curvature. This is a central distinction. Much of the history of architectural representation has focused on drawing techniques that are abstractions of the process of vision: orthography and perspective. While these forms of representation can produce images that appear to be divorced from any notion of vision, they operate by associating every measurable dimension with a particular two-dimensional view. Preston Scott Cohen’s work, which has developed an array of generative derivations of orthography, stereotomy, and perspective produces dense arrays of line-work that despite their level abstraction, result in the production of measurable views of objects (Cohen 2001). In the case of the wedge, views are no longer necessary. The dimensions of the vault are calculated within the wedge and do not correspond to a particular view of the object. The graphic product of the wedge is a receding grid of parallel and non-parallel lines. In order to construct a view of the forms generated by the wedge, the information must first be plotted as a set of polar coordinates within an orthographic drawing. Orthography is used here not as generative drawing practice, but rather as a visualization of information obtained from the calculations within wedge.

THE EUCLIDIAN ORIGINS OF THE WEDGE

This drawing practice, as deployed by Guarini, Durer, and Lobkowitz is a clear example of a sixteenth and seventeenth century computational drawing in architectural practice. However, it is the intent of this paper not only to argue that computational drawing is extant within a historical lineage of Western architecture, but rather that it is constitutive. In order to do so, this wedge must be positioned within a text that much of architectural representation and language is founded on, Euclid’s Elements (300 BCE). Problematically, there is not a single reference to vaulting in Euclid. However, in a seemingly disassociated book of Elements, in which there is not a single mention of curvature, Euclid provides a proposition which is the basis for Guarini’s, Durer’s and Lobkowitz’s respective constructions. In book six, proposition two, Euclid states, “If a straight line be drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally” (Euclid 10). This statement, absent of reference to ellipses, is the generator of the wedge. The proposition, described as the “intercept theorem” establishes a proportional relationship between non-parallel lines via the use of a parallel projectors (Field and Gray 1987). It establishes a process, in which any set of two or more lines can be proportionally divided (Figure 6). If we take this process and begin with the minor and major axis of an ellipse we can see that this proportional relationship will provide the same result as the one offered by Guarini for the development of ellipsoid vaults. Similar analysis, shows this the likely origin of Lobkowitz’s constructions. In all cases, Euclid’s proposition has been utilized to construct a drawing capable of calculating variable two and three-dimensional objects. It is not a drawing of a particular static object, but rather graphical device capable of computing relationships between a variable inputs.

In the late 1990s and early 2000s the phrase non-Euclidean appeared in publications to signal the advent of a new calculus based form of architectural representation. Complex curvature no longer required the aggregation of compound circular curves. Instead, curvature could be understood as continuous entity that was responsive and variable to a series of weights (Lynn 1999). More recent writings have gone as far as to describe computational drawing itself by describing it in terms of what is argued to be is antithesis, Euclidean geometry (Freeland 2012). Non-Euclidean geometries, of which there are more than one, are based on their non-agreement with Euclid’s parallel axiom. In non-Euclidean geometries, such as hyperbolic and elliptical geometries, positive and negative curvature are deployed in lieu of the zero curvature of the Euclidian plane. They exist as alternative spatial programs to Euclidian space and are not exclusionary (Hilbert 1952). This positions Euclidean geometry as one of many options rather
than a negative against which the space of computation can be foregrounded. Therefore, if the wedge is a form of computational drawing utilizing a Euclidian propositions, computation as a representational tool within the discipline of architecture can be argued to be independent of either a particular geometry or instrumental regime. Achim Menges has pointed out the way in which biologic and material entities can be used to computational ends, thereby foregrounding the importance of process in lieu of instrumentation (Menges 2012). Bernard Cache has described the Tower of the Wind in Athens as a computer (Cache 2012). Both instances reposition the assumed relationship between digital instrumentation and computation by offering examples that fall outside of digital practice.

Positioning a particular form of geometry or instrumentation outside of computational practice becomes increasingly problematic when considered within a larger historical framework. Euclid’s Elements was written in approximately 300 BCE. Its geometric and esthetic framework formed the basis of architectural representation. While architectural history has focused largely on the vision based forms of representation that are derived from Euclid, there are an array non-visual drawing practices based on Euclidian principles. These practices are not linked to a particular era of instrumental thinking, such as the variety of nineteenth century machines produced to develop complex curvature (Witt 2010). Instead they are computational drawing practices that are neither dependent on notions of vision nor instrumentation for their operation. Relegating Euclid to the periphery separates current computational practices from their historical lineage in architectural production. It mistakenly ties the development of line of inquiry to the development of instruments, when in fact the line of inquiry is so entrenched in architectural production that it has moved through an array of instrumental regimes and continued to hold sway over the discipline. Euclidian propositions have the potential to be integrated into current lines of inquiry positioning architectural history and the classical tradition, not only as predecessors to current modes of production but also active participants.

ARCHITECTURAL EDUCATION AND EUCLID

This research developed through the construction a series of drawings that attempt to both understand the operative logic of Euclid’s wedge as well as to speculate on alternative constructions. All of the drawings were constructed digitally on a two-dimensional plane using the wedge combined with orthographic projection. The drawings allowed the internal logics of the wedge to be understood and documented (Figure 7). Furthermore, while the wedge’s appearance in Guarini and Durers’ text suggest the potential of variable outputs, additional drawings were required to understand the array of potentials
and limitations inherent in the relationship between the wedge and curvature. Guarini states that this figure can be used to calculate the form of any sectional profile that has a center, but he does not demonstrate this (Guarini 1735). In order to test this statement hyperbolic, parabolic, and ellipsoid vaults were generated from single conic sections. The transverse sections generated through the use of the wedge where then orthographically tied to longitudinal sections generated from a second computational wedge. There correspondence and alignment was tested through the construction of full surface developments of the each of the vaults (Figure 8). Further drawing showed that the vaults could be stretched elongated and compressed by manipulating the internal geometry of the wedge (Figure 9). Symmetry and vertex location are not fixed parameters. Instead, they can also be manipulated to introduce variation into the curvature of the vault (Figure 10). The drawing project has elucidated some of the variable outputs that can be constructed with Euclid’s wedge as well as its limitations. It has also helped to structure methods for integrating classical geometry into the curriculum of a digital education in architecture.

The courses in which this research has been introduced range from the introduction to architectural drawing within our graduate program to upper-division vertical electives focused on studying the methodological relationships between contemporary practice and classical geometry. In all classes the work is structured to ground students understanding of both geometry and architectural drawing within a historical framework. Euclidian propositions center the operations in the studio while the rubrics of orthography and descriptive geometry are introduced as malleable operations. Students begin by intensively exploring two-dimensional Euclidian constructions and later use them as the basis for the generation of orthographic projections of three-dimensional volumes (Figures 11) and (Figure 12). Digital modeling is introduced as an extension of the two-dimensional operations and is projectively linked to its two dimensional point of origin (Figures 13) and (Figure 14). Euclidean procedures continue to govern the work, but are more intensively explored through the use of the model. Students are therefore introduced to architectural drawing not as linear history leading from Euclid to the digital present, but rather an overlapping and integrated framework in
which classical geometry and digital operations coexist. This project argues that the understanding of historical drawing practices are central to the continued development of architectural representation and instrumental practice. It positions current computational practices as an extension of the computational work extant within the history of architectural representation.

CONCLUSIONS AND FURTHER RESEARCH

The flat, two-dimensional field of representation continues to be an efficacious and generative site of action. Form can remain elusive within the two-dimensional surface and emerge only when it is assembled from the information provided by the drawing. The project focused largely on two-dimensional drawings as generative procedures for the production of curvilinear forms. It tested the relationships between contemporary notions of computation and historical methods of drawing. It relied exclusively on digital tools, two-dimensional representation and paper models. As the project continues to develop, the drawing techniques and methodologies need to incorporate more diverse methods of fabrication. The drawing techniques used within the work were originally developed in the sixteenth and seventeenth centuries in conjunction with fabrication techniques. These techniques operated at the scale of the model as well as building. The project will need to develop a similar feedback loop between fabrication and procedural drawing. It must begin to incorporate a more diverse array of fabrication strategies. This would further inform the potentials in the overlap of contemporary instrumentality and historical modes of representation.
PLines – The surface development of an irregular volume (Aldaud, Woodbury University, 2013)
14 PLines – Euclidian principles extended into the digital model (Alaud, Woodbury University, 2013)
REFERENCE


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