ABSTRACT

The topic of form-finding for minimal surfaces (surfaces with the minimum amount of surface area connecting between fixed boundary conditions and a mean curvature of zero) is not new, nor original. Historically this has been done through the discovery of intensive equations that could calculate mathematically minimal surfaces, or through dynamic physical models such as Frei Otto’s seminal soap film experiments, which have the property of adjusting to find a “minimal surface.” Engineers and mathematicians have had a long legacy with these highly efficient and amazingly elegant forms. More recently the emergence of computation within the world of architecture has enabled novel form-finding solutions through coding. However, our interest lies not in the mathematics and coding that computation has enabled, but rather computation as a catalyst for a material intuition. It is through the emergence of visual programming and physics-based engines in associative modeling have enabled designers, rather than mathematicians, engineers and programmers, to simulate physical and material forces in a visual and intuitive manner.

This paper focuses on the application of tool, exploration of technique and subsequent development of an informed design intuition for generating “near” minimal surface structures in the design, development and construction of the Pure Tension Pavilion. In short, we will examine how the Rhino 3D plug-in for Grasshopper along with accompanying physics engine Kangaroo was utilized to explore the development of an intuitive design process for form-finding, informing geometric articulation, and rationalizing the structure into buildable components.
INTRODUCTION
The topic of form-finding for minimal surfaces is not new, nor original. Minimal surfaces are surfaces with the minimum amount of surface area connecting between fixed or flexible boundary conditions, a mean curvature of zero and a constant in-plane stress at every point. All three conditions need to be satisfied for a stable minimal surface. Either one of these conditions on its own does not produce a minimal surface. Historically this has been done through the discovery of intensive equations that could calculate mathematically minimal surfaces, or through dynamic physical models such as Frei Otto’s seminal soap film experiments, which have the property of adjusting to find a “minimal surface.” Engineers and mathematicians have had a long legacy with these highly efficient and amazingly elegant forms. More recently the emergence of computation within the world of architecture has enabled design researchers such as Axel Killian and Sean Ahlquist to explore novel form-finding solutions through coding. However, our interest lies not in the mathematics and coding that computation has enabled, but rather computation as a catalyst for a material intuition. It is through the emergence of visual programming and physics based engines in associative modeling have enabled designers, rather than mathematicians, engineers and programmers, to simulate physical and material forces in a visual and intuitive manner. This opportunity allows designers to explore tools (Kangaroo for Grasshopper) that enable techniques (Dynamic Relaxation) to develop design intuitions (a real-time understanding of force and form) for the generation of minimal surface structures that are informed by visual understandings of physical forces.

The terms tool, technique and intuition in this paper are defined as:

- Tools enable designers to automate existing methods of design to make them faster, more powerful and more precise.
- Techniques emerge when designers develop specific design methodologies for using tools to enable results not possible without them.
- Intuition is developed through the understanding of tool and the mastery of technique to enable an informed sensibility that allows designers to work with agility, dexterity and precision.

This paper focuses on the application of tool, exploration of technique, and subsequent development of an informed design intuition for generating “near” minimal surface structures in the design, development and construction of the PURE TENSION pavilion. In short, we will examine how the Rhino 3D plug-in for Grasshopper along with accompanying physics engine Kangaroo was utilized to explore the development of an intuitive design process for form-finding, informing geometric articulation and rationalizing the structure into buildable components.

TOOL: KANGAROO FOR RHINO/GRASSHOPPER
With the introduction of the Grasshopper plug-in for Rhino, designers were given the opportunity to visually engage with hardwiring the associated parameters of geometry. The introduction of Kangaroo has further extended the software program to include a real-time dynamic physics engine, thus allowing designers to visually and intuitively interact with “virtual” physical forces applied to their designs. The combination of Grasshopper and Kangaroo enables real-time feedback between variable geometric conditions and dynamic force conditions, while also enabling the potential for real-time analysis of the geometries.

TECHNIQUE: DYNAMIC (MESH) RELAXATION
Dynamic relaxation is an actual algorithm originally developed by A. Day and pioneered by Barnes/Wakefield/Lewis. It approximates a mesh to a series of lumped nodes at the vertices, and assigns a weighted mass/stiffness to each node. Based on the out of balance force and Newton’s second law, the node accelerates proportional to the out of balance force and mass. Wither viscous or kinetic damping is then used to modify the displacements and the geometry is gradually updated until the displacements and out of balance force are small and the structure has reached a point of static equilibrium. The power with DR is its ability to handle massive displacements which makes it ideal for form finding when the initial geometry is far away from the form-found.

In short, pre-defined topologies of particle and spring networks are “relaxed” against fixed anchor points to discover the geometry where all forces are in equilibrium. Each time step of the process entails that each individual spring in the network simultaneously attempts to minimize its length towards a given “rest length.” With the boundary conditions fixed, some springs will elongate and others will shorten, pushing and pulling the particle network causing the entire topology to “relax” or minimize, eventually finding a state of equilibrium.

Though the results do not include true material or physical properties, in Kangaroo, low-poly mesh topologies such as a cube or even a plane can be converted into spring networks, with each mesh edge operating as a spring (with an assigned rest length) and each vertex operating as a particle (Figure 1). Desired boundary conditions are defined by selecting the corresponding vertices (such as the naked edges of a mesh). This allows designers to easily model and manipulate a relatively simple polygonal topology, while allowing Kangaroo to discover it’s form found equivalent.
Diagramming dynamic spring-particle systems (Huang 2013)

Catalogue of low-poly to near-minimal surface translations (Huang 2013)
INTUITION: A NOSE FOR NEARLY MINIMAL SURFACES

Intuition is inherently informed by our collateral experiences, memories and implicit thought. Design intuition draws on our entire experience, not only on what we consciously isolate as relevant information\(^9\). In our case, design intuition is defined as the intangible understanding of a complex system that enables designers to make instinctual design moves that are inherently informed by the previously described deeper understanding of the system with which they are operating. This requires a deep familiarity with both the tool and the technique in a wide variety of conditions to produce a wide variety of results, so as to allow the designer to develop an inherent ability to anticipate, coordinate and manipulate the relationships between tool and technique to produce desired outcomes. This informed sensibility, or “nose,” can be equated to the notion of craft and the informed sensibility a craftsman carries in working with matter.

In an attempt to develop a deeper understanding of dynamic (mesh) relaxation as a design technique, a case study was conducted in an attempt to reverse-engineer a catalogue of known mathematically generated minimal surface types such as the catenoid, costa, rheimann, etc. (Figure 2).

By intuitively approximating the perceived topologies of these surfaces through low-poly meshes, and relaxing them to discover their form-found equivalents, a few things became clear:

- The further the edge (spring) is from the anchor, the greater the relaxation.
- The denser the mesh, the smoother the curvature.
- By manipulating anchor points, one could radically alter the resulting form-found geometry.
- By decomposing the topologies into a series of polygonal parts, one could re-combine the parts to create new topologies.
- By associating anchor points with “live” Grasshopper geometry, one could parametrically redefine boundary/anchor conditions and interact with the real-time form-finding process to manipulate topologies.
- The process was capable of visually approximating all known minimal surfaces, but the surfaces produced were not truly minimal.

As a result, an intuitive feel for low-poly to “near” minimal surface translations defined a technique for exploring how to anticipate, model, and manipulate new minimal surface topologies which could be adapted for different design intentions.

If one were to form-find using traditional structural analysis principles of dynamic relaxation of a constant surface stress, one would either achieve a converged solution that is a minimal surface with a constant surface stress, or no solution would be discovered as the minimal surface exists too far away from the starting geometry. To achieve some of the forms in Kangaroo, such an extreme non-uniform pre-stress would be required it would be impossible to regenerate the form in practice, as once the installed the form would wrinkle or move to a new point of equilibrium far away from the architectural ideal.

The development of this intuition was absolutely critical, as with this intuition as an added understanding in the design process, designers are able to make pre-geometrical decisions\(^10\) that have informed geometrical outcomes.

APPLICATION: THE PURE TENSION PAVILION

The application of the previously described tool-, technique- and intuition-driven process for form-finding was extended through the design, development, and construction of the PURE TENSION pavilion. Armed with an understanding of how to “sculpt” near minimal surfaces, and given the lightweight nature of tensioned membrane structures, the PURE TENSION competition to design a portable pavilion for the new PURE TENSION plug-in electric car was seen as the perfect opportunity to apply the experiment (Figure 3). The challenge of the brief was clear—to design an iconic and portable structure that represented the three power modes of the car (diesel, hybrid and pure electric).

Additionally, the design team established a series of additional self-defined design goals as constraints for the development of the project:

- the pavilion should be rapidly deployable and enabled by a hand assembly process.
- the disassembled pavilion should be able to flat pack to fit inside the trunk of the car.
- the assembled pavilion should be embedded with flexible photovoltaics in order to charge the car.

Though computational design processes were used to inform every facet of this project, including solar incident analysis and the gradient mapping of photovoltaics, this paper will focus exclusively on the development of geometry through intuitive and informed processes of form-finding and rationalization.
GEOMETRIC DEVELOPMENT

Given the intuitive understanding for manipulating the form-finding process gained from the previous experiments, the design team established three criteria for defining, constraining and manipulating the topology of the mesh to be developed:

- A continuous boundary condition (a closed edge) was required to serve as a semi-rigid perimeter frame to tension against (anchor points)
- The combination of open (planar polygonal mesh) and closed (cylindrical polygonal mesh) topologies would enable a more articulate and expressive topology as illustrated in the minimal surface studies
- The act of constraining the anchor points of the form found topology to a parametrically defined boundary curve would allow a real-time design process

Based on these criteria and constrained by the challenges of the design brief the design team explored a range of design options through digitally form-found models, with corresponding analogue study models as a proof of concept. The stretched nylon and bent wire models were manually constructed from unrolled patterns of the 3D mesh topology, a preview of the process for fabricating the full scale result.

The final design proposal features a free-form continuous perimeter frame which touches the ground at three points with three corresponding apices defining three access points to enter the structure (Figure 4). The three-sided nature of the pavilion emphasizes the three modes of the cars operation (diesel, hybrid, “pure”). A continuous hyperbolic skin is stretched between the undulating frame, with two catenoid-like funnels stitched into the topology to add structural tension, create added internal volume and define distinctive surface conditions and double curvature.
ANALYZING INTUITION

As noted previously, the resulting mesh surfaces of the design experiment did not produce true minimal surfaces, but rather “nearly minimal” surfaces. For a successful installed fabric structure, the actual elastic properties of the fabric have to be modeled and the required numerical pre-stress regime has to be determined in the as-built structure. This means the fabric stretches by different amounts in two orthogonal directions (warp and fill) which are dependent on the as-built directions of the fibers in the material. Kangaroo does not account for elastic material properties and produces a pre-stress regime that is highly variable. Thus a properly engineered form-finding solution was required to verify the intuitively modeled geometries.

The engineered form-finding and finite element analysis was carried out through NDN Membrane (Figure 5). NDN is structural analysis software that is stiffness matrix–based with an advanced solver that is designed to handle large scale displacement theory for elastic static analysis of fabric structures. In addition it also includes a physics-based solver for form-finding where the form is found based on assigned values of surface pre-stress (and sometimes inflation pressure) without the elastic properties of the fabric. In form-finding, the solution is found without including the elastic properties of the fabric; it is defined purely by a constant force/stress in directions defined by the user and the boundary conditions and geometry specified by the user which may be both fully rigid or elastic/flexible. Once form-found, we take that equilibrium shape and elasticate it with real elastic properties and/or pre-stress values and then a) find the equilibrium elasticated shape for no external load—if form-found correctly there will be little movement/deviation as the structure geometry and pre-stress is already in equilibrium; and b) find the working stresses and resulting boundary forces on the elasticated model form applied loads such as wind.

Using the Kangaroo-generated geometry as a starting point, different starting pre-stress levels were assigned in the NDN model in warp and fill directions. As fabric is not an isotropic material, this means the elastic stiffness of the fabric is different in two orthogonal directions. The goal of this exercise was to arrive at an elasticated NDN model where:

- the pre-stress regime is possible to replicate in practice, for instance the means of tensioning the fabric is important to ensure one can achieve the variation and magnitude of pre-stress necessary to achieve the theoretically form-found shape
- the fabric is not significantly slack in large areas, which would mean the fabric would wrinkle

4 Geometric development of the mesh topology (Huang 2014)

5 Structural finite element analysis in NDN Membrane (Lewis 2014)

6 Internal Pre-stress Regime Analysis in NDN Membrane (Lewis 2014)
stress should be above zero to ensure non slack areas which could lead to “wrinkling of the fabric,” an undesirable aesthetic, and an upper extreme where the increase in-plane stress under an imposed load should be less than an allowable limit placed on the minimum breaking strength of the material. Within those boundary limits, a suitable internal pre-stress regime had to be found.

External Boundary Stiffness: The internal load path is resolved at the boundary by aluminum arch members. These members resist the loading through a combination of flexural and compression stiffness. The members must have enough material to ensure that the stiffness is suitably high to ensure the fabric does not go slack, and the required internal pre-stress regime load path can be met in practice (Figure 6).

The form-found shape developed requires a non-uniform internal pre-stress regime. It is not a minimal surface where an internal pre-stress regime is constant in all directions. To achieve the desired aesthetic of the relaxed form-shape in the installed state required several engineering performance criteria to be met.

Internal Pre-stress Regime: As the structure could only resolve imposed forces internally exclusively through in-plane pre-stressing tensile forces, this required that two boundary limits had to be met: the lower extreme where under no external load, the in-plane stress should be above zero to ensure non slack areas which could lead to “wrinkling of the fabric,” an undesirable aesthetic, and an upper extreme where the increase in-plane stress under an imposed load should be less than an allowable limit placed on the minimum breaking strength of the material. Within those boundary limits, a suitable internal pre-stress regime had to be found.

• the pre-stress level required in the as-built condition is in equilibrium so when actually built the fabric does not distort the boundary frame and end up in an undesirable as-built equilibrium position/geometry which is not aesthetically nor structurally desired.
An atypical form-finding approach was adopted due to the extreme non-uniform internal stress regime of the fabric required to achieve the desired aesthetic. After rationalizing the geometry to a suitable layout of seam lines and fabric panels, the surface was immediately elasticated and allowed to relax (Figure 5). It was apparent that there are large areas where the fabric is essentially slack. The typical form-finding approach is to form-find non-elasticly to equilibrium, that is, elastic stiffness of the material is not accounted for until a form-found equilibrium shape is found purely geometrically for user defined pre-stress values in different directions that do not change with the geometry during form-finding.

From the base line, a feedback loop was established whereby the pre-stress regime found at the end of the form-found elasticated approach was used as the starting pre-stress values for subsequent form-found runs until the final elasticated pre-stress regime was within the upper and lower boundary limits set by the user, and the form-found geometry had not deviated excessively from the desired theoretical form-found shape.

In short, the delta between the intuitive Kangaroo model and the engineered NDN model was 2% at its greatest deviation. The delta relates to how much the geometry of the as-built elasticated NDN analysis model deviated from the Kangaroo model as a metric comparison between selected central node points of the two models.

RATIONALIZATION & FABRICATION

As the project moved from digital to physical, the translation of virtual free-form geometry to rational and buildable geometry was critical. This development was directed in two parallel explorations: the rationalization and fabrication of the pre-stressed boundary frame and the panelization and tailoring of the tensioned membrane. Major hurdles to be addressed were time and budget.

To reduce costs, significant savings were made in the production of the perimeter frame through two considerations. First was the shift away from the original composite tube to a CNC bent aluminum pipe system with a swaged slip-fit assembly. The second was the rationalization of the freeform geometry from hundreds of arcs into five rational arcs (Figure 7), thereby reducing the cost and time of the CNC pipe bending (Figure 7). A parametric model was developed in Grasshopper which allowed the design team to define the perimeter geometry by an upper and lower diameter (spread), vertical height and 2 driving arc radii. This model split the frame into precisely 24 pieces with the length of tube constrained by the dimension of the trunk of the car (the intention was to transport the pavilion by the car itself), calculated the resulting tangent arcs, arc lengths and rotational angles, all of which were the parameters for driving the fabrication and assembly of the frame.

With the perimeter frame being rationalized this created a slight, albeit significant, delta between the perimeter of the original freeform geometry and the newly optimized version. To accommodate for this difference, the anchor points of the Kangaroo mesh were constrained to the rationalized arc model and re-relaxed to find a new equilibrium state that was defined by the new perimeter. Any and all changes to the parametrically variable perimeter frame model were followed by real-time updates from the Kangaroo model adjusting to its new constraints.

Whereas the geometry of the perimeter frame was driven by a rationalization process, the geometry and fabrication of the tensioned membrane was driven by the direction of the seams, a tailoring process. The warp direction of the fabric lies in the direction of the seams—thus the patterning of the mesh, and ultimately the fabric, influences the geometry and the internal pre-stress regime.

By utilizing Weaverbird, a topological mesh editor plug-in to Grasshopper, the mesh topology was able to be sub-divided in a number of configurations and subdivision iterations (Figure 8) in order to determine the appropriate density and shape of mesh faces that would allow for the most fluent and efficient patterning of the tensioned skin.
Warp stresses lie parallel to the warp direction, fill stresses are orthogonal. This demanded that seam lines had to be sufficiently spaced and aligned so that any excessive curvature could be accommodated by the single curvature of the unfolded fabric cutting pattern. In turn, for structural, aesthetic and fabrication requirements, it was decided that a quad mesh was the most effective and efficient manner to panelize and pattern the fabric.

While the digital design of the skin was able to produce a fluent form through intuitive form-finding, the precision of the physical form was enabled by a high level of handcraft applied to a highly empirical process of physical form-finding and on-site as-built adjustments (Figure 9). This is where the intuition and craft of the fabricators and their team of seamstresses took over. Digital geometries were physically recreated to identify seam locations, connection details, and the integration of the photovoltaic panels. The 3D mesh topologies were exploded into non-uniform quadrilaterals to allow for the digital mesh to become successfully unrolled into hundreds of “quilting” tiles that were then cut from the CNC fabric cutter. Tolerances were built into each tile so that sewing adjustments could be made on the 1:1 scale mock-up.

Following the erection of the perimeter frame, the tiles were sewn together and stretched onto the frame in two mirrored sections with zippered connections and spandex sleeves to wrap the frame. Areas of slack were identified and tailored to eliminate slack through a highly intuitive process of visual adjustment and “eyeballing.” The photovoltaic panels were attached to the skin via a substrate of a scaled-down series of “quilting” tiles cut from black vinyl. The photovoltaic panels were then wired together in parallel and its appendages fished through and concealed by fabric “conduits” along the perimeters of the “quilting” tiles.
CONCLUSION

It is clear that the design and realization of the Pure Tension Pavilion (Figure 10) was reliant on intuition. It was conceived through the development of a physically informed yet highly digitally driven design intuition. It was constructed through a process of a digitally informed yet highly physical material intuition.

The evolution of computational design processes have brought forward a new graphic and interactive way of understanding geometric logic and physical simulation, thus allowing designers to engage form-finding and geometric rationalization as intuitive methods of designing. It is through the understanding of these processes that an informed understanding of their outcomes can be developed and a new and novel design intuition can be developed. As Albert Einstein once alluded:

“This is no logical way to discovery... There is only the way of intuition, which is helped by a feeling for the order lying behind the appearance.”

NOTES


IMAGE CREDITS

Figures 1, 2. Huang, Alvin (2013).


Figure 10. Mosca Partners (2014). Pure Tension Pavilion.

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