ABSTRACT

Historically, stereotomy consisted of the geometric methods developed to design and construct with large stone volumes. While massive masonry construction is used rarely now, architects still routinely develop complex volumetric assemblies. Recognizing this, our research pursues a new approach for contemporary stereotomy: construction with solid or hollow volumes made from any material. This idea was developed through two interconnecting research phases that employ computational design methods, digital fabrication and lightweight composite materials. The first shows how architects can use FEM buckling simulations to create complexly-curved, displaced surfaces. In this investigation, we study the multiple sinusoidal buckling modes of a primitive plane and create a catalog of intricate volumetric tessellations. The second phase of research is design-based. During this phase, the algorithmically generated catalog is used to develop techniques for designing hollow volumes that stack into intricate assemblies. To prove the concept, several models are constructed by 3D printing plastic units, and a large free-standing wall is built from fiber-reinforced polymer (FRP) composites laid-up in CNC milled molds. These prototypes show the architectural possibilities of laminar stereotomic elements designed with FEM surface buckling.
INTRODUCTION

Buckling is an instability failure model—one of the fundamental mechanical phenomena governing structural design. Buckling deforms configurations of material through displacement. The phenomenon is capable of transforming axial tectonic structures and curved shells into exotically sinusoidal new configurations. Buckling is disastrous for structural integrity, but it produces wavelike displacements that are formally very similar to doubly-curved surfaces designed by architects using NURBS or subdivision surface modeling. Our research began from this observation and a curiosity to learn whether the intentional cultivation of buckled surfaces using computational FEM analysis tools might be valuable for the design of architectural constructions. Using Karamba 3D, we studied surface buckling patterning and developed a catalog of intricate tessellations.

To explore a relevant architectural issue, we subsequently tested whether these tessellations could develop contemporary stereotomic and volumetric construction techniques. Stereotomy—the descriptive geometric methods and construction techniques once essential to successful construction in cut stone—has been marginalized to the status of historical footnote by the dominance of tectonic frame construction. While substantial load-bearing stone construction is unlikely to be revived, contemporary approaches to volumetric construction are on the rise as architects experiment with fiber-reinforced polymer (FRP) composites and other plastics.

The Blobwall pavilion by architect Greg Lynn, made from rotationally molded plastic, and the Surface-to-Volume installation by architect Tom Wiscombe, made from composites, are two recent examples of this trend. Both point to a renewed interest in volumetric construction but now with surfaces that flaunt their thinness. Our buckling design research explored this new development—stereotomic construction without stone solids—through the production of several prototypes.

BUCKLING SURFACES FOR DESIGN WITH FEM ANALYSIS TOOLS

Buckling is not often pursued as a source of design inspiration, and architects rarely encounter FEM tools. To situate these topics within our research investigations, we first briefly describe the phenomenon of buckling, precedents that use buckling for design, and explain how our buckling research was implemented using the FEM analysis tool Karamba 3D.

BUCKLING

Buckling is best known in architecture for compromising long, slender columns; however, the phenomenon can also occur in truss and shell structures. It is also found abundantly in nature—in the rippling of the edges of leaves for example—in countless materials, and in an extreme range of scales (including the nanoscale). Buckling is widespread, since relatively low forces are required to instantiate it if favorable conditions exist. Moreover, buckling onset primarily correlates to just two variables—stiffness and geometric proportion. This generic character of buckling, when coupled with its signature sinusoidal patterns, makes the phenomenon attractive to research with computation for design purposes.

Buckling is structural failure resulting from mechanical instability. Buckling in a long slender column is the classic case of this failure mode. Even a relatively low application of load in compression will force material to displace laterally in a column whose length is too long for its cross-section, leading to catastrophic failure. This is true even if the column is made from a high-strength material like steel. In contrast, a sufficiently short steel column with the same cross-section will not buckle but instead crush at a much higher load.

Buckling has long been studied as an independent structural phenomenon. The inventive Swiss mathematician Leonhard Euler first captured it with an equation in 1759, determining that the buckling load for a column is dependent on the inverse square of its length. We still use Euler’s observations and equations. For example, the term Euler buckling load identifies the amount of force required to buckle a structure. Bracing a structure increases its resistance to buckling and also increases the complexity of the sinusoidal shape it will assume if buckling occurs. When buckling is considered as a device for architectural invention, this increase in sinusoidal complexity is significant.

AN OVERVIEW OF BUCKLING RESEARCH FOR DESIGN

Buckling is typically avoided in structural design. However, materials science engineers are now considering how instability failure may be used to develop deployable and collapsible structures at extremely small scales. This area is known as “extreme mechanics.” Representative research is pursued by Katia Bertoldi at the Harvard School of Engineering and Applied Sciences. Bertoldi’s research group cultivates a deep understanding of buckling for potential design applications. By strategically embedding weak spots in materials, they are designing...
polyhedra—Buckliballs—that reliably collapse and expand in response to external stimuli. Bertoldi predicts that the resulting material applications could range from the pharmaceuticals to buildings (Nature 2012).

An older study of buckling by the French structural engineer, theorist and educator Robert le Ricolais (1894–1977) is the direct inspiration for our research. Teaching in the 1960s and 1970s, le Ricolais generated an array of diverse structural prototype models with his graduate students. These models—preserved for study at the Architectural Archives of the University of Pennsylvania—include a tube structure buckled to reveal the intricate pattern of sinusoidal deformation found in higher modes of buckling. Higher mode in this context simply means that braced nodes are required to produce an S-curve shape in the members. (A column that is not braced will just “bow” out of plane when buckled and not produce a complete sinusoidal curve.) Photos of the model before and after buckling reveal that a strategic application of bracing was used to cultivate the desired displacements (Figure 1). Le Ricolais considered this model to show that the “order of destruction follows the order of construction” (le Ricolais 1997). The resulting buckled tube is no longer a load-bearing structure. But its sophisticated, voluminous figure suggests that strategic buckling can be a tool for other design purposes.

Our research focuses on surfaces—not the tectonic assemblies studied by le Ricolais—but the approach is deeply influenced by his desire to reveal hidden structural principles in architectural form. Working before the era of widespread computing, Le Ricolais was limited to the use of physical models for testing buckling behavior. Producing more complex deformation patterns would have required extensive labor. Our research continues the development of latent architectural design material from buckling by using computational tools unavailable to le Ricolais.

**BUCKLING SIMULATION FOR DESIGN WITH FEM ANALYSIS TOOLS**

To study buckling computationally, we used Karamba 3D, a tool developed by Clemens Preisinger for the German-Austrian structural engineering firm Bollinger Grohmann Engineers. Karamba 3D is an FEM structural analysis plug-in for Rhinoceros 3D and Grasshopper. Since Rhino is commonly used by architects, Karamba3D greatly improves the opportunity to study structural phenomena as source material for design. Karamba can analyze wireframe models modeled directly in Rhino or parametrically developed in Grasshopper. Once processed, the multiple deformation states of the original wireframe model can be output as editable geometry for further design work and fabrication development.

Karamba’s buckling algorithm does not calculate the exact Euler buckling load for an analyzed wireframe; instead, Euler shapes—referred to as Eigen-modes—are simply calculated in the order that they would be instantiated in a given structural wireframe. According to the Karamba reference manual, “the Eigen-values represent a measure for the resistance of a structure against being deformed to the corresponding Eigen-form” (Preisinger 2013). Thus the Eigen-mode number assigned to a specific Euler buckling deformation state produced in Karamba is simply an indication of how likely a structure would be to conform to a particular deformation pattern were buckling to occur.

For an analyzed wireframe, the buckling shape produced by the Eigen-value “1” in Karamba will always correspond to the actual deformation pattern that would occur in a real structure. This pattern is a result of the geometric configuration of the wireframe and any applied loads. An Eigen-mode of “2” corresponds to the second most likely Euler buckling shape that the wireframe could assume. Multiple buckling “frequencies” can be calculated by Karamba for a structure, but only the first mode follows the natural path of least resistance. Buckling shapes have distinctive, wave-like shapes because the phenomenon is closely related to vibration but without inertial forces. As an instability failure, buckling operates like a wave of displacement that overwhelms an unstable structure.

The number of buckling shapes calculable for a model analyzed in Karamba corresponds to the number of nodes in the wireframe multiplied by the degrees of freedom—in translation and rotation—allowed for each node. A simple wireframe with nine nodes, each with six degrees of freedom, results in fifty-four possible buckling shapes. Only the first calculated mode could occur in an actual structure, and the remaining fifty-three shapes are virtual buckling shapes. These excess shapes create a library of complex geometric forms, many of which are a provocative departure points for design.

**CREATING TESSELLATIONS FROM BUCKLED SURFACES**

Since our buckling research focused on surfaces, rather than the tectonic wireframes that can be buckled by Karamba, we had to first determine methods for reliably producing displaced surfaces. We then analyzed these buckled surfaces to understand their properties and combined them to form intricate tessellations.

**METHODOLOGY AND INITIAL OBSERVATIONS**

Surface buckling can be simulated and visualized using either polyline wireframes or meshes. NURBS surfaces, however, are more desirable for fabrication, but calculus-based interpolated to-
FABRICATION AGENCY

I. BUCKLING SHAPES

In-plane buckling shapes for a wireframe model produced by FEM analysis in Karamba 3D. These deformations do not push out of the plane of the original wireframe topology. To solve this problem, we used Karamba to buckle wireframe models approximating surfaces. By creating two directional sets of curves from the buckled line segments of these wireframe models, we could reliably create NURBS surfaces. The “Network Surface” command in Rhino produced the best results, since lofting either just the u- or v-direction curves derived from the wireframe lost the finer surface displacements produced by Karamba’s algorithm.

Since hundreds—even thousands—of virtual buckling modes can be produced from relatively simple models, we confined our research to simple planar, single-layer gridded wireframes with square cells. Each node in the studied wireframes was allowed the entire range of movement, six degrees of freedom.

II. INITIAL OBSERVATIONS

The sinusoidal patterns produced by Karamba’s buckling algorithm increase in complexity as Eigen-modes with higher numbers are calculated. A similar progression is seen in a simple buckled column that produces an S-curve shape if the center point is braced and more complex sinusoidal curves if additional points are braced. Karamba buckles digital wireframes with a similar progression. Based on the nodes and members in the model, each calculated buckling mode offers a displacement pattern that is slightly more complex than the preceding one. The variety and complexity of the sinusoidal configurations generated from a simple wireframe immediately suggested that computational buckling could be useful for design (Figure 2).

The pattern of a buckling mode is established by the directions and speeds with which individual nodes are displaced relative to one another. Once the pattern of movement has been calculated, the amount of overall displacement can be controlled with a slider on the algorithm’s Grasshopper component. In this way, the magnitude of the buckling effect acting on the base geometry can be scaled.

We noticed that not all displacements move out of the plane of the original structural wireframe. Many buckling modes produce only in-plane displacement; this phenomenon occurs most frequently in the higher Eigen-values computed for a wireframe. While not a focus of our research, in-plane displacement patterns offers an area for future design research (Figure 3).

III. COMPLEX SYMMETRIES

When the nodes of a wireframe are not selectively restrained in their degrees of freedom, buckling using Karamba is fundamentally symmetrical. Similar to vibration, the “frequencies” of buckling modes are wave-like curves with structured patterns of crests and troughs. In the virtual buckling modes of a single wireframe, a variety of complex patterns of symmetry and symmetry types can occur. We found several symmetry types: bilateral across a single axis, rotational around a central point, diagonal from corner to corner, and multi-axis symmetries.

The symmetry type present in a particular buckling mode was revealed most clearly by overlaying the deformed surface with a flat plane (Figure 4). When compared against this reference surface, another unique symmetry was revealed. Since buckled surfaces are volumetric, the surface deformations create pockets of cupped space. Often, if a buckled surface creates a
bulging crest on one side of an axis of symmetry, the same dis-
placement will occur as a trough on the opposite side (Figure 4,
surface 025). This amounts to a 3D rotational symmetry that is a
unique feature of buckled surfaces.

TESSELLATIONS

It soon became evident that the many symmetry types exhibited by
buckled surfaces constitute a form of tiling. Since we only analyzed
single-layered, flat wireframes, the tiles generated covered a two-di-
mensional plane. The translations, rotations, scalings and mirrorings
that allow a set of tiles to cover a 2D plane are of great theoretical
interest to mathematicians. Such tiling systems are also of great in-
terest to architects who must routinely subdivide large surfaces into
many smaller elements for design and fabrication. Our research
revealed that buckling offers a new method for producing tiles with
depth in relief.

The displaced undulations of a buckled surface do not immediately
form tiles, but we determined that tiles could be made by intersect-
ing the displaced surface with a plane. Splitting a buckled surface
with a plane made two sets of surfaces that interlocked along their
edges. To better reveal the tiling pattern, the surfaces below the
cutting plane were assigned a white color, and surfaces above the
plane were colored black (Figure 5). The graphic tessellations pro-
duced using this technique could be used as a subdivision strategy
for panelizing architectural surfaces with volumetric tiles.

We extended this technique by overlaying one buckled surface
mode with another buckled surface mode instead of a flat plane.
In these cases, one surface was toned black and the other white
to make the tessellation patterns legible. A sampling of a few of
the tessellations produced using this technique reveal surprising
variety. Some combinations produce large, meandering interlocking
surfaces while others form small, discrete tiles. Some combinations
produce a sense of directionality across the surface while others
produce repetitive figures (Figure 6). Since a single base wireframe
can form a library of hundreds or more distinct displacements, the
possible combinations grow exponentially to form an extensive cat-
alog of potential tessellations for design.

STEREOTOMIC DEVELOPMENT OF
TESSELLATIONS AND FABRICATION OF
PROTOTYPES

After identifying that overlaid buckled surfaces can form tessella-
tions, we identified several surface pairs with interlocking patterning
that seemed suitable as templates for stereotomic assemblies, fur-
ther developed into self-supporting architectural wall prototypes.
STEREOTOMIC DESIGN DEVELOPMENT AND INITIAL PROTOTYPES

The selected overlaid surfaces required further editing before becoming viable as self-supporting assemblies. First, the surfaces were intersected and then split with the resulting curves. This produced four sets of surfaces. Two of the interlocking sets were affiliated with the front side of a potential wall assembly and two with the back. Though the edges connecting the sets were complexly curved in space, they did not produce structural stability since they had “zero” thickness. The geometry at this stage was not self-supporting.

To overcome this problem, two approaches were used. For the first prototype, the two sets of surfaces associated with the “back” of the wall were deleted. The edges of the remaining surface tiles were then extruded perpendicularly from the overall plane of the wall. This procedure produced structural flanges which were joined to the original surfaces. To give the individual pieces thickness for fabrication, the surfaces were capped to temporarily form closed solids. The “Shell” command in Rhino was then used to create thin-shell volumes—open to the rear of the design—with a wall thickness of 1/16”. The final pieces stacked and nested into a free-standing stereotomic assembly (Figure 7).

The design was subsequently 3D printed in ABS plastic. The resulting prototype is self-supporting and consists of interlocking, friction-fit stacked volumes (Figure 8). The pieces were later faired with plastic modeling putty, sanded, and painted to match the rendering.

A second approach was used to design two additional prototypes. For these designs, instead of discarding the rear surface sets, they were moved, perpendicularly, away from the front sets to make a gap. The gap was then bridged by lofting the edges of corresponding surface sets on either side of the wall. This technique produces both structural flanges and closed, volumetric solids (Figure 9). These geometric solids can be 3D printed without further editing. However, for the final 3D prints, some of the surfaces were shelled and left open on the back or side to showcase interiors of the hollow volumes.

LARGE-SCALE, FIBER-REINFORCED POLYMER (FRP) PROTOTYPE

Our stereotomic assembly research culminated in the construction of a large-scale composite prototype measuring 6’ tall by 10’-6” long. This self-supporting wall consists of thirty-two interlocking elements made from E-glass and a high-strength epoxy resin matrix. The composite surfaces were laid-up in open-face, CNC-
milled foam molds that were smoothed with standard wood filler. The molds were coated with a Styrofoam protector and release agents before the composite lay-up process. To achieve a black-and-white color scheme similar to the painted 3D prints, pigmented polyester gel-coats were sprayed into the molds as a first layer (Figure 10).

After the gel-coat layer was dry, it was roughed up with eighty grit sand paper. Several structural layers of reinforcement and high-strength epoxy resin were then laid-up to an average final thickness of 3/32" (Figure 11). When the pieces were de-molded, the color gel-coat layers pulled away from the faces of molds as a finished surface. White components were left open on the rear side of the final wall, so each piece only required one open-face mold. The black components are closed volumes. Each element required two open-face molds to produce halves that were then fused into a single hollow shell. Both the black and white pieces appear massive on the front side of the wall (Figure 12). But on the rear of the wall, the white surfaces reveal the thinness of the elements and the overall lightness of the construction (Figure 13).

The interplay of open and closed volumes in the FRP wall highlights the lightweight intricacy that is possible with a thin-walled approach to stereotomic construction. The wall also shows that the voluptuously displaced surfaces produced from FEM buckling can be productively used to develop a free-standing volumetric design.

CONCLUSIONS

Historically, stereotomic techniques allowed architects to design with and cut the volumetric stone solids required to efficiently construct walls, arches, and vaults. Our research aimed to update and extend the repertoire of stereotomic techniques and concepts available to architects. FEM surface buckling was investigated as a potential source of material for such an extension.
Our investigations showed that FEM surface buckling is suitable for this ambition for two main reasons. First, the surface displacements of out-of-plane buckled surfaces are inherently volumetric. The undulating, wave-like deformations of buckling create pockets of captured space, or poché, a classic feature of stereotomic architecture. Second, overlaid buckled surfaces form interlocking tessellations. These interlocking patterns can be developed into volumetric pieces that nest and stack into intricate stereotomic assemblies.

Interesting questions are raised by this research. The wall designs we developed all use extruded flanges to make stacked or interlocked pieces. But historically, stereotomy used projection systems to produce complexly skewed volumes for arches and vaults. Could FEM buckling simulation and its tessellations also be used to design intricate arches and vaults? What would happen if curved wireframes were substituted for flat wireframes? Another question concerns the graphic patterning produced with overlaid buckling modes. Could these have applications that are independent from the stereotomic systems developed in this research? For example, might they be applied to reorganize the visual perception of volumes in an architectural composition?

Such questions may organize further investigations. For the moment, the designs produced through our work with FEM buckled surfaces provide an initial model for contemporizing stereotomy with computational design techniques, digital fabrication methods and lightweight composite construction.
REFERENCES

IMAGE CREDITS
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