ABSTRACT
This paper outlines a framework and conceptualization of combinatorial design. Combinatorial design is a term coined to describe non-parametric design strategies that focus on the permutation, combination and patterning of discrete units. These design strategies differ substantially from parametric design strategies as they do not operate under continuous numerical evaluations, intervals or ratios but rather finite discrete sets. The conceptualization of this term and the differences with other design strategies are portrayed by the work done in the last 3 years of research at University of Southern California under the Polyomino agenda. The work, conducted together with students, has studied the use of discrete sets and combinatorial strategies within virtual reality environments to allow for an enhanced decision making process, one in which human intuition is coupled to algorithmic intelligence. The work of the research unit has been sponsored and tested by the company Stratays for ongoing research on crowd-sourced design.
INTRODUCTION—OUTSIDE THE PARAMETRIC UMBRELLA

To start, it is important that we understand that the use of the terms parametric and combinatorial that will be used in this paper will come from an architectural and design background, as the association of these terms in mathematics and statistics might have a different connotation. There are certainly lessons and a direct relation between the term ‘combinatorial’ as used in this paper and the field of combinatorics and permutations in mathematics. As it will be explained further, the term combinatorial design encapsulates notions of both permutation and combinatorics and uses the studies of discrete finite sets of units and their possible arrangements by an algorithmic or intuitive process.

The term parametric, as understood for this paper, will be demarcated by the series of geometrical operations that concern the continuous variation and differentiation of form by the manipulation of ratios, intervals and intensities as a form to increase or reduce detail. It is in this description in which the term parametric, with its association of infinite continuous numbers, can be opposed to the term combinatorial, associated with finite discrete numbers.

Parametric design has received a great deal of attention since Patrik Schumacher elevated the design techniques to a form of architectural style (Schumacher, 2011). From a formal perspective, it is only natural that a particular set of mathematics would provide a path of least resistance for certain forms, and with a massive adoption for tools that work in such a paradigm, the yield of the claimed style would grow.

The tools of parametric design have flourished within a NURBs geometry environment, where as opposed to vertices, faces and edges from a mesh environment, the notions of curvature and parameters describe the building blocks for geometric constructions. A point in a curve represents an infinitesimal location along a domain which can be easily mapped to a parametric slider. This simple operation can be understood as a ratio or intensity described by an infinitesimal number. These notions do not only apply to NURBs geometry, as meshes as well rely on the positioning of points in a continuous space. The location of vertices can be described by a parametric function, and the resolution of a mesh geometry can also be described by a parameter.

Architecture and design have developed a large repertoire of tools and operations that operate in such a framework, creating and detailing geometric forms out of parametric functions. But the design discipline has quickly denominated under the same umbrella, doing a disservice to the further research and study of
taxonomies of design strategies (such as combinatorics) that do not follow the same paradigm.

**Combinatorial design**

A different set of techniques will be described here as combinatorial design: the use of a finite set of units or operations that can be arranged in different configurations. It’s important to be clear that combinatorial design is not just the study of possible permutations of parameters, what Kostas Terzidis calls ‘Permutation design’ (Terzidis, 2014), where any given variable of a design problem establishes a degree of freedom that can be catalogued and cross referenced to other variables, yielding the solution space of a given system. In fact, Terzidis rejects how intuition and experience can play a role in the design process, favoring a framework of optimization performed by a deep search of algorithms over the permutation space. He explains:

“Traditionally, such arrangements are done by human designers who base their decision making either on intuition (from the point of view of the designer) or on random sampling until a valid solution is found. However, in both cases the solution found may be an acceptable one but cannot be labeled as “the best possible solution” due to the subjective or arbitrary nature of the selection process. In contrast, an exhaustive list of permutation-based arrangements will eventually reveal the “best solution” since it will exclude all other possible solutions.” (Terzidis, 2014)

Combinatorial design is a design strategy that starts from the definition and individuation of parts, describing an open ended series of relations with one another. These parts will be coupled and aggregated to generate larger assemblies, describing meaning, performance and function at different scales of configuration. The system will always remain open ended and malleable, allowing for the replacement of parts within it.

The open-endedness of the system implies, in contrast to Terzidis ideas, that there is no possible optimization, as the solution space of permutations grows with each unit added at an exponential rate, becoming computationally impossible to search for an optimum.

The malleability of the system implies a temporal relevance, a contingent emergence of a pattern out of contextual conditions. No pattern might be the optimal one for a long period of time, but rather an ad-hoc solution harvested by designers or as we will see, a crowd.

**GRANULAR ASSEMBLIES**

What is described here as combinatorial design could be associated with what Ian Bogost calls ‘Unit Operations’. Bogost explains:

“I will suggest that any medium – poetic, literary, cinematic, computational – can be read as a configurative system, an arrangement of discrete, interlocking units of expressive meaning. I call these general instances of procedural expression unit operations.” (Bogost, 2006)
For Bogost, meaning emerges from the coupling of units without belonging to a larger holistic system. He is able to describe units in their autonomy to a larger structure rather than parts of a whole. His distinction between wholes and multitudes allows for the existence of units without any overarching structure. He explains:

“A world of unit operations hardly means the end of systems. Systems seem to play an even more crucial role now more than ever, but they are a new kind of system: the spontaneous and complex result of multitudes rather than singular and absolute holisms” (Bogost, 2006)

In the studies of combinatorial strategies, the research team has addressed these multitudes in design as granular assemblies. While the granularity could be understood as a tectonic condition, it encapsulates the autonomy and spontaneous interactions of units. The idea of assemblies addresses the properties of re-configurability of parts and potentially their reversibility. It describes a temporal condition, one of a contingent, almost convenient, performative configuration.

It is Bogost who also explains the struggle that units need to maintain their individuation:

“Unit-operational systems are only systems in the sense they describe collections of units, structured in relation to one another. However, as Heidegger’s suggestion advises, such operational structures must struggle to maintain their openness, to avoid collapsing into totalizing systems” (Bogost, 2006)

This is a further distinction between the formal expression of combinatorial systems and parametric ones. While the former uses the language of patterns and arrangements, the latter uses the language of gradients and flows that transition into one another. Parametric systems are designed via articulation of flows while combinatorial ones are designed by the patterning of units.

Three years of combinatorial research, Polyomino studio.

The Polyomino agenda started in 2013 at University of Southern California, working with students from the Master in Advanced Architectural Studies. The research brief for the studio is that of defining and designing a combinatorial building system, one that could produce a series of architectural conditions. The projects would be developed both physically and digitally, allowing material experiments to inform a simulation within a game engine.

By studying games and interactive experiences, the project was able to develop a digital platform with all the material constraints to explore the combinatorial possibilities. Often, many of the tools of aligning, orienting and snapping complex geometric shapes to one another require a series of commands in any software platform. By pre-writing code that could execute quickly and effectively these alignment calculations, the team is able to create user experiences for intuitive combinatorial design.

The games, developed as simulations, would instantly offer alternatives of growth for the designer and provide the data to make decisions every step of the way.
Evolution of Polyomino

The Polyomino agenda was developed over 3 years. In each iteration of the studio, the team would analyze the developments of the previous years and innovate and experiment testing different assumptions. The first iteration of the studio focused on the symmetry breaking operations of a truncated octahedron, which became the boundary geometry for more complex forms which would establish structural connections with a specific number of neighbors. Each unit was designed in great detail, attempting to communicate and guide the assembly and valid topologies through the geometry and curvature of each unit. This is the model that was carried forward for the further development with the company Stratasys in the Polyomino series released in 2016.

The second iteration of the studio allowed for a broader study of perfect packing, in the interest of exploring granular assemblies that could host new topological networks. One of the projects decided to abandon the perfect packing framework altogether to allow for the unit assemblies to possess a larger functional difference. This strategy proved to be highly productive as the combinatorial space grew, offering a larger possibility of neighbors and rotations. The project required a deep study of permutations defining the building blocks of larger tectonic assemblies. The project reduced the complexity and information of the unit itself, but increased the differentiation of assemblies. The combinatorial strategy was exacerbated and became a central advancement for the 3rd iteration of the studio.

The third iteration of the studio was an attempt to interrogate in a more radical form the framework of combinatorial design. Three projects were developed, each with an attempt to question what had become a familiar setup.

The first project was developed as a study of ‘systems and exceptions’, where a perfect packing arrangement allowed a small polyhedron to propagate and define large granular tectonic assemblies. This operation was repeated in different areas of a test project but without an underlying structure that would connect these different ‘islands’ together. The idea here was to allow for a small percentage of bespoke units to work as the connecting tissue of the perfectly packed parts. The implications of these strategy is far reaching, as it suggests a possibility to couple the power of serially repeated units working in a combinatorial framework with unique units machined and detailed for tolerance and to operate as a system of exceptions for a formal rigid system.

The second project from this series attempted to introduce analog connections to operate as an alternative to discrete rotations. This operation certainly starts blurring the boundary...
between combinatorial and parametric design, but was a proof of concept to allow for gradient intensity arrangements to coexist within a combinatorial framework. This was achieved by designing a sliding joint that would allow two units to describe a continuous set of positions to one another. This breaks the notion of a finite set, as the analog connection offers an infinitesimal set of positions between two units. Nevertheless, the combinatorial strategy prevailed within the design assembly. The sliding connection became an opportunity for differentiation without overtaking the main framework of the project agenda. The units, as expressed in the figure below, offer two discrete face to face connections, each with 4 possible rotations, and a 2 sliding faces, to allow for the bundling of units parallel to each other.

The third project of this series stayed within a purely combinatorial space growing upon the lessons of previous years. The project defined a smart simple unit that could connect to itself in a large arrange of permutations. In the traditional Polyomino methodology, a large cataloging of permutations defined the ‘vocabulary’ of arrangements that created meaningful patterns. These arrangements were analyzed for their recursive capabilities, structural capacity, porosity and even formal delineation - all design criteria that could be deployed as a granular field condition for a given project. This project proved that a simple combinatorial system can yield a great deal of differentiation providing the basis for the development of open ended building systems for use in architecture.

These operations could not have been performed by a permutation algorithm, as what became meaningful in terms of configurations was at all times a human synthesis mediating among diverse criteria. The framework is indeed computational, but has been designed to allow for a human-machine symbiosis, allowing human designers to interact with generative aggregates in an intuitive way.

**Evolution of Polyomino**

For the collaboration with the company Stratasy, the research unit envisioned the use of gaming technology and virtual reality to generate a frictionless combinatorial environment where users could immerse themselves into a design experience. The deployment of a gaming platform implies a real-time feedback with a simulation where tools provide enough information to generate a feedback loop with the designer. As explained in the ACADIA paper of 2014 (Sanchez, 2014), the use of gaming platforms has a great potential to allow for new designers to emerge from crowdsourced operations. As Jeremy Rifkin describes it, the ‘prosumers’ (Rifkin, 2014), consumers that become producers of their own goods, can potentially drive a new explosion of manufacturing.

This was the objective of the series of projects developed with Stratasy. Framed in the notion of ‘from gaming to making’, the Polyomino prototypes establish a bridge between a massive entertainment platform such as video games and a growing maker movement empowered by tools like 3D printing.
The challenges of the virtual reality platform have to do with the framerate requirements while dealing with complex geometry and the user experience at the moment of design. The team makes the distinction between a 3D modeling software in the form that we know them in computers, and a game experience that inserts a player into a constrained space with particular combinatorial operations available. This is an area that is still in early development as virtual reality is a very recent technology and the research group sees great potential in future development.

One of the most radical innovations of 3D printing has nothing to do with form but rather with forms of distribution. By digitizing objects to digital files, transferred through the web and manufactured on local machines, we bypass the whole distribution chain that is highly inefficient and has a great environmental impact. The team envisions how users would be able to use gaming platforms to adapt and create new objects that they could print locally.

The modular design of the Polyomino print is a reflection on how modular combinatorics will play a role into the customization of products, where a user can decide to exchange modules to adapt to diverse conditions. Polyomino uses magnetic connections to allow an intuitive snapping into place of units, allowing the user to decide diverse patterns and arrangements. The project uses color and the new technology developed by Stratasys with the J750 to print a large range of units in different digital materials defined by their color and flexibility. The model establishes an analogy of how materials work at an atomic scale, allowing for the bonding of certain molecules to one another and generating larger assemblies.

CONCLUSION
The work developed over the past three years in the framework of combinatorial design has developed a rich repository of strategies and a clear framework to continue further research. There are indications that industry will continue pursuing systems and products with a strong modular open-ended architecture, as the business model coming from a serialized repetition of units is still the strongest and most economical form of fabrication. The addition of combinatorial strategies adds a new layer of heterogeneity and customization where families and sets of products could be combined and re-organized by an interconnected community.

The tools and infrastructure developed for achieving these design strategies also resonate with the zeitgeist of the manufacturing industry, where the issue of distribution of goods is one of the largest industries that could be radically transformed by the digitalization of physical goods with the advent of local and cheaper 3D printing centers. Gaming platforms could take an educational role and help to minimize the friction between creativity and production.

This future is currently in the making and pushed by those institutions and individuals who are building the tools to allow its implementation.
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Team 3: Jiawen Ge, Yuchuan Chen, Yu Zhao, Yu Zhao, Qian Liu

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REFERENCES

IMAGE CREDITS
Figure 1: Polyomino Studio year 3 – Team 1: Jingbo Yan, Dechen Zeng, Wansu Zhang, Xinran Ding.
Figure 2: Polyomino Research – Plethora Project with Yuchen Cai, Setareh Ordoobadi.
Figure 3: Polyomino Research – Plethora Project with Yuchen Cai, Setareh Ordoobadi.
Figure 4: Polyomino Studio year 2 – Hanze Yu, Kaining Li, Siyu Cui
Figure 5: Polyomino Studio year 2 – Hanze Yu, Kaining Li, Siyu Cui
Figure 6: Polyomino Studio year 3 – Team 1: Jingbo Yan, Dechen Zeng, Wansu Zhang, Xinran Ding.
Figure 7: Polyomino Studio year 3 – Team 1: Jingbo Yan, Dechen Zeng, Wansu Zhang, Xinran Ding.
Figure 8: Polyomino Studio year 3 – Team 3: Jiawen Ge, Yuchuan Chen, Yu Zhao, Yu Zhao, Qian Liu
Figure 9: Polyomino Studio year 3 – Team 3: Jiawen Ge, Yuchuan Chen, Yu Zhao, Yu Zhao, Qian Liu
Figure 10: Polyomino Studio year 3 – Team 3: Jiawen Ge, Yuchuan Chen, Yu Zhao, Yu Zhao, Qian Liu
Figure 11: Polyomino Studio year 3 – Team 2: Aditya Gulati, Ashley Kasuyama, Zhixue Zheng, Qianqian Xu
Figure 12: Polyomino Research – Plethora Project with Yuchen Cai, Setareh Ordoobadi.
Figure 13: Polyomino Research – Plethora Project with Yuchen Cai, Setareh Ordoobadi.
3D Printed piece developed out of the gaming platform. The units have been printed separately and magnets have been added to the face connections to allow the growth of the voxel structure.
Virtual reality gaming software – Player is able to place different kinds of units out a pre-established inventory, pick color and rotate them to define the necessary connectivity with other units.

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He has taught and guest lectured in several renowned institutions across the world, including the Architectural Association in London, the University of Applied Arts (Angewandte) in Vienna, ETH Zurich, The Bartlett School of Architecture, University College London, and the Ecole Nationale Supérieure D’Architecture in Paris.

Today, he is an Assistant Professor at USC School of Architecture in Los Angeles. His research ‘Gamescapes’, explores generative interfaces in the form of video games, speculating in modes of intelligence augmentation, combinatorics and open systems as a design medium.