Surface Disclination Topology in Self-Reactive Shell Structures

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1 Thin plywood prototypes of triangle and square based surface disclinations

ABSTRACT

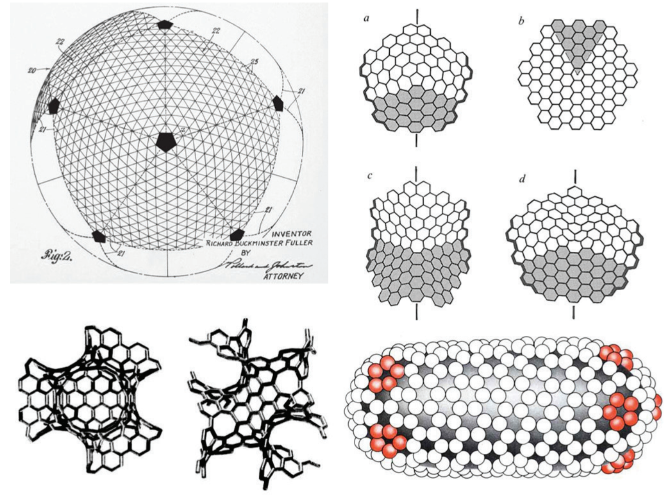
This paper discusses recent developments on the geometric construction and fabrication techniques associated with large-scale surface disclinations. The basic concept of disclinations recognizes the role of "defects" in the composition of materials, the strategic placement of which shapes the material by inducing curvature from initially planar elements. By acknowledging the relationship between geometry and topology that governs disclination based form-finding and material prototyping, this work consciously explores its potential at the architectural scale. Basic geometric figures and their topological transformations are documented in the context of digital modeling and simulation, fabrication, and a specific material palette. Specifically, this work builds on recent efforts by focusing on three particular areas of investigation: a) enhancing the stability of surface disclinations with a synthetic fibrous layer, b) aggregation via periodic tilings, and c) harnessing snap-through instability to increase bending stiffness in thin surfaces.

INTRODUCTION

The work presented in this paper builds on methods that were developed in 2019 and subsequently explored in a second sponsored graduate research studio conducted early 2021, as a design-build educational exercise resulting in refined digital modeling and fabrication strategies and new knowledge associated with surface disclinations. Here, the term more broadly refers to the topological concept that introduces angle deflections in sheet materials resulting in out of plane buckling, while a more specific term, *wedge disclinations* (wd) refers to a particular variant that topologically induces elliptic or hyperbolic geometry in elastic surfaces and lattice structures (Harris 1977; Weeks 1985; Bruscia 2020).

The notation used here to describe this variant is [+/-x°] wd, referring to the overall sector (or wedge) of inserted material into the original shape as a rotation angle—otherwise described as "angular excess" (producing negative curvature) or "angular deficit" (producing positive curvature) (Terrones and Mackay 1993). An additional terminology used in this paper, "2-circle" or "3-square," describes a [-360°]wd or [-720°]wd respectively. This casually refers to the overall amount of surface area, while the degree notation refers more specifically to the inserted amount. The physical form-finding process for surface disclinations involves the rotational bending of pre-cut sheet materials whose edges reconnect back upon themselves. Strain is introduced to the sheet, and the sheet bends in reaction. The strain is then releived and distributed evenly across the now stable surface (Harris 1977). Nothing is added, but the surface takes on a new form; in a sense, it is reacting against it's own natural tendency to become flat again. The term "self-reaction" is used here to describe this behavior.

Wedge disclinations are found in various sculptural and woven work. In architecture, Buckminster Fuller's geodesic dome is a two-dimensional hexagonal framework with several wedge disclinations occurring as well-placed pentagons (deWit 1971). Surface and lattice transformations caused by placing singularities within tri-axial meshes (Ayres et al. 2020) share common references in the natural world; the defect in the molecular arrangement of carbon molecules are present in thin periodic structures such as virus shells, the pattern of fingerprints, and in the pelts of striped animals (Harris 1977). Because carbon atoms are arranged into a hexagonal arrangement, it is possible to build complex molecular structures with curvature using disclinations, as demonstrated in the work on controlled shape change in graphite by Terrones (1993), Mackay (1985), and



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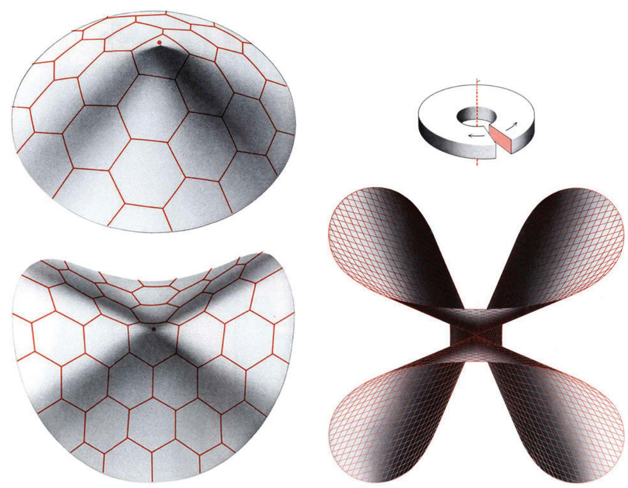

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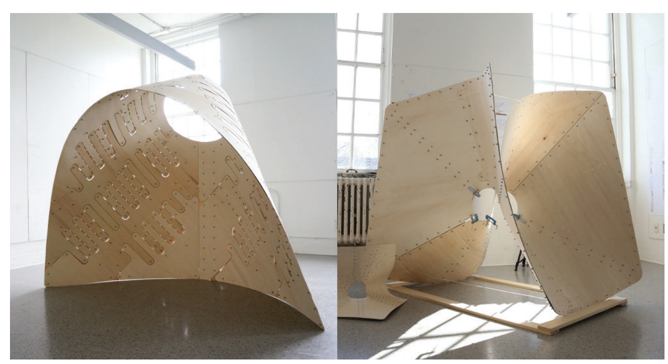

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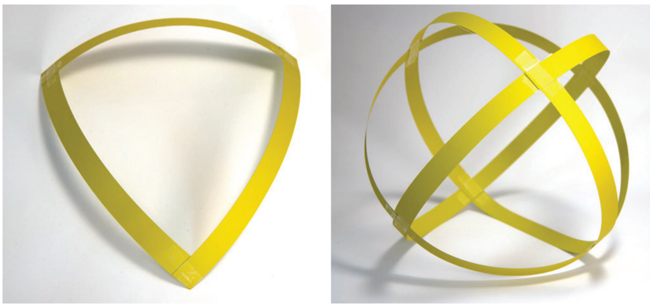
2 Disclinations in hexagonal lattices (clockwise rom top left): Buckminster Fuller (1954), Iijima et.al (1992), Terrones and Mackay (1993), W. Harris (1977)

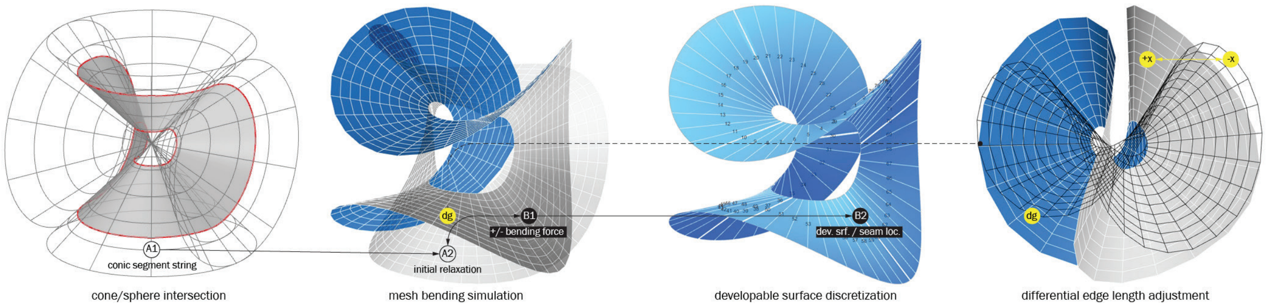
3 Left: [+60°] and [-60°] wedge disclinations (wd) in thin sheets (W. Harris 1977); Right: [-360°] wedge disclination (wd) in a thin sheet

4 Left: [+180°]wd plywood prototype, and Right: [-288°] wd plywood prototype



5 Gauss-Bonnet bending of a 90° equilateral triangle (Harriss 2020)

6 Gauss-Bonnet bending through curved and serrated seams (Delp 2011)



7 Basic geometric construction and bending simulation of a 2-circle [-x°]wd

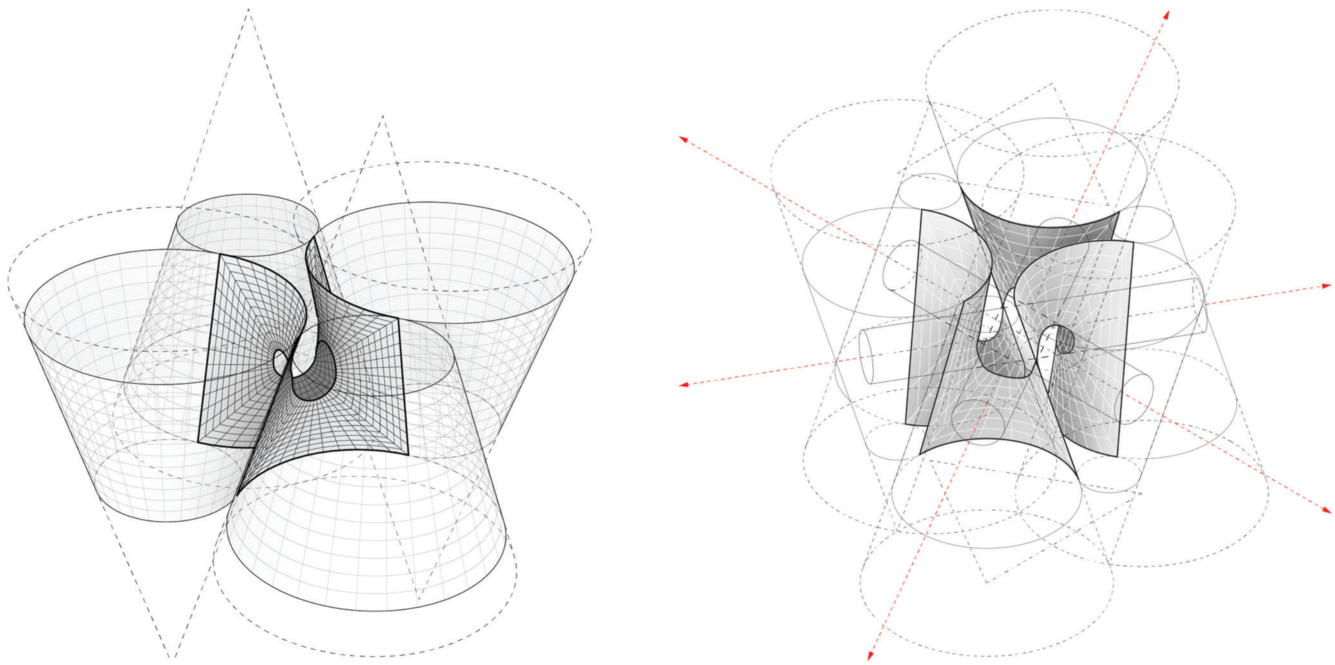
Iijima et al. (1992) (Figure 2). The observation of the effect of disclinations in thin surfaces was illustrated by crystallographer William F. Harris (1977), from which this work adopts the general concept, terminology, and notation (Figure 3).

Previous work demonstrated that [+x°]wd produced a variety of conic shells with positive global curvature, and [-x°]wd gave rise to a variety of saddle-like shells with negative global curvature. Both were prototyped at a large scale; the [+x°]wd forming a tilted conic shell, and the [-x°]wd resulting in a 8-sided saddle-like form, found by combining two pentagons (Bruscia 2020) (Figure 4). Although the prototypes showed that topologically induced bending using the disclination method could scale, the shells were only moderately stable and were subject to shearing during the assembly process. The advancement of this work had three primary goals intended to address these shortcomings; a) surface reinforcement, b) expansion via tiling, and c) stability via snap-through buckling, all of which are discussed in the following sections.

STATE OF THE ART

Disclination topology may be described more formally with the Gauss-Bonnet theorem, and we find related work in mathematics and sculpture. Paper strip models are used to teach mathematical concepts such as discrete Gaussian curvature, face-defects, and angle deflections; the paper model as a tool to demonstrate them. For example, we can look to Ilhan Koman's developable sculptures to see how angle deflections describe local bending behavior in surfaces (Akleman and Chen 2006). Inversely (and serendipitously), we have discovered the connection in our work to these mathematical concepts by first exploring surface disclinations intuitively. Mathematician Edmund Harriss provides a clear explanation of Gauss-Bonnet: that the sum of the angles in a triangle is 180°, and by changing the corner angles of the triangle, it must bend into positive or negative curvature (Harriss 2020). For example, consider that the corner angles in an equilateral triangle are 60°. By increasing the corner angles to 90°, we increase the overall sum. Constructing this triangle from paper strips will force it to buckle with positive curvature, and eight of them constructed the same way will create an octahedral sphere (Harriss 2020) (Figure 5). Paper bending visually demonstrates this transition from Euclidian to elliptic and hyperbolic polygons by combining them into spherical platonic solids or minimal surface approximations (Akleman et al. 2010; Harriss 2020).

Another example is seen in the work of Kelly Delp and Bill Thurston (2011), whose technique pushes the curvature away from vertices and into the edges. Considering again the equilateral triangle, a 30° sector of a circle replaces each side, forming a 'fat octahedron' when the curved seams are forced to connect. Meandering profiles along the curved edges are added, forcing the parts to bend and interlock, evenly distributing the curvature across the seam and smoothing the polyhedron into a sphere (Figure 6). Simply forcing the profiles to come together without gaps automatically determines the localized bending and global curvature. The method is easily adapted to create models



8 Geometric construction techniques for a 2-square [-x°]wd (left) and a 3-square [-x°]wd (right)

of negative curvature and combinations of both, as shown in a variety of their models (Delp and Thurston 2011). The serrated profiles presented in this paper were designed to evenly distribute bending forces across the seam, but they themselves do not induce the curvature. In the discussed prototypes, serrated seams function to combine smaller parts into structures with dimensions that exceed off-the-shelf materials. However, both the techniques introduced above, as well as the disclination techniques described below, result in surfaces that are formed automatically during assembly. The wooden surfaces are not steam-bent or forced into molds; they begin flat, and are shaped entirely by disclination topology and active bending forces.

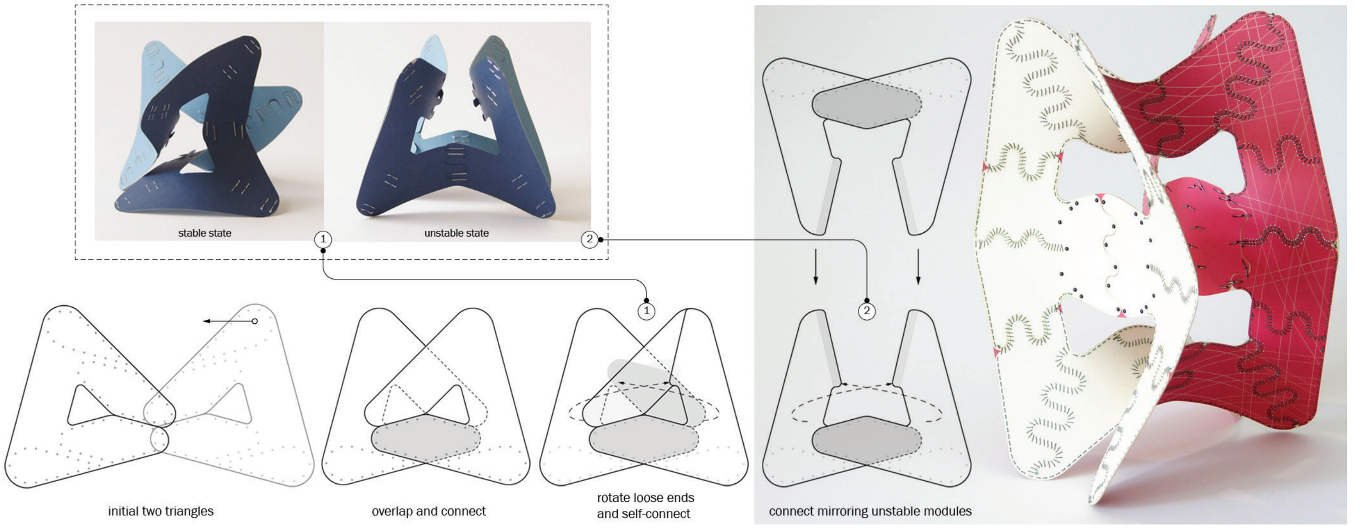
DESIGN AND FABRICATION METHODS

While it can be argued that the [-x°]wd can itself be discussed as an efficient method for fabricating curved surfaces from developable sheet materials, the specific application of those surfaces remains one of the goals of future work. Instead, this work is concerned with the methods for developing sculptural shell structures that are intended only to demonstrate how the potential structural qualities of surface disclinations translate to a larger scale. Recent developments in the modeling and simulation of surface disclinations are discussed below, paying particular attention to the geometric construction of base mesh topologies.

Geometric Construction of Disclination Topology

Three techniques to digitally construct and simulate surface disclinations were studied previously (Bruscia 2020). Of the three, the "double-cone intersection" technique has shown to be the most versatile, especially when used in combination with the "expanded concentric polylines" technique for added control over the result (Figure 7). Initially devised to prove that the surfaces are made up of strings of conic sections and thus maintaining their developability on-screen, new variations of this technique can now accommodate a wider range of base geometric figures. The subsequent mesh bending simulation is useful to arrive at more "natural" digital representations.

Bending simulation is achieved using a physics simulation platform, for example Kangaroo2 (Piker 2013) which models physical behavior qualitatively on triangle meshes using a 3-point approach. The following sections focus on the geometric approximations of disclination topology, with the assumption that a variety of platforms may be used to apply bending simulation. The work in this paper was developed by using default algorithmic sequences for mesh bending in Kangaroo2 as a proof-of-concept. Once the mesh topology is modeled correctly, the concentric mesh edges are assigned a numerical input to dynamically change their length. The radiating mesh edges are forced to remain at their original lengths as the concentric edges shrink or grow, forcing the mesh buckling to increase or decrease. For example, a numeric range to adjust the amount of curvature could be $[0.5 < 1 < 2.0]$ with 1 referring to the original length. The amount of material insertion based on the differential growth of mesh edges depends on the initial state of the mesh. Assuming that the geometric



9 Counter-reactive stability by combining (2) opposing 2-triangle [-x°]wd modules forced to connect between their stable states

construction shown in Figure 4 produces a surface that is an exact approximation of the 2-circle [-360°]wd, the angular excess can be found by simply multiplying the edge length factor (x) by 360. If x=0.25, the angular excess is 90°. The following paragraphs include few examples to illustrate the relationship between the desired geometric profile and the digital modeling process.

Topology Based on the Circle and Ellipse

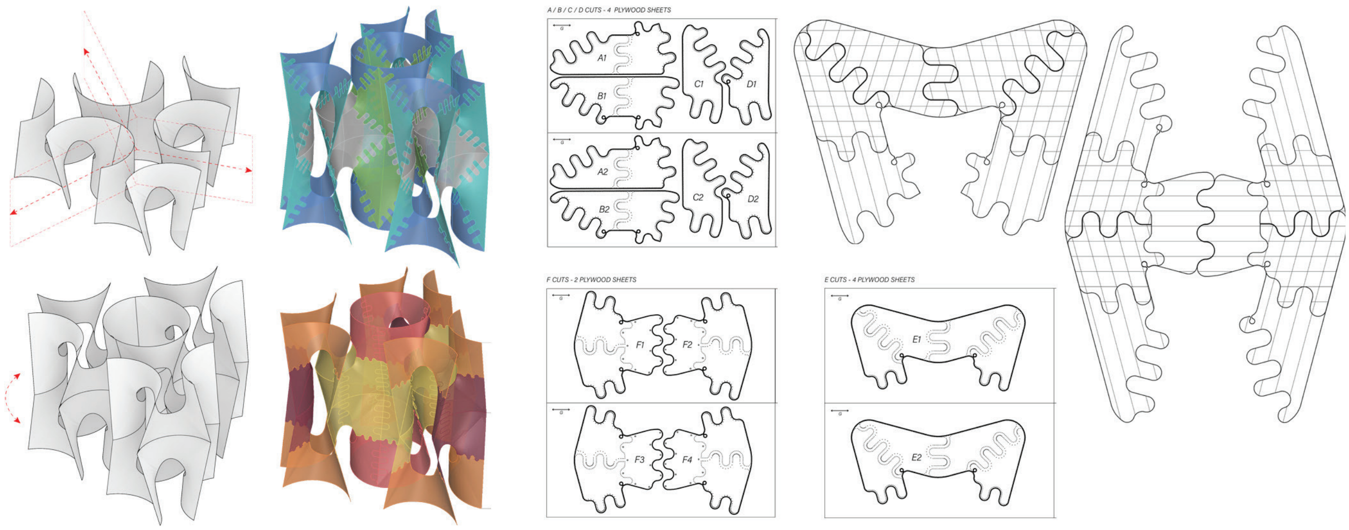
The simplest geometric construction intersects two spheres (one large, and one small) with two double-cones whose apices meet at the same point. The intersections form circles that can be split using the tangents between the cones. Selecting every other circle segment will string together the inner and outer profiles of a 2-circle [-360°] wd. The profiles may then be used to draw a ruled surface and/or a quad or triangle mesh between them. The exact process can also be used to model a 3-circle [-720°]wd by adding one more double-cone to the cube, such that a circular cone base is drawn along each of the cube's 6 faces. Geometrically constructing the topology for a surface disclination based on the ellipse generally follows the same process. Instead of a cube to organize the cones, the faces of rectangular prism organizes the cones which are intersected with ellipsoids instead of spheres. As an alternative to the rectangular prism, a hexagonal prism may be used to construct a 3-ellipse surface disclination with 3-fold symmetry.

Topology Based on the Square

Square-based surface disclinations follow the same topologically induced bending behavior as the circle and ellipse, but a different arrangement of cones is needed to model the 3-dimensional cubic profile. Both 2-square and 3-square structures begin with mirrored sets of vertical cones that touch along a straight tangent line (Figure 8). The method chosen to ensure tangency will determine input parameters that may be numerically controlled to dynamically change the outer edge tilt and overall proportions. For example, we begin with three circles whose center points rest at the vertices of an equilateral triangle, and whose radii are the distance between the triangle vertices and the triangle center point. Three cones are drawn from the base circles, whose intersections create the apices of three opposite cones. For both 2- and 3-square [-x°]wd, it is advantageous to cut the set of cones with horizontal planes that are equidistant from the cone apices. This will prevent the structure from self-intersecting, and spacing them apart to the same distance as one side of the equilateral triangle will ensure more cubic proportions. The tangent lines between the set of 6 cones are used for the outer, rectilinear edges of the surface. The center profile is drawn from a separate process, however, for instance the profile of a 3-square circle disclination may be used and centered among the set of vertical cones. Finally, the center and outer profiles are connected as meshes or NURBS surfaces for further development.

Topology Based on the Triangle

Triangle-based surface disclinations are unique because they tend toward a stable state that appears twisted, and in a sense, incomplete. Between states, when the bending forces are at their highest and the surface is trying to release its energy by rotating back, is when the 2-triangle [-x°]wd has the appearance of being stable due its perceived symmetry. After many attempts to discover how to repeat and control this behavior, it was eventually



10 (Left) 3-square [-x°]wd modules blended into two continuous layers with staggered seams; (Right) CNC cut-sheets and grain orientation of the 2-module, 4-triangle [-x°]wd

harnessed as a strategy for increasing the overall stiffness by connecting two opposing surface disclinations at their mid-way and most unstable position, thereby maximizing the self-reaction (Figure 9). The simulation of such behavior remains an area of interest for future investigation as it could hold interesting potential in the design of thin and lightweight sheet materials in larger structures. The geometric construction borrows the same cone arrangement as the 2-square [-x°]wd, but with two major differences: to find the inner profile, folded planes are intersected with the cones, and to obtain the outer profile, planes are extended off of the cones from where they meet tangentially.

Fabrication and Assembly

Three large-scale prototypes were constructed from rotary cut 4ft x 8ft x 1/8in (3mm thick) 2-ply sheets of Okoume BS 1088 Lloyds approved marine-grade plywood, and rotary cut 4ft x 8ft x 1/8in 3-ply sheets of bendable barrel grain exterior grade Lauan plywood. To maintain bending continuity throughout the entire surface, each prototype consists of two layers of material for a total thickness of 1/4in (6mm). The 2-layer approach avoids awkward lapping such that all seams have the same visual hierarchy and each side of the surface is smooth (Figure 10). Deeply serrated edges allow the material to bend evenly, although a very small area of single-layer thickness remains where the seam terminates at the outer edge. A relief hole is placed in these locations to distribute the bending forces deeper into the 2-layer surface. The prototypes were assembled as a sequence of parts that were gradually bent and forced into position, automatically taking shape as parts are connected. The form emerges naturally as a result of the topologically induced bending, and the seams between parts are located so that each part can be cut from a single 4ft x 8ft sheet and with a desirable grain direction that is assigned based on whether it is advantageous achieve a tight bending radius or to locally increase the stiffness. Each of the three demonstrators represent a different base geometry, aggregation, and stiffening strategy, but all demonstrate the topologically induced bending process of [-x°]wd.

RESULTS AND DISCUSSION

Disclination based on the Ellipse

The 2- or 3-circle [-x°]wd is perhaps the most clear demonstration of disclination topology, but its stability varies depending on proportions, surface depth and overall dimensions, and seemingly requires the most additional stiffening after assembly. The studio sought to test the limits of this disclination variant, and a form finding study based on the ellipse led to the design of a free-standing ribbon that stands 9ft (3m) tall (Figure 11). The total surface area of material used is about 199 square feet, including both layers of 1/8in Okoume plywood for a total weight of approximately 56 lbs.

Smaller scale investigation revealed that the self-reactive bending could not provide enough stability for the structure to resist its own weight. This was addressed by adding a carbon fiber exoskeleton that increases the structural depth of the surface, allowing it more quickly spring back. The structure easily carries its own weight and wiggles confidently under minor loading, and has remained symmetrical without tilting or sagging several months after completion. The exoskeleton consists of ~1125 ft of 12k T700 carbon fiber roving impregnated with epoxy resin cured in room temperature. The fibers are looped around

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792 nylon threaded rods that extend 20mm from each side of the surface, holding fibers approximately 7mm to 15mm away. Fiber roving may be added to further reinforce specific areas of the surface and the threaded connections can vary in length to create a gradient of deep and shallow areas along the surface. The lightweight CFRP roving ensured that the materials work as a uniform system, assisting and not overtaking the active bending.

Disclination based on the Square

Polygon-based surface disclinations are able to more easily aggregate due to their points and edges. Aligning modules by overlapping edges gives rise to periodic tilings of disclination modules, shifting the focus from the development of the singular object to that of larger structures. The studio aimed to provide a clear example by tiling the 3-square [-x°]wd disclination both horizontally and vertically. The 3-square disclination is particularly conducive to tiling due its 6 "sides"—three curves along the ground plane, and 3 above rotated 60°. 3-square modules may thus be arranged in a hexagonal tiling and easily stacked in the z-axis. Three modules centered at the vertices of an equilateral triangle will form a cylindrical space, however the aggregate surface is always continuous and open. The finished prototype stands at 8in tall and consists of ~430 square feet of material including a combination of 1/8in Okoume and barrel grain Lauan plywood fastened together with 3/16in (4mm) aluminum rivets and #10-32 SS bolts along demountable seams (Figure 13).

Given the overall footprint, this structure is extremely stable. However, a stiffened proof-of-concept was built using a single 2-square module. 3in wide carbon fiber tape was manually applied to the surfaces after bending into a completed assembly, consisting of 5.7oz IM2 unidirectional layers, topped with a layer of 3K carbon and fiberglass biaxial tape. The pattern was manually drawn onto the surface using a strip of PETG that was pinned between CNC-cut holes and laid out along the geodesic path.

Disclination based on the Triangle

Interestingly, the 2-triangle [-x°]wd is more conducive to tiling periodically when in the tensed state. This provided a key opportunity to harness the structural potential of disclination topology; in order for one 2-triangle module to hold its position, a second module must counter the natural tendency for it to rotate/twist. The second module is designed with identical geometry but with the tendency to rotate in the opposite direction. When connected, the two modules balance each other by providing an equal and opposite reaction, as observed in both small scale models and the larger prototype that blends the modules into a single, continuous surface. While the prototype maintained this mutually self-reactive behavior, the simulation requires a different approach to demonstrating the natural tendency toward a state that differs from its initial form. Some work has been done to experiment with adding 'bias' to the meshes by introducing diagonal edges in opposing directions, but as of the writing of this paper this remains a work-in-progress.

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11 3-ellipse [-x°]wd prototype with standoff CFRP roving for added stability

12 2-module, 4-triangle [-x°]wd prototype

13 6-module, 18-square [-x°]wd tiling prototype, with 2-square [-x°]wd CFRP tape test

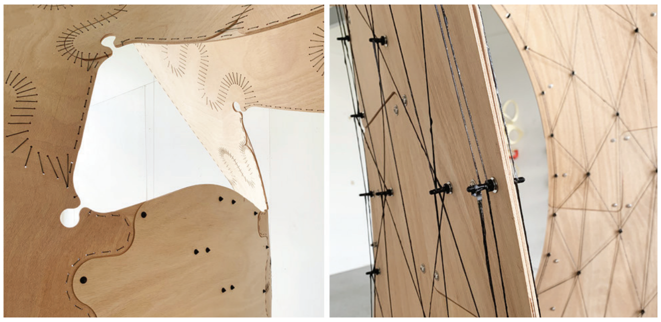
The approximately 8ft tall, 2-module (4-triangle) prototype was designed and built as a blended counter-reactive surface (Figure 12). Twenty-six parts assemble into the 2-layer plywood surface that is ~145.6 square feet of total material weighing ~41lbs. The parts and layers are manually sewn together simultaneously using braided Spectra twine through a CNC drilled hole pattern. This method has shown to be the most successful in maintaining layer adhesion without air gaps, while having enough flexibility in the joint to allow the layers to adjust while bending. In-depth analysis of fibrous joints applied to segmented timber shell structures is documented in Bechert et al. (2016) and Garufi et al. (2019).

Partially assembled portions exhibited the tendency to twist as expected and canceled each other out in the finished assembly. The structure is confidently stable and the internalized counter-reaction seems evident in the way it resists external forces pushing on it. While this prototype was intended to test the structural potential of counter-reactive bending, a portion of the surface was built separately as a proof-of-concept for additional fiber-reinforced stiffening, experimenting with 2in wide, 9oz T700 unidirectional carbon fiber tape topped with a layer of biaxial tape of the same specification.

The tendency for the surface to rest in multiple semi-stable states reveals a particularly interesting strategy for increasing the overall stiffness of the structure by using characteristics inherited from surface self-reaction. By attempting to understand what determines this behavior, we recognized that surface disclinations exhibit snap-through buckling, always maintaining a degree of increased stiffness between stable states. It has been observed throughout this research to date that physical models will behave differently under subtle changes, for example upon which side the material overlap occurs, assembly sequence, and accidental material "training" or fatigue that results from forcing the card stock to buckle and bend. This has not been a hindrance to larger scale prototyping however, and the counter-reactive assembly studied in the triangle-based prototype has inspired continued investigation into "functionalized instability" for increasing the structural capacity of very thin sheet materials (Schleicher et al. 2015).

CONCLUSION

In summary, the work outlined in this paper shows the continuation of research focused on developing new fabrication and form-finding techniques based on the topological and geometrical properties of surface disclinations. Yet to be explored at this scale previously, the results illustrate the feasibility of applying this particular form of topological bending at a large scale, and point toward paths forward that have both educational and material value. Specifically, the results demonstrate how the goals of this phase were addressed by demonstrating proof-of-concept approaches to: a) adding stability by integrating a lightweight carbon fiber reinforcement b) periodic tilings by blending disclinations into larger surfaces, and c) how snap-through instability may be harnessed to pre-tension disclination modules and



14 Sewn plywood layering and CRFP standoff details

15 Single, continuous surface of the square-based [-x°]wd tiling

16 Spatial pockets of the square-based [-x°]wd tiling

increase the overall bending stiffness in planar, off-the-shelf building materials.

In addition to the exercise of enacting disclinations from an architectural point of view, the work may contribute to a variety of contexts that are also seen here as areas of future improvement: (1) moldless shell fabrication wherever developable materials are advantageous, (2) the discourse on bending-active shell structures from thin sheet materials, (3) the geometric construction of base mesh topologies for computational analysis and simulation, and (4) design-based exploration of the relationship between mathematics, materials, and form. Linking the surface disclination fabrication technique with the Gauss-Bonnet formula is also a desirable area of further investigation, as it may open the door to a new design technique that expands the disclination repertoire to include spherical portions to the catalog of conic segments, and lead to a mathematical application of surface disclinations in architectural form rationalization and construction.

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REFERENCES

Akleman, Ergun and Jianer Chen. 2006. "Insight for Practical Subdivision Modeling with Discrete Gaussian-Bonnet Theorem." In *Proceedings of Geometry, Modeling and Processing 2006*, edited by M-S. Kim and K. Shimada. Pittsburg: Springer-Verlag Publishing. 287–298.

Akleman, Ergun, Jianer Chen, and Jonathan L. Gross. 2010. "Paper-Strip Sculptures." In *IEEE International Conference on Shape Modeling and Applications (SMI)*, edited by J-P. Pernot et.al., Aix-en-Provence. IEEE Computer Society. 236–240.

Ayres, Phil, Ji You-Wen, Jack Young, and Alison Grace Martin. 2020. "Meshing with Kagome Singularities." In *Advances in Architectural Geometry 2020*. Ecole de Ponts ParisTech and Université Gustave Eiffel. Accessed May 10, 2021. https://thinkshell.fr/advances-in-architectural-geometry-2020-on-line-library/.

Bechert, Simon, Jan Knippers, Oliver David Krieg, Achim Menges, Tobias Schwinn, and Daniel Sonntag. 2016. "Textile Fabrication Techniques for Timber Shells. Elastic Bending of Custom-Laminated Veneer for Segmented Shell Construction Systems." In *Advances in Architectural Geometry 2016*, edited by S. Adriaenssens et al. Zurich: vdf Hochschulverlag AG an der ETH Zurich. 154–169.

Bruscia, Nicholas. 2020. "Structural Papercuts; Scaling Disclinations in Self-Reactive Surfaces." In *ACADIA '20: Distributed Proximities; Proceedings of the 40th Annual Conference of the Association for Computer-Aided Design in Architecture*, edited by B. Slocum, V. Ago, S. Doyle, A. Marcus, M. Yablonina, M. del Campo. Online+Global. 536–545.

Delp, Kelly, and Bill Thurston. 2011. "Playing with Surfaces: Spheres, Monkey Pants, and Zippergons." In *Proceedings of the Bridges Conference 2011*. Coimbra: Tessellations Publishing. 1–8.

17 Selected paper form-finding models

deWit, Roland. 1971. "Relation between Dislocations and Disclinations." *Journal of Applied Physics* 42 (9): 3304–3308.

Garufi, Dominga, Hans Jakob Wagner, Simon Bechert, Tobias Schwinn, Dylan Marx Wood, Achim Menges, and Jan Knippers. 2019. "Fibrous Joints for Lightweight Segmented Timber Shells." In *Research Culture in Architecture: Cross-Disciplinary Collabortation*, edited by. C. Leopold, C. Robeller, and U. Weber. Basel: Birkhäuser. 53–63.

Harris, William F. 1977. "Disclinations." *Scientific American* 237 (6): 130–145.

Harriss, Edmund. 2020. "Gauss-Bonnet Sculpting." In *Proceedings of Bridges 2020: Mathematics, Art, Music, Architecture, Education, Culture.* Phoenix, Arizona: Tessellations Publishing. 137–144.

Iijima, Sumio, Toshinari Ichihashi, and Yoshinori Ando. 1992. "Pentagons, heptagons and negative curvature in graphite microtubule growth." *Nature* 356: 776-778

Mackay, Alan L. 1985. "Periodic Minimal Surfaces." *Nature* 314: 604–606.

Piker, Daniel. 2013. "Kangaroo - Form Finding with Computational Physics." *AD, Architectural Design* 83 (2): 136–137.

Schleicher, Simon, Andrew Rastetter, Riccardo La Magna, Andreas Schönbrunner, Nicolas Haberbosch, and Jan Knippers. 2015. "Form-Finding and Design Potentials of Bending-Active Plate Structures." In *Modelling Behaviour. Design Modelling Symposium 2015*, edited by M. Ramsgaard Thomsen et. al. Cham: Springer International Publishing. 53–63.

Terrones, Humberto, and Alan L. Mackay. 1993. "Hypothetical Curved Graphite." *Nanostructured Materials* 3: 319–329.

Weeks, Jeffrey R. 1985. *The Shape of Space. How to Visualize Surfaces and Three-Dimensional Manifolds.* New York: Marcel Dekker, Inc.

IMAGE CREDITS

Figure 2: Clockwise from top left: Buckminster Fuller 1945, Iijima et.al. 1992, Terrones and Mackay 1993, W. Harris 1977
Figure 3: W. Harris 1977
Figure 5: E. Harriss 2020
Figure 6: Delp and Thurston 2011
Figure 8: Left: Kaur 2021, Right: Hayden 2021
Figure 9: Hills, Kaur, Copete, Starr 2021
Figure 10: Left: Hayden, Cleary, Goembel, Barzideh 2021, Right: Hills, Kaur, Copete, Starr 2021
Figures 15 and 16: Hayden 2021
All other images by the author.

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