BEYOND DISTANCE

New Criteria for Spatial Configuration of Design

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Abstract. Simple Euclidean distance has thus far been the dominating concept in analyzing architectural and urban spaces. This paper demonstrates that distances between spaces cannot be measured solely in terms of simple Euclidean Distance, but instead other kind of distances (e.g. City block, Chebyshev, Minkowski, Canberra distance or Angular separation) are shown to offer new meaningful insight into space and its denotation. Several issues are raised in light of these new measures such as how much are these measurement techniques influenced by what counts as "space"? In addition is there a difference between the physical distance and the human perception of distance? More importantly, how do these methods alter design or offer a new process of designing? Applications and analysis is applied to classical examples of architecture.

1. Introduction

In the sixties, seventies and throughout the eighties, there was a great deal of excitement about what computers and quantitative methods such as operation research techniques can offer to architectural design. Several researchers went on to formulate the “optimum design” by minimizing traveling distance between building spaces, maximizing natural light and ventilation while minimizing cost, etc… In fact, even earlier on there were attempts to optimize the design of space. In the 1920s the modern movement including the BAUHAUS had tried to conduct systematic research in this area. The “Frankfurt Kitchen” was the first research in the regular work happening in a household and the man kilometers traveled
inside. Neufert was a student at the BAUHAUS and then carried this research further under the NAZIs as they were very interested in economizing on construction costs.

Problems about how to quantify other aspects of design soon became obvious. Questions on how to measure and model the way people interact with the space as well as how do they see, experience and perceive the space still remain open until today. Recently there has been a growing interest in modeling people movements in the built environment, driven by the advances in numerical simulation technologies and theories and computer-game-design.

Although the motivating force behind this interest has been issues like emergency egress analysis and urban design traffic modeling, a number of important conclusions and models about the interaction of occupants with architectural space have been discovered. Simultaneously, research on space syntax has also been growing. Space syntax research treats the built environments as a complete system that can be analyzed configurationally, to identify its fundamental pattern and structure through quantitative measures and techniques such as axial maps, isovists, integration measures, etc…. These two areas of research can be amalgamated to bring forward new kinds of design criteria that advance a distinctive approach to the analysis and design of architectural spaces. This paper discusses these advances and puts forth a novel proposition to include specific measurable criteria in the design evaluation process based on the space syntax and occupant movement models. As an illustration this paper demonstrates that distances between spaces cannot be measured solely in terms of simple Euclidean Distance, but instead other kind of distances (e.g. City block, Chebyshev, Minkowski, Canberra distance or Angular separation) are shown to offer new meaningful analysis into space and its denotation.

Distance measures are proposed here as a new criteria for spatial configuration of space. The paper presents analysis examples from real buildings and conclusions are drawn to illustrate the use of the new criteria. It is shown that by considering the different meaning of distance there is an opportunity to incorporate effective criteria for spatial configuration of architectural design.

2. Measures Of Distance Applied To Architectural Space

In order to assess the meaning of distance in floor plans we need to first define the space we are working in. One could consider the space in terms of the physical dimensions, i.e. simply being the 2 dimensional or 3 dimensional. However, the space may have more than just the physical dimensions, i.e. social, functional, etc… The social for example could
represent the use of the space, i.e. conventionally speaking the social activities in the bathroom are a distance apart from those in the bathroom, while the living room and the dining room may be more proximate in terms of the “social distance”. The social distance can be a function of the proxemics of the main activities that take place in the space. The term proxemics is used to describe the measurable distances between people as they interact. Usually there are four different distances used to refer to the activities of occupants; intimate distance for embracing, touching or whispering (15-45 cm, 6-18 inches), personal distance for interactions among good friends (45-120 cm, 1.5-4 feet), social distance for interactions among acquaintances (1.2-3.5 m, 4-12 ft) and, public distance used for public speaking (over 3.5 m, 12 ft).

The same may be true when we are talking about the functional distance. The kitchen and the bathroom both need plumbing connections and therefore the “functional distance” between them may be close. In addition one may consider the openness of the space as another dimension. If a space has several opening and windows to the outside or to other spaces, it would be given a certain score for that dimension. Spaces with similar openness would therefore have the same dimension, e.g. outside space will have the highest value, and while closed inside spaces would have the lowest openness value.

Given these different dimensions of space, one can represent each space in the building as points in n dimensions. A space for example may have n coordinates each assigned based on any of the dimensions described above. Dissimilarity (or similarity) between any two objects can be based on these coordinates and be measured by one of several types of distances. Euclidean Distance for example is the traditional concept of distance. The Euclidean is what a typical occupant would conceive as being distance. It is calculated by basically evaluating the root of square differences between coordinates of the two spaces considered.

\[ d_{ij} = \sqrt{\sum_{k=1}^{n} (x_{ik} - x_{jk})^2} \]  

(1)

City block distance is also known as Manhattan distance. Its is widely used in the domain of space syntax to evaluate issues such as integration, proximity, etc. (Hillier 2005a, 2005b, 2004, 1996, 1984). City Block distance is also sometimes referred to as the boxcar distance and the absolute value distance. This distance represents the sum of the orthogonal distances between two spaces on an imaginary grid placed over the space and is therefore evaluated by the absolute differences between the coordinates of the two spaces,
The Maximum value distance which is also called the Chebyshev distance is the maximum value of the absolute magnitude of the differences between the various coordinates of the spaces. In the traditional sense, it represents the furthest distance of all the coordinates in space, e.g. it examines whether the spaces are further apart horizontally or vertically.

\[ d_{ij} = \max_k |x_{ik} - x_{jk}| \]  \hspace{1cm} (3)

Minkowski distance is considered the universal concept of metric distance, with a parameter \( \lambda \) used to denote different kinds of distances. For example, when \( \lambda = 1 \) Minkowski distance is equivalent to the city block distance and when \( \lambda = 2 \), it is equivalent to the Euclidean distance. In addition when with \( \lambda = \infty \) reduce to the Chebyshev distance. It is calculated using the formula,

\[ d_{ij} = \left( \sum_{k=1}^{n} (|x_{ik} - x_{jk}|)^\lambda \right)^{\frac{1}{\lambda}} \]  \hspace{1cm} (4)

Canberra distance normalizes the various dimensions that make up the distances between the spaces. For example using the Canberra distance one would be able to normalize the x-dimension of space based on the coordinated of the two spaces being considered. The Canberra distance is calculating by summing the fraction differences between coordinates of the two spaces and as such, evaluated so that each falls between 0 and 1. The Canberra distance has the unique property that if one of coordinate is zero, the particular term of the distance is equal to 1 (in spite of the other value) and thus the distance will not be affected1. This means that this distance can be very sensitive when the coordinates of the spaces are near to zero.

\[ d_{ij} = \sum_{k=1}^{n} \frac{|x_{ik} - x_{jk}|}{|x_{ik}| + |x_{jk}|} \]  \hspace{1cm} (5)

The Odum or Bray-Curtis distance is derived from the Steinhaus similarity measure and has the property of offering a normalized value between zero and one. Also called Sorensen distance, Bray Curtis distance is normalized by dividing the absolute difference of the coordinates of two spaces by the summation of the coordinates. As such, if all coordinates are positive, its value is between zero and one. This is important when one wants to assign equal weights to all the dimensions of space. For example, social space coordinate values will be normalized and thus have the same

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1 When both coordinate are zeros, a special case of \( 0/0 = 0 \) has to be defined
impact on distance as the physical distance between the spaces. The Bray Curtis distance is calculated using the following formula,

$$d_{ij} = \frac{\sum_{k=1}^{n} |x_{ik} - x_{jk}|}{\sum_{k=1}^{n} (x_{ik} + x_{jk})}$$

(6)

Angular Separation is another method measuring distance and is determined by calculating the angle between two vectors representing the coordinates of the spaces being considered. Angular separation measures similarity and therefore a higher value indicates closeness of the two space being considered. In addition, the values of angular separation always fall between -1 and 1 (i.e. also can be considered a normalized measure of distance).

$$S_{ij} = \frac{\sum_{k=1}^{n} x_{ik} \cdot x_{jk}}{\left( \sum_{k=1}^{n} x_{ik}^2 \cdot \sum_{k=1}^{n} x_{jk}^2 \right)^{1/2}}$$

(7)

Notice that equation 7 is exactly the same as the Coefficient of Correlation calculated often for statistical analysis. This leads to several other measures of similarity that are rooted in statistics such as the Correlation Coefficient. The Correlation Coefficient is a standardized Coefficient of Correlation based on the average value of the coordinates and is calculated using,

$$S_{ij} = \frac{\sum_{k=1}^{n} (x_{ik} - \overline{x_i}) \cdot (x_{jk} - \overline{x_j})}{\left( \sum_{k=1}^{n} (x_{ik} - \overline{x_i})^2 \cdot \sum_{k=1}^{n} (x_{jk} - \overline{x_j})^2 \right)^{1/2}}$$

(8)

where $\overline{x_i}$ and $\overline{x_j}$ are the average values. Another measure of similarity from the statistics realm is the Mahalanobis Distance. This distance, which is also sometimes called quadratic distance, is important because it can measure the distance between two groups of spaces and not just two individual spaces. This means that one can evaluate the distances between two different buildings or two areas within the same building by combining more than one space together. Mahalanobis Distance uses the mean values of the coordinates, $\overline{X_i}$ and $\overline{X_j}$ as well as the pooled sample covariance.
matrix of the two groups of spaces or buildings (Q) to calculate distance using,

\[ d_y = (\bar{X}_i - \bar{X}_j) \cdot Q \cdot (\bar{X}_i - \bar{X}_j) \]  

(9)

In addition to the above dimensions, a virtual-dimension can be added. Fifty years ago one would physically walk or drive somewhere to find out information. 20 years ago one would have made a phone call and requested a company to send a brochure, whereas today the same information can be downloaded from the Web. Consequently people spend more and more time on the Internet and therefore this could be started to be considered a space with all the architectural and space organization issues pertaining. This leads to the issue of how much are these measurement techniques influenced by what counts as "space"? In addition is there a difference between the physical distance and the human perception of distance? More importantly, how do these methods alter design or offer a new process of designing? While these are deep questions and answering them sufficiently would exceed the scope of this paper, applications of these measures of distance on real buildings can begin to shed some light.

3. Application To Examples From Classical Architecture

The measures of distance described above may shed light on the space syntax of architectural environments. Space syntax studies of spaces thus far involved the computation of the visibility polygon or isovist. An isovist is the space visible all around a particular viewpoint and then calculating various shape measures of this isovist of visibility polygon (Batty 2004). The isovist (Benedikt 1979) itself is determined by drawing rays from the viewpoint of interest at very small resolution at equal angular intervals. Where those rays hit an obstruction a point is created. The shape of the enclosing polygon for those rays is then analyzed to obtain particular shape measures. Such analysis has revealed important information about the space and its meaning. The idea of space syntax originated actually in architecture as well as planning; however space syntax application to architecture has been surprisingly ignored albeit a few research efforts (Hillier 2004). The various measures of distance may be used to answer questions about what impact different space design choices will have on the final architecture created. To a deeper extent one can start to perhaps address the inherit question of space syntax analysis, “what is the effect of the space itself on that activities that happen within it”.
In order to examine that effect, let’s consider an example from classical Islamic architecture, the Sultan Hassan Mosque and Madrassa. This is one of the extraordinarily examples of Islamic architecture located in Cairo, Egypt. In figure 1, the plan was modeled on a vector based CAD program and the centroids of all the spaces of the mosque have been connected. The final representation is a connectivity graph of the spaces in the mosque. The distance measures considered above are actually measures of proximity of each space with respect to every other in terms of different dimensions. In a building with n spaces one would have K number of connections where K = n(n-1)/2. This is in the case of a fully connected graph, i.e. when all the spaces are accessible from each other space. From the connectivity graph, one can see that the spaces in the Sultan Hassan mosque do not create a lot of loops and the spaces are compartmentalized. The simple Euclidean distances between the various spaces of the mosque can be calculated and values can be added to the arcs (links) in the graph. The space with the minimum average distance would clearly be the most central of them all. In that case, it would obviously be the court (The “Sahn”, or the court, of the Mosque is almost square, about 34m long and 32m wide) in the middle. In addition the simple Euclidean distance, an m-dimensional distance can be calculated for the graph in Figure 1. This would consider other dimensions of the space such as the social, or openness. When this 4-dimensional space
is considered, one would find that the spaces seem closer than they actually are. The spaces that are closer physically are also closer in terms of the other dimensions and design suddenly seems a lot more compact than in the raw plan (Figure 2). In contrast to the design of the Sultan Hassan is the Falling water by Frank Loyd Wright, where the idea was to create the perfect open plan (“the destruction of the box” as Wright himself would put).

Figure 2. The Connectivity Graph Imposed on the Floor Plan of the Sultan Hassan

Figure 3 shows a table of the simple pair wise distances between the spaces in the Falling Water. The distances make up the elements of the diagonal matrix. These are calculated from the centroids of the different spaces and a partial connectivity graph is shown in Figure 4. The shading represents bands of distances with the furthest distances being shaded the darkest. From this simple representation, one can start to see how the spaces are arranged and the distribution of the Euclidean distances between these spaces. For example, a table that has an almost uniform shade except for a few darker shades represents a plan that is spatially compact with a few spaces extending from the mass. On the other hand a table with values that are equally distributed among the various shaded would represent a spatial configuration similar to that of the Sultan Hassan Mosque described above.
The several other distances were also calculated and these are shown in Figure 5.

There are other kinds of distance that start to reveal more information about the spatial arrangement of the building. An important point to consider is how to unify the units of the different dimensions to maintain a consistent description of the architectural spaces considered. The Bray-Curtis solves this issue since all the distances are normalized. For example, the Bray-Curtis distance shows that when one considers all the dimensions of the space and the distances are normalized, the space appear actually closer than when considering the simple Euclidean distance. Furthermore, a close analysis reveals that the Minkowski distance of the spaces is insensitive to λ.

### Figure 3. Simple Euclidean Distance

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Figure 4. A partial Connectivity Graph for the Falling water

Figure 5. Other Distances

Part 1
Another way to represent the information is to plot a line weighted connectivity graph. In this graph, the weights of the lines connecting the various spaces will indicate the extent of the actual distance being presented. So even though two spaces may appear far physically, they may be closer in terms of the other measure of distance being represented. Another type of space analysis that space syntax considers is the dominant lines of visibility, which are considered to act as subtle cues subconsciously followed by walkers (Batty 2004, Conroy 2005). The correlation between the distances considered above the dominant lines of visibility can be another important aspect of research.

4. Conclusion

This paper demonstrates that distances between spaces cannot be measured solely in terms of simple Euclidean Distance, but instead other kind of distances (e.g. City block, Chebyshev, Minkowski, Canberra distance or Angular separation) can offer new meaningful analysis into space and its meaning. Several issues are raised in light of these new measures such as how to interrupt the results of these measurement techniques when they are applied to architectural spaces. Applications and analysis was applied to classical examples of architecture. A possible extension of this work is to applying schedule (graph) complexity measures on the spaces in a building as well as applying further analysis techniques such as Principal Coordinate Analysis.
References


