

THE USE OF SMART GEOMETRY IN ISLAMIC PATTERNS

Case Study: Mamluk Mosques

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Abstract. It is noted that architects need new and quick methods designing the historic architectural styles, as well as restoring the historical urban areas particularly the Islamic ones. These designs and restorations should adapt to the basics of the Islamic style used; general concept, module and features. Smart Geometry provides advanced design concepts and increases alternative variations. Parametric design softwares also add more rules and relations on the design process. Obviously, the Islamic module and proportions are used as design generators that result in extracting a number of alternatives easily in a little time. Generative Components (GC) is the parametric software used to achieve the desired objectives of this research.

1. Introduction

Geometry is an essential design generator in the Islamic architectural style. Islamic styles are distinct for using the art of geometry in their creative designs. In the Islamic designs, geometry represents the order, harmony and beauty in calculations, scale and proportion. It clearly exists in the design of plans, façade, ornaments and patterns. It expresses many concepts of Islam such as the unity and the oneness of Allah, the perfection and the infinity of creation in the universe, the containments and the continuity. The main proportions of the Islamic style depend on the square proportions (ex: Islamic proportion $1:\sqrt{2}$ (Williams, 2006), in which the square is the basic module shape that generates the other geometric forms such as the famous Islamic star, octagonal Islamic rose. In addition, square gives the basic axes and symmetry in the main internal spaces such as the Courtyard in the mosques. So far the role of geometry has been fundamentally dependent in which in it contains, regulates and supports the module of the elements.

Islamic geometry and proportions are following special shape grammar rules and relations which should be taken into consideration while the design process or the restoration of historical buildings. One can see the Islamic geometry simple in shape but precisely it is composed of a number of complex relations. And when the rules and the relations increase, the complexity in manipulating the geometry manually increases. That requires more time and effort to execute or modify. Therefore, parametric design strategies are employed to aid in solving this complexity. Parametric design develops the geometry to be more related and dependent. Consequently geometry is introduced in a new smart one.

In the traditional CAD softwares, if one of the related geometries is deleted or changed, the modeled relationship may be lost. However, Smart geometry, as well as the parametric design, can convert lines, arcs, shapes, solids, and surfaces into a set of algebraic and geometric constraints that could easily generate those patterns according to a specific relations and proportions. Thus, a number of alternatives are presented to use the most suitable solution in a short time.

2. The Aim of the Research paper

2.1. PROBLEM DEFINITION

Manipulating the rules and relations of the Islamic pattern manually can consume a lot of time. Therefore there is a need to relate the geometry of the Islamic pattern such that it becomes easier to be changed or modified. Mamluk style is the selected Islamic style to apply the parametric design on it; especially mosques in AlDarb AlAhmar, Cairo.

2.2. AIM

The main aim is using the parametric design to handle the forms of Mamluk architectural style to generate a number of alternatives that can help in designing new alternatives of the same prototype or in the restoration of the urban historic areas. The aim will be achieved by analyzing the basic theories and design constraints of Mamluk architectural style. Moreover, some case studies of the precedents examples of the Mamluk architecture should be analyzed. Synthesis of the parameters needed; $\sqrt{2}$ geometric relationship, design module, symmetry, proportions, and axis of the Qibla. And consequently, different variations of Mamluk style forms emerge by changing the previous parameters.

2.3. HYPOTHESIS

The researcher hypothesized that the implementation of the parametric design tools in Generative Components (GC) would lead to the creation of a number of alternatives of the Mamluk style. And that it could be applied on the Mamluk monuments found in AlDarb AlAhmar area.

The main hypothesis of this study is that the relations and rules that arise between the parameters of the Mamluk style will lead to generate a number of alternative that follow the style theories and its design guidelines in an efficient way.

3. Smart Geometry

Definitely, Smart geometry offers a rule-based geometry, in which geometry can be defined by a set of rules and relations. Smart geometry links geometric shapes with variables. It is associated with parametric environment in which geometry lines, arcs, circles, solids, surface can be related, transformed (Day 2005), embedding fixed or variable parameter (Araya 2006, Kilian 2006). Thus controlling and manipulating the geometry increase and consequently new controlled forms of the buildings increase as well. Moreover, Smart Geometry, or parametric design, provides advanced design concepts and increases alternative variations.

Parametric design confirms using the system that provides a set of relationships between objects that form a greater entity. A system binds rules and relations in the architecture design as well as the geometry to maintain the proportions and apply the basic theories of architectural style. Hence, the building is analyzed, decomposed to its basic dependent points that build up the final form. So that if a point is moved or deleted, this modification reflects on the other dependent points or geometries (that is called for part to part system), and also reflects on the behavior of the whole form as well (that is called for part to whole system). Any architectural style can be represented as a system of dependencies.

In the following section the paper will discuss the concept of the Mamluk architectural style according to the design drivers (generators), parameters (variables) and rules that control its geometry.

Generative Components (GC) is the implemented parametric platform that is compiled for the purposes of achieving the objectives of this paper. GC system is “a model-oriented end-user programming environment which combines direct interactive manipulation design methods based on feature modeling and constraints, with visual and traditional programming techniques. It turns user-defined geometric and behavioral design models into new classes (or types) of objects (or components).” In which a component can be as simple as a line (a geometric primitive) or as complex

as a double curves adaptive glazing panel arrayed over a complex building façade. A component can also be a numeric value (a single parameter) or a complex equation linking a number of parameters to the driving properties of geometric components (Aish 2003). All the modifications in Generative Components (GC) is shown by parallel representation of views in real time; modeling, scripting, transactions, symbolic view, law curves and even fabrication unfolding (as shown in Figure 1). Generative Components (GC) captures and graphically presents both design components and abstract relationships between them.

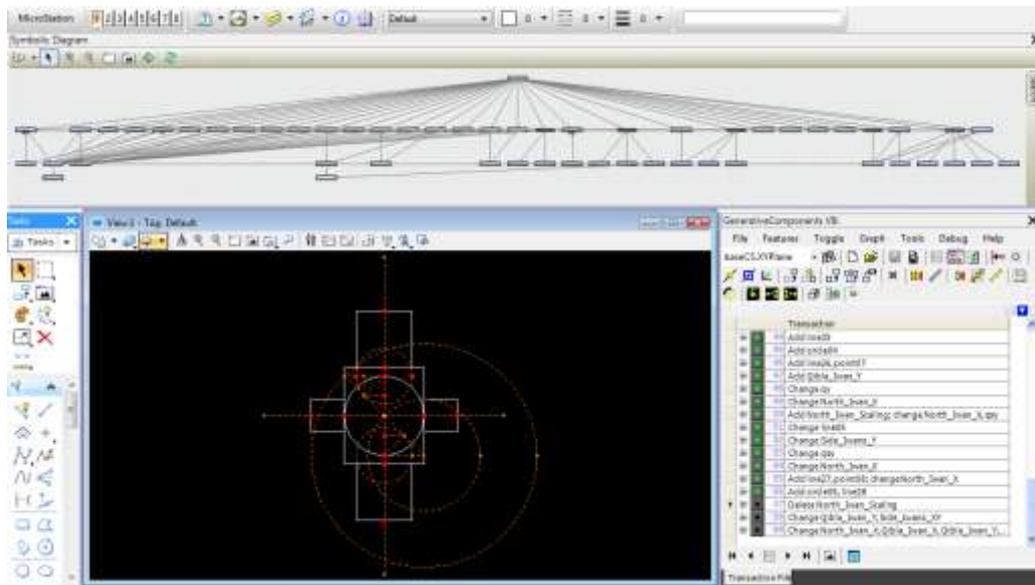


Figure 1. the interface of the Generative Components (Reseacher 2010).

Now architects can build the parametric system of rules and variables for any building prototype depending on its theories and design criteria of each case. This system can be edited afterwards by any end user.

4. Concept of Parametric Shape Grammar

According to Stiny (Stiny 1985) any architecture style has its own shape grammar. There are two principle kinds of shape grammar: standard grammar and parametric grammar. While in standard grammars most attributes of shapes are constant, in parametric grammars they tend to be more flexible and variable. Because of its flexibility, parametric grammar is more practical and popular than the standard one.

The designs of any style are analyzed by decomposing them into a vocabulary, which represents the lexical level, and rules, which represent the syntactic level of their structure. Spatial relations, or arrangements of vocabulary elements in space, are identified in terms of the decompositions of the original designs. As a result of the re-synthesis of the decomposed elements, the shape grammar generates the original designs and possible new designs that did not exist before. New designs are generated by restructuring new combinations of the vocabulary elements in accordance with the inferred spatial relations, or by re-assembling the same elements using different rules or sequences (Buthayna Eilouti, Amer Al-Jokhadar 2007).

“Parametric shape grammar allows the lengths of lines and the angles between lines to be varied in shapes. Values are assigned to the variables in these schemata to produce specific rule” Stiny, 1985.

Although shape grammars have good predictive, generative, derivative, and descriptive powers, they do not focus on the historical, social, functional, or symbolic aspects of the architectural compositions.

A set of rules are applied to a set of given vocabulary shapes in order to reproduce existing shapes and come up with emergent ones. Different types of shape grammars can be defined according to the constraints that are used in their applications. They vary according to the format of rules, variable parameters, constant proportions, and the order of rule applications. All components of shape grammars (vocabularies, spatial relationships, parameters, attributes, rules, transformations, and initial shapes) provide a foundation for a science of form-making and for a theory of systematic architectural design and composition methodology through algorithms that perform arithmetic calculations on geometric shapes.

In addition, parametric design introduces a fully organized controllable geometry and adapts it to specific circumstances. In which, parameters provide the relations between geometries and functions encoded as constants or variables values of its design either numerical or mathematical equations or formulas. Parametric design has three main concepts: **Parameters (Variables), Constraints, and rules**. Definitely, parameters (variables) could be independent and dependent. Constraints could be either dimensional or geometrical which act as a design driver and increase the dynamic variations. While the rules link the data inputs together, parameters and constraints. Parametric design can offer manipulations and explorations in real time simply by moving a slider bar of the parameters.

Stiny; 1985 proposed a framework to define a language of design, constructed by means of shape grammars. To set up a well-controlled relationship, it passes through four phases to define the shape grammar that generates a certain prototype (Colakoglu 2001) as shown in Figure 2:

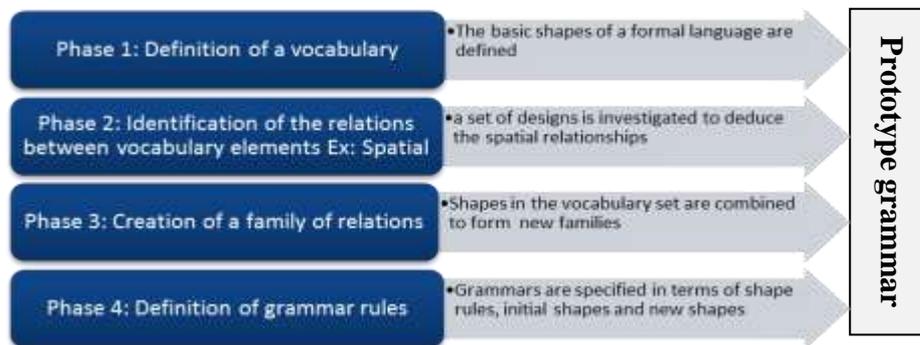


Figure 2. four phases is presented to define a shape grammar for a prototype (Colakoglu 2001).

Phase 1: This phase starts with the definition of the initial shapes. It is located by a certain shape grammar using suitable rule schemata. Then variable values are added to the shapes. This allows the lengths of lines and the angles to be varied in shapes. Values are assigned to the variables to produce specific rule schematas (Stiny 1985).

Phase 2: geometric relations and constraints are being built up to be analyzed from the previous case studies to define them into algebraic and geometric constraint that build the style form. Constraints are non-geometrical entities that limit and control the behavior of an attribute or group of attributes by putting restrictions on the possible parametric variations (Hernandez 2006). In addition, they represent a powerful way to drive solution space and focus the design (Kilian 2006). For example, geometry is constrained by length, angle, radius, tangency, parallelism, perpendicularity, concentric shapes, symmetry, while others might be specified by midpoint of a line. For example: If a line is constrained to be tangent to an arc, then changing the location of the arc it maintains the tangency automatically. These are called geometric constraints (as shown in Figure 3).

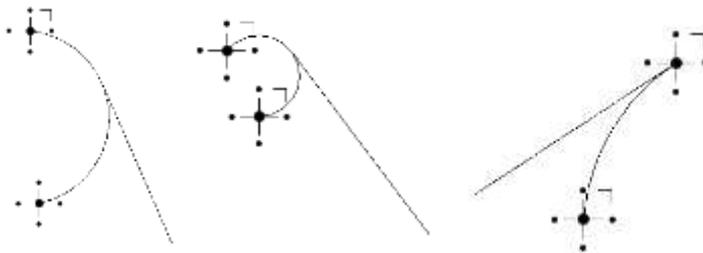


Figure 3 line is parametrically tangent to a curve (Abd Elsalam 2010)

Phase 3: Families of the initial shapes are created to derive the related rules. In this phase a common module between families should be defined (as shown in Figure 4).

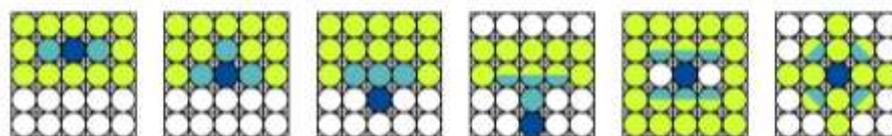


Figure 4 translating families of office building plan to the same datum (Abd Elsalam 2010)

Phase 4: Derive the suitable rule (algorithm) to link between parameters. The rules should be based on the conceptual as well as the formal components of a design, and on the relational, morphological, and topological associations among all components. (Buthayna Eilouti, Amer Al-Jokhadar 2007).

5. Mamluk Shape Grammar

5.1 MAMLUK CONCEPT GENERATORS

Mamluk architecture represents a significant period of Islamic architecture. It displays most of the aesthetic principles that underlie Islamic architecture. Mamluk architecture reflects designers strive to achieve harmony between people, their environment and Allah (Himmo 1995). In addition to the confidence derived from its military successes and is one of the most distinctive Islamic styles of building. The biggest factor affecting design and layout was the lack of space in an increasingly crowded area. Buildings were often built on an irregular-shaped plot because of the shortage of space. Many Mamluk buildings which seem to be square and symmetrical are built on irregular ground (Petersen 2002). Shapes in Islamic architecture are strongly related to the study of mathematics and other sciences. Compositions in the Islamic architecture have been transformed into highly abstracted shapes on which principles of rhythmic repetitions, unity, symmetry, and variation in scale were applied to create ordered yet dynamic effects.

However, when the great monuments and precedents of Islamic architecture are examined, their formal structure reveals a complex system of geometrical relationships, a well-designed hierarchy of space organizations and a highly sophisticated symbolic articulation of ornaments. In the Islamic perspective, the method of deriving all the organizational proportions of a building form from the harmonious recursive division of a basic shape is a symbolic way of expressing the oneness of God and his presence everywhere (Himmo 1995). Mamluk style can be mathematically

analyzed and syntactically systematized to formulate a powerful compositional language that may help in the understanding of the Mamluk style and the aesthetic principles of Islamic architecture.

The frame work in this paper represents the parametric design that links and relates a number of Mamluk architectural vocabularies in a set of relations. The parametric rules are being deduced. The deduced rule schemata depend on the basic theories, constraints and design guidelines of designing Mamluk style. Consequently a number of alternatives having the same style features are emerged.

5.2 VOCABULARY OF THE MAMLUK STYLE

Mamluk architects repeatedly used certain features in their buildings that shared some common attributes in size and type. Thus, close relationships can always be found among these buildings in their proportions, geometric shapes, topological relationships of spaces and formal compositional aspects.

5.2.1 *Plan vocabulary*

The most common formal prototype of the early Mosques is the four-Iwan plan. Obviously, the general case is the hypostyle plan takes a rectangular shape that consisted of a Courtyard (sahn) surrounded by a number of riwaqs in which the number of riwaqs at the north-eastern and south-western sides is larger than that on the north-western side. The prayer hall is the deepest hall and its riwaq ran parallel to the qibla wall except for three at the centre which were higher and arranged perpendicular to the mihrab. The main entrance is a cubic structure projecting out of the centre of the north-western wall, and two other gates, smaller in scale, project from the north-eastern and south-western sides of the mosque (Yeomans 2006).

The vocabulary of the Mamluk style refers to the main generators of the Mamluk style design principles; these include symmetry, unit-to-whole ratio, repetition, addition and subtraction, geometry and module structure, hierarchy, complexity, and orientation of parts. The major principles are briefly described as shown in Table 1:

Table 1 the main Principles of the mamluk style (Buthayna Eilouti, Amer Al-Jokhadar, 2007; Yeomans, 2006).

The concept	This concept was achieved through
Asymmetric balance	<ul style="list-style-type: none"> • The use of multiple axes of various directions to control all topological relationships of the vocabulary of Mamluk madrasa components. • central symmetry in which major axes meet in a focal point that is typically located at the center of the courtyard and controls the layout of the Iwans • Balance is achieved by geometry when a dominant form, balances a massive cuboid that is dominant by its size. • some levels of symmetry found in the organization of the internal layout • symmetry controls the organization of internal components of the floor plan rather than the overall exterior shape
Rhythm	<ul style="list-style-type: none"> • The number of riwaqs • The form of addition or division of a whole, or simply represented as a series of ratios. Elements of the plan were repeated by using the ratios of 1:1; or: $\sqrt{2} = 1:1.41$.
Orientation	<ul style="list-style-type: none"> • The major axis of orientation is dictated by the angle of Qibla direction (the prayer or Mecca direction). • Grouping elements which have the same function on the same orientation axis. • Respect the site layout and the street outline
Hierarchy	<ul style="list-style-type: none"> • Ordering primary components (such as the main courtyard) in the most central position and the secondary elements (such as cells around courtyard) in the less dominant locations.
Simplicity vs. Complexity	<ul style="list-style-type: none"> • all interior spaces are simply shaped • The overall layout has a complex form. This complexity refers to two features of the layout which prevent the fragmentation of the secondary components: <ol style="list-style-type: none"> 1. The change of angle was used as a planning technique to create a new axis of orientation to organize the minor vocabulary elements. While the primary components were directed to the Qibla direction, the secondary ones were made more dynamic by changing their directions to distinguish them visually. 2. The large central space acts as a reference point or datum line for all other plan components.
Regularity	<p>Square ratio Most the spaces use the square proportions because it gives stability, purity, symmetry, justice and rationality</p>
Portal emphasis	<p>portal space has been emphasized by a minaret above it</p>

Understanding the morphology and the design process of individual Mamluk buildings may enhance the explanation of town and city growth in Islamic urban areas such as AlDarb AlAhmar. By projecting component structure and relations in a Mamluk building on various buildings and their interrelationships in town planning, new layers of interpretation can be added. For example, the previous characteristics such as space hierarchy and angle manipulation can be observed as planning techniques in some Islamic cities.

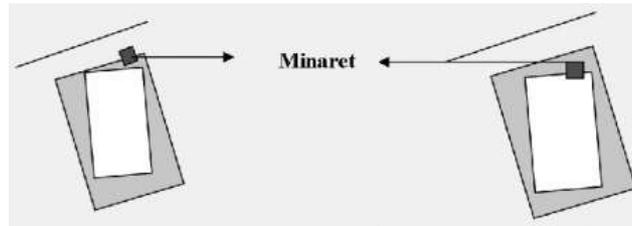


Figure 5 A minaret that is: a) aligned with the street, the rest follows the angle of Mecca orientation, b) aligned with Mecca orientation (Buthayna Eilouti, Amer Al-Jokhadar, 2007)

5.2.2 Façade vocabulary

- The geometric features and proportions of the great and small arches of building's Sahns (central courts of madrasa) are similar;
- The position of Iwans may be identified by the fenestrations of the façades of the madrasa without actually entering the building;
- Externally, fenestrations were set back from the wall planes in recesses. The dimensions and shapes of the openings and recesses reflect the position and size of the hidden Iwan and provide the details necessary to identify the Qibla (Mecca orientation) Iwan;
- The façade treatment exhibits skillful monumentality scale and articulation of openings and detailing of ornaments;
- The horizontal stripes found on the façade to give an illusion of the square proportions. Although the proportions are rectangular.

5.3. THE PARAMETERS

Within this language, architectural vocabulary elements along with spatial organization rules and aesthetic principles of the components and their compositions are defined. Multiple aspects of the formal language can be parameterized. Some common vocabularies, in the following case studies, are being selected to be applied in the parametric design. There are three main parameters are used in the paper application to generate the Mamluk style; the Qibla Axis, the courtyard area and proportion, the iwans that change in size according to courtyard proportion.

5.4. RELATIONSHIP BETWEEN SPACES

Arab mathematicians continued to move forward understanding the Basic concepts of geometry, developed by the earlier mathematicians. They implement mathematics into geometry. Geometric transformations, projections, an area relating to determining the direction of the Qibla, a particular interest of Al-Biruni around the turn of the eleventh century, spherical geometry and, from this, latitude, longitude and, of course, the sciences relating to the astrolabe. Related to this is the development of geometries based on the diagonal of a **square** $\sqrt{2}$ where the side is 1 unit rather than on the Golden Section ¹(1:1.414 compared with 1:1.618) (Lockerbie 2010) as shown in Figure 6.

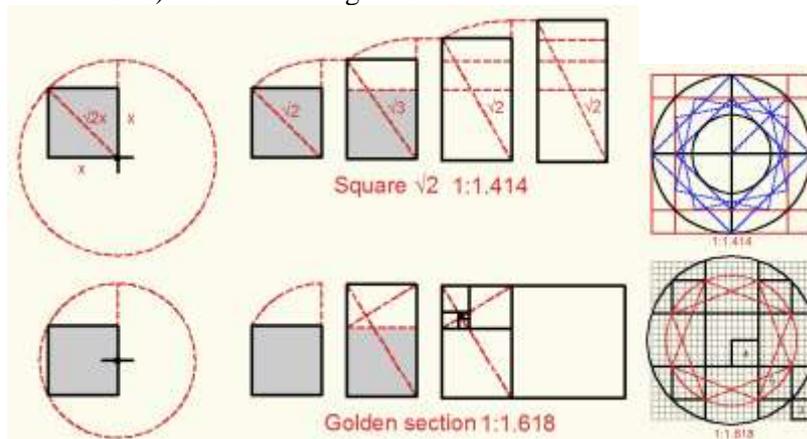


Figure 6 shows the difference between the $\sqrt{2}$ relation and golden section (Reseacher 2010)

Proportions of a square generate octagon and hexadecagon. Proportions of an equilateral triangle generate hexagon and dodecagon. Right triangles (isosceles right triangle and 30-60-90° triangle) are used for the rules of symmetry and proportion (Uluengin 2000) as shown in Figure 7.

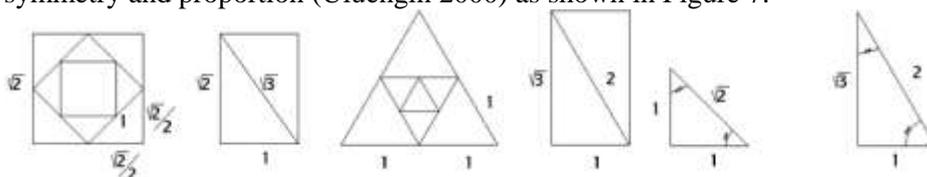


Figure 7 geometrical relationships used mostly in islamic architecture (Uluengin 2000)

Mamluk architects follow these mathematical relations and apply it on the Islamic patterns used in architecture and arts. The Mamluk mosques depend on the mathematical relations and ratios that govern the main space

¹ Golden section is a relationship applied by Fibonacci in which it equals approximately 1.61803

in the mosque which is the courtyard geometry. Consequently, the other spaces; iwans and Qibla iwan follow the proportions of the courtyard.

A set of four Mamluk Mosques in AlDarb AlAhmar in Cairo are being examined. Moreover, the case studies are being analyzed according to the geometrical relationships of the Islamic culture ($\sqrt{2}$ relationship). However, there is no clear reference for the geometric relations in the mosques. It is the researcher personal trial to apply the general Islamic rules for geometry on these plans. The case studies include Alsultan Hassan Complex, Alsultan Qalwoon Complex, Amir Khayrbek Complex and the complex of Um Elsultan Shaaban as shown in Figure 8.

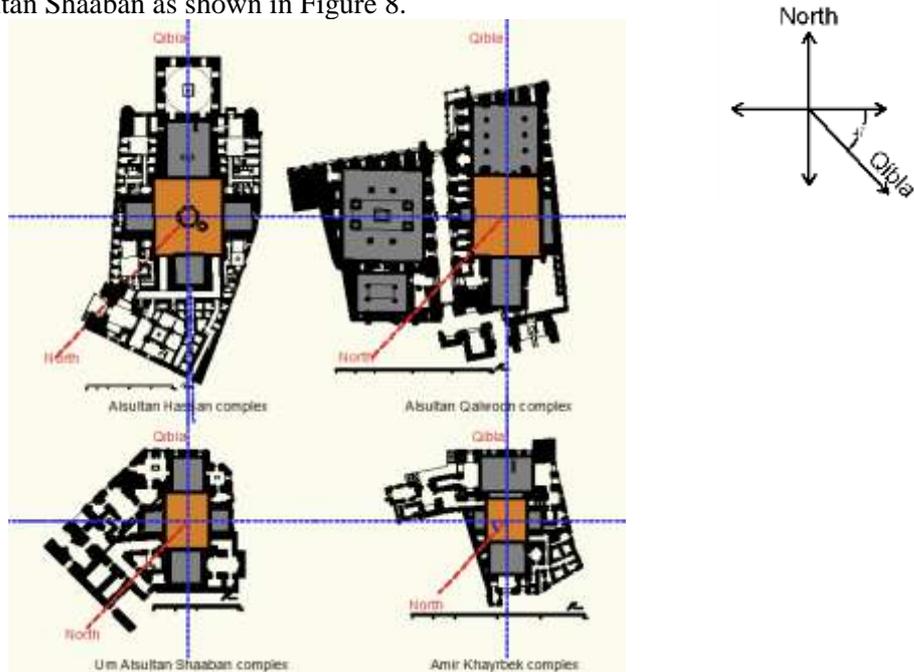


Figure 8 the four mosque and indicating the courtyard and the four surrounding iwans (Researcher 2010), source: (www.archnet.org 2010)

The following table describes the vocabulary and of the four case studies and the proportions of each vocabulary as shown in Table 2.

It is deduced from the table that there are some common Proportions between the four case studies. However, they vary in between a specific range. These values of this range will be translated into a slider bar in the application.

Obviously, the court is almost square shaped; and it tends to be longer in the Qibla direction by 0.1 or 0.2 the length of the court side.

Table 2 Vocabulary and proportions of the four case studies (Researcher 2010)

	Alsultan Hassan	Alsultan Qalwoon	Um Elsultan Shaaban	Amir Khayrbek
Total Area (m2)	8000	1810 Mosque only	1164	887.5
Courtyard (sahn)				
• (Area:Total area)	1:7	1:4.95	1:8.25	1:9
• (Area: Area of iwans spine)	1:2	1:2.3	1:1.8	1:3
• (Length:Width)	1:1.11	1:1.2	1:1.27	1:1
• Radius of the court (min Width)	16	8.85	5	4.8
Side iwans (Area: Court area)	1:5.7	1:13.5	1:5.74	1:6
North Iwan	1:5.7	1:3.38	1:2.36	1:1.5
Qibla iwan (Area: Court area)	1:2	1:1.14	1:2.4	1:1
(Length:Width)	1:1.26	1:1	1:1.1	1:1.4

5.5. FAMILIES OF THE MAMLUK MOSQUES

The families of the Mamluk mosques vary in shapes; however they respect the $\sqrt{2}$ relationship. Since the families are being unified according to a specific relationship, deducing the common general rules become easier as shown in Figure 9.

6. The Application

This application represents a sample of the design of Mamluk architecture that is applied by parametric design. The paper focuses on the rules used to define a style that is based on $\sqrt{2}$ relationship. The application steps are implemented by the Generative Components (GC) platform in modeling and scripting to generate different design variations. The derived Rules represent the constraints of the courtyard proportions, the Qibla iwan proportions with respect to the courtyard and the other iwans proportions. Design generators have been mentioned in the previous section. The following part explains the parameters and the rules implemented in the application.

6.1. PARAMETERS

X, Y signifies the origin point which is usually the center of courtyard. These proportional and locational values of shape are translated into parametric relations to derive the suitable rules. **Y Axis** act as the Qibla direction, such that it is away from the north direction by an angle equals 47 degree. Courtyard Area is calculated from the total area of the layout according to the ratios mentioned in table 2.

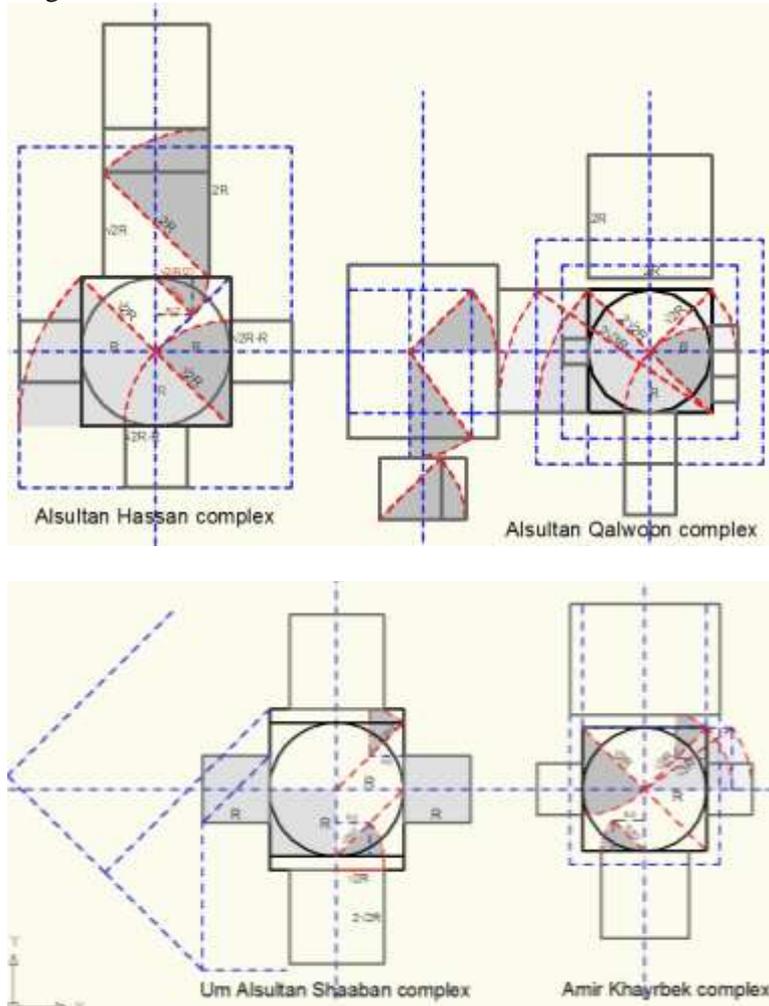


Figure 9 shows the $\sqrt{2}$ relationship in the four case studies (Reseacher 2010)

Firstly, a main parameter is defined to be the base of all the other parameters. And it is concluded from the previous analysis that the radius of the circle of the central **courtyard** is the main parameter. This variable is named by (**Radius_court**). The architect can build his relations of the mosque after determining the radius of the main court. There is two main steps should be done manually before deducing the radius of the courtyard.

- The ratio of the courtyard area to the total area of the layout of the mosque according to the ratios mentioned in the previous table
- Deducing the side length of the square. Then dividing it by 2 to give the radius of the circle.

The courtyard can be scaled in the Qibla direction as found in some examples of the Mamluk style. It is optional to add the scaling factor (**Scaling_court**), for example (0.11, 0.2, and 0.27), to the (**Radius_court**).

Iwans:

- **Qibla Iwan** is the largest iwan in which its shape is either square shaped or stretched in the Qibla direction (Y direction) by a ratio $\sqrt{2}$ of a half of its side.
- Side iwans are symmetrical squared shape in most examples; the Y-direction side length varies according to its relation with the (**Radius_court**). It sometimes equals to half (**Radius_court**) or equal $2\sqrt{2} * (\text{Radius_court})$.
- North iwan is square shaped and it ranges between half (**Radius_court**) and $\sqrt{2} (\text{Radius_court})$ as.

The researcher has synthesized the parameters analyzed in the application in which each parameter depends on a number of other parameters as shown in Figure 10. Finally, seven parameters are derived to control the parametric model of the basic concept of organizing the internal spaces of the Mamluk mosque. The parameters are classified into the Courtyard proportions in the X and Y direction, and the four iwans proportions. Table 3 represents the name of the parameter used in the scripting, the function of each parameter and the range of its value. Given these seven parameters as inputs, a group of relationships increase flexibility to change the number of generated variations.

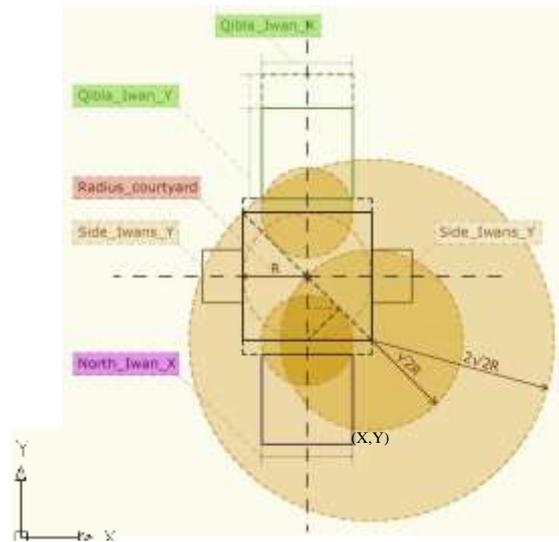


Figure 10 The general parametric model that is generated in GC (Reseacher 2010)

Table 3 shows the parameters, its definition and its value range

Parameter	Definition	Value
North_Iwan_X	The North Iwan side length in the x- axis direction	$\text{Sqrt}(2)*\text{Radius_courtyard}/2$ Or $\text{Sqrt}(2)*\text{Radius_courtyard}-\text{Radius_courtyard}$
Qibla_Iwan_X	The Qibla Iwan side length in the x- axis direction.	$\text{Sqrt}(2)*(\text{Radius_courtyard}/2)$ Or Radius_courtyard
Qibla_Iwan_Y	The Qibla Iwan side length in the Y- axis direction.	$2*\text{Radius_courtyard}$ Or $\text{Sqrt}(2)*\text{Radius_courtyard}$
Radius_courtyard	The radius of the central court (in which all parameters depend on it.	0 to 20 by 0.1 step
Scaling_courtyard	The increase of the courtyard in Y- direction	0.11 to .33 by 0.11 step
Side_Iwans_XY	The factor of the increase in X-direction of side iwan.	1 to 2 by 0.5 step
Side_Iwans_Y	Side iwan length in Y-dir.	$(\text{Sqrt}(2)*\text{Radius_courtyard})-\text{Radius_courtyard}$ Or $\text{Radius_courtyard}/2$

The parameters are linked together by suitable rules and represented in a slider bar interface. Slider bars dynamically generate and update different combination of proportion by varying dimensions (as shown in Figure 11). The slider bar is used as alternatives generator in the application of this

research later on. The interface could be easily edited by an end user of the application as well as the programming designer.

Name	Value	Analog Value
North_Iwan_X	11.737...	<input type="text"/>
Qibla_Iwan_X	11.738	<input type="text"/>
Qibla_Iwan_Y	23.476	<input type="text"/>
Radius_courtyard	16.6	<input type="text"/>
Scaling_courtyard	0.22	<input type="text"/>
Side_Iwans_XY	2	<input type="text"/>
Side_Iwans_Y	6.876	<input type="text"/>

Figure 11 the interface used to control the application case study (Reseacher 2010)

Rule schemata

In order to reconstruct a floor plan of a Mamluk madrasa, Four main rules are need to be defined. The derived rule schemata are based as follows:

- a. **1st rule:** calculates the total layout and specify Qibla direction as Y-direction.
- b. **2nd rule:** The court is determined as an inscribed circle into its square shaped in which the radius f the circle equals to half its side length.

feature Radius_courtyard Bentley.GC.GraphVariable

```
{LimitValueToRange= true; RangeMaximum = 20.0; RangeStepSize = 0.1;}
```

feature Scaling_courtyard Bentley.GC.GraphVariable

```
{ LimitValueToRange = true; RangeMaximum= 0.33; RangeStepSize= 0.11;}
```

- c. **3rd rule:** the Qibla iwan

feature Qibla_Iwan_X Bentley.GC.GraphVariable

```
{ LimitValueToRange = true; RangeMaximum = Radius_courtyard; RangeMinimum = Sqrt(2)*(Radius_courtyard/2);}
```

feature Qibla_Iwan_Y Bentley.GC.GraphVariable

```
{LimitValueToRange = true; RangeMaximum = 2*Radius_courtyard; RangeMinimum = Sqrt(2)*Radius_courtyard;}
```

- a. **4th rule** : side iwans:

feature Side_Iwans_Y Bentley.GC.GraphVariable

```
{LimitValueToRange= true; RangeMaximum = Radius_courtyard/2; RangeMinimum = (Sqrt(2)*Radius_courtyard)- Radius_courtyard;}
```

feature Side_Iwans_XY Bentley.GC.GraphVariable

```
{LimitValueToRange = true; RangeMaximum = 2; RangeMinimum= 1; RangeStepSize = .5; }
```

feature Side_Iwans_X Bentley.GC.Point

```
{DistanceFromOrigin= Side_Iwans_Y*Side_Iwans_XY;}
```

feature North_Iwan_X Bentley.GC.GraphVariable

```
{LimitValueToRange = true;RangeMaximum =
Sqrt(2)*Radius_courtyard/2;
RangeMinimum =(Sqrt(2)*Radius_courtyard)-Radius_courtyard;}
```

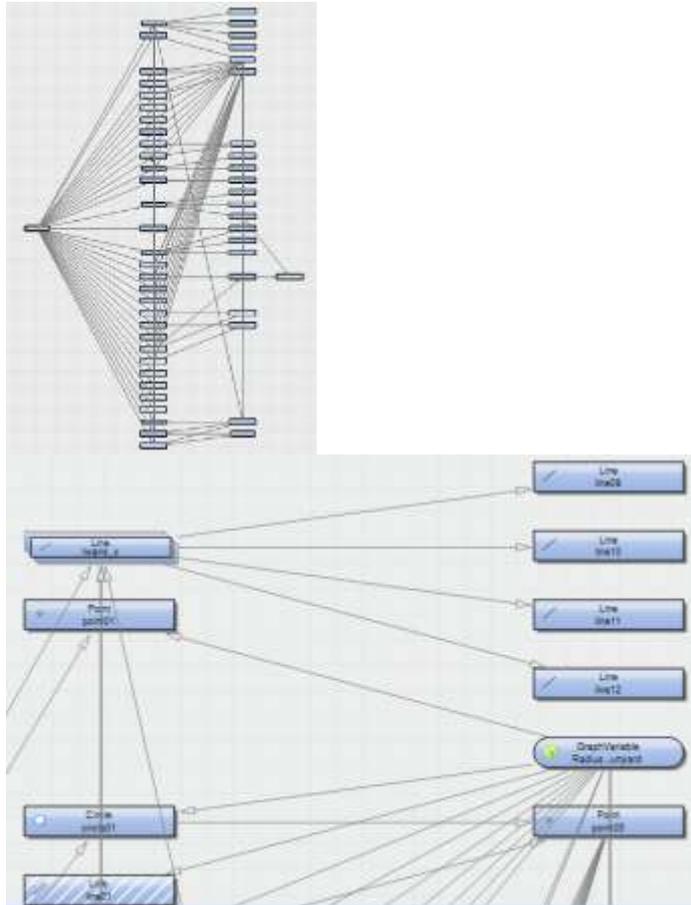
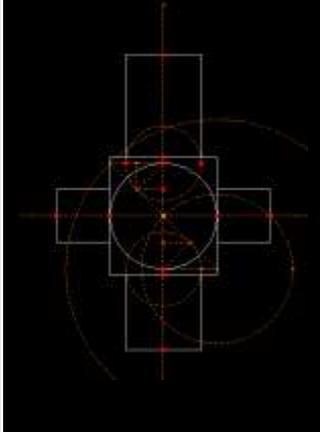
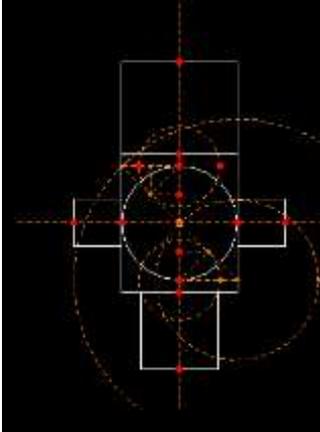
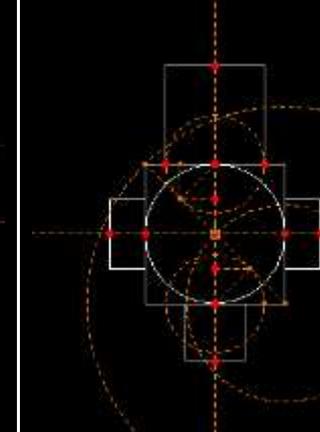


Figure 12 A sample of the relationship tree in Generative Components software (Reseacher 2010)

Alternatives generations

Moving slider bars results in a number of variations as shown in the Table 4 by moving slider bar to the selected radius of the courtyard and then change the other iwans according to the previous rules.

Table 4 Different generated variations of Mamluki mosque (Reseacher 2010)

Generate model																																																			
	<table border="1"> <thead> <tr> <th>Name</th> <th>Value</th> </tr> </thead> <tbody> <tr> <td>North_Iwan_X</td> <td>11.737972...</td> </tr> <tr> <td>Qibla_Iwan_X</td> <td>11.737972...</td> </tr> <tr> <td>Qibla_Iwan_Y</td> <td>32.227594...</td> </tr> <tr> <td>Radius_courtyard</td> <td>16.6</td> </tr> <tr> <td>Scaling_courtyard</td> <td>0.11</td> </tr> <tr> <td>Side_Iwans_XY</td> <td>2</td> </tr> <tr> <td>Side_Iwans_Y</td> <td>8.2857594...</td> </tr> </tbody> </table>	Name	Value	North_Iwan_X	11.737972...	Qibla_Iwan_X	11.737972...	Qibla_Iwan_Y	32.227594...	Radius_courtyard	16.6	Scaling_courtyard	0.11	Side_Iwans_XY	2	Side_Iwans_Y	8.2857594...	<table border="1"> <thead> <tr> <th>Name</th> <th>Value</th> </tr> </thead> <tbody> <tr> <td>North_Iwan_X</td> <td>7.7947313...</td> </tr> <tr> <td>Qibla_Iwan_X</td> <td>13.490678...</td> </tr> <tr> <td>Qibla_Iwan_Y</td> <td>18.783170...</td> </tr> <tr> <td>Radius_courtyard</td> <td>11.6</td> </tr> <tr> <td>Scaling_courtyard</td> <td>0.22</td> </tr> <tr> <td>Side_Iwans_XY</td> <td>2</td> </tr> <tr> <td>Side_Iwans_Y</td> <td>4.8048773...</td> </tr> </tbody> </table>	Name	Value	North_Iwan_X	7.7947313...	Qibla_Iwan_X	13.490678...	Qibla_Iwan_Y	18.783170...	Radius_courtyard	11.6	Scaling_courtyard	0.22	Side_Iwans_XY	2	Side_Iwans_Y	4.8048773...	<table border="1"> <thead> <tr> <th>Name</th> <th>Value</th> </tr> </thead> <tbody> <tr> <td>North_Iwan_X</td> <td>3.0651803...</td> </tr> <tr> <td>Qibla_Iwan_X</td> <td>5.2325901...</td> </tr> <tr> <td>Qibla_Iwan_Y</td> <td>10.465180...</td> </tr> <tr> <td>Radius_courtyard</td> <td>7.4</td> </tr> <tr> <td>Scaling_courtyard</td> <td>0</td> </tr> <tr> <td>Side_Iwans_XY</td> <td>1</td> </tr> <tr> <td>Side_Iwans_Y</td> <td>4.4922792...</td> </tr> </tbody> </table>	Name	Value	North_Iwan_X	3.0651803...	Qibla_Iwan_X	5.2325901...	Qibla_Iwan_Y	10.465180...	Radius_courtyard	7.4	Scaling_courtyard	0	Side_Iwans_XY	1	Side_Iwans_Y	4.4922792...
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Conclusion

Parametric design facilitates the exploration of the design process and produces morphology analysis to better understand the beauty underlying its design. The developed grammar with its systemized components (vocabularies, mathematical and topological relations, rules, and initial shapes) enhances knowledge of design process through a rule-based framework that performs relations, parameters, and combination, as well as set operations of shapes and through connecting style attributes with principles of visual composition. It depends on the basic concept of Islamic geometry which is $\sqrt{2}$ relationship and accordingly specifies the Mamluk mosque components according to its relation with the central courtyard of the mosque.

In order to develop the parametric shape grammar, this research adopted two main methods: the case study analysis and the applicable rules in parametric software.

The generated parametric module is fully controlled by slider bars and allows exploration new patterns in the design of beautiful Mamluk buildings.

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