COMPUTATIONAL INTERPRETATIONS OF 2D MUQARNAS PROJECTIONS IN 3D FORM FINDING

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Abstract. In the scope of this study, we developed an algorithm to generate new 3D geometry (interpretation) of a given or generated planar projection of a muqarnas in a digital 3D modelling software (Rhinoceros), its visual scripting environment (VSE) Grasshopper and also the Python programming language. Differing from traditional methods, asymmetrical form alternatives are examined. In other words, 2D projections of muqarnas were only used as an initial geometrical pattern for generative form finding explorations. This study can be considered an attempt to explore new relations, rules and vocabulary through algorithmic form finding experiments derived from 2D muqarnas projections.

1. Introduction

As an architectural element, it is possible to come across Muqarnas in a wide geographical zone from Spain to India. There are different opinions about the origin and development of it, tracing back to the 4th or 10th century (Dold-Samplonius, 1992). One of the earliest, major and written resources on muqarnas is Al-Kashi's "Key of Arithmetics" book, which covers arithmetic and geometric definitions of muqarnas construction for artisans/ master masons (Al-Kashi, 1977 [1472]). Harb's (1978) study provided another basic foundation for researchers in this field. The focus of the related studies increased from early 1990s and can be read in three decade-based directions: 1990s as descriptive- and typology-based approaches, 2000s as geometrical decoding and 2010s as exploratory computational studies.

The common denominator of most of the studies on muqarnas is its decorative potential. A few recent studies examined the structural, functional and performative potentials (Hensel, 2008; Abbasy-Asbagh, 2013) and algorithmic reconstructions (Yagdan, 2000; Harmsen, 2006) of muqarnas. Another common tendency in muqarnas studies is using pre-redefined typology and definitions. However, 3D muqarnas formation might provide new insights to the mathematical understanding of form beyond its aesthetic and ornamental qualities in the digital age. In other words, muqarnas formation has potentials to be decoded by a series of rules, algorithms or topological relations instead of only considering the pre-defined components.

In traditional construction practice, artisans/master masons have been gaining the practical knowledge by constructing on site or from definitions in manuscripts. There were constraints derived from the material selections, construction methods and building on site. The 2D patterns were designed firstly and then used as a base for 3D constructions. A collection of these 2D drawings can be found in Notkin's (1995) study. Dold-Samplonius (1992) also mentions the usage of predefined elements and their assembly. These constraints and reasons might have affected the emergence of limited amounts of novelty in muqarnas formations. We acknowledge that in the scope of this paper we neglect the material usage and the technique in historical muqarnas construction.

2. Related Studies on Muqarnas

It is not possible to approach studies on muqarnas in separated periods with concrete properties. Keeping this in mind, in order to gain a comprehensive understanding, our goal is to investigate the related studies in three decade-based periods: Description (1985–1995), Decodification (1995–2005) and Interpretation (2005–2015). A property, method or approach that we introduce might be seen in all three periods; however, we made assumptions based on the distinction of the characteristics.

In the first period, manuscripts of Al-Kashi and his calculations were studied (Özdural, 1990; 1991). These calculations and definitions constituted a basis for the further study (Harmsen, 2006). Some 2D (Figure 1A) and 3D (Figure 1B) geometric relations by Al-Kashi were introduced by Özdural (1990). Those typological definitions are helpful for explaining the production process in detail. On the other hand, the aspects of design and compositions were neglected in Al-Kashi's concrete descriptions (Özdural, 1990).

The construction process of muqarnas has usually been started by drawing the plane projections (Harmsen, 2006). Plane projections of muqarnas can be considered as simple un-interlocking patterns. In these cases, the arrangements of muqarnas elements were mostly symmetrical and the niches were also equal.

TABLE 1. Decade-based thresholds of muqarnas studies.

periods	approaches	dominant characteristics	studies
1985-1995	-Descriptive Approach	-Attempt to understand through measuring - Al-Kashi calculations on the muqarnas became popular	Tabbaa (1985); Özdural (1990); Özdural (1991); Dold-Samplonius(1992); Notkin (1995)
1995-2005	- Component-based modulation	- Decoding through pre-defined elements - Cells and filling elements - 3D interpretations from 2D patterns	Yaghan (2000); Yaghan (2003); Dold-Samplonius and Harmsen (2005);
2005-2015	- Panelling - Layering - Diagramming	- Graph-based representation; - Decoding by algorithms; - Exploring performative and structural potentials.	Harmsen (2006); Hensel (2008); Abbasy-Asbagh(2013);

Another categorization by Al-Kashi, which has been used in many muqarnas studies, is the assumption of "cell" and "intermediate elements". The upper part of each cell involves "roofs", vertical surfaces involve "facets" and the filling elements complement the compositions (Occhinegro, 2016).

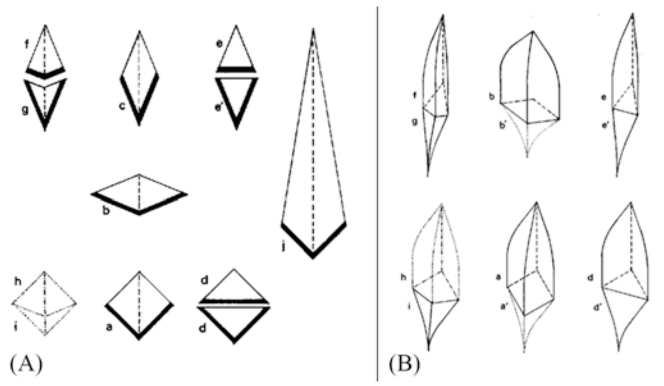

Figure 1. (A) Ceiling elements drawn according to Al-Kashi's decriptions;
(B) 3D elements of muqarnas (Özdural, 1990, pp. 38–40).

Dold-Samplonius and Harmsen's (2005) study can be considered as a 3D interpretation of the second period. They examined how different combinations of cell elements might constitute a row from the pre-defined elements such as square, intermediate half rhombus, intermediate biped, rhombus.

3D interpretation samples of muqarnas can be seen in Figure 2.

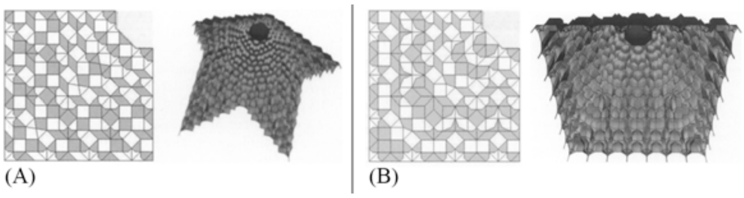

Figure 2. Redrawing and interpretation of muqarnas based on the plate found by Harb (Dold-Samplonius and Harmsen, 2005: p. 91–92).

In relation with the muqarnas codification, some categories such as scale, level of complexity, cornice or vault, the place that muqarnas settle (square-base/central column/eight arch), form of the niche were used by Notkin (1995), and point based-line base assumption, number of the layers and elaboration tools were studied by Yaghan (2003).

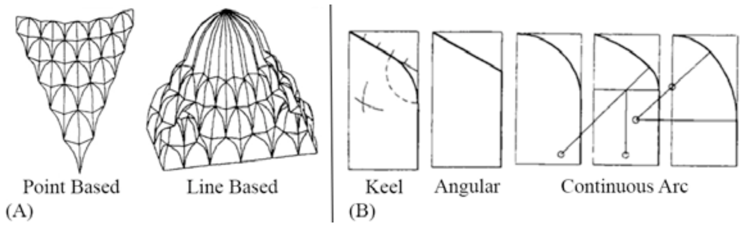

Figure 3. (A) Base type of a muqarnas; (B) Elaboration Tools (Yaghan, 2003: p.75).

3. Investigating Computational Potentials of Muqarnas

In this section we introduce our explorations on muqarnas in three parts. The first part focuses on the digital and physical modelling and encoding exercises; the second and the third parts focus on exploration of 3D form-finding algorithms, which have been derived from outcomes of previous exercises.

3.1. ENCODING MUQARNAS – MODEL-BASED EXPLORATIONS

Further to investigating the studies in the literature, we exercised with physical models. We modelled a scale-free line-based muqarnas located in Friday Mosque, Isfahan. The model starts with an octagon frame. Operations such as selection of midpoints, translations in Z direction, adding new points and connecting the vertices were used. Translation of the 3D model's information into a computer environment involved some reductions and interpretations (Figure 4).

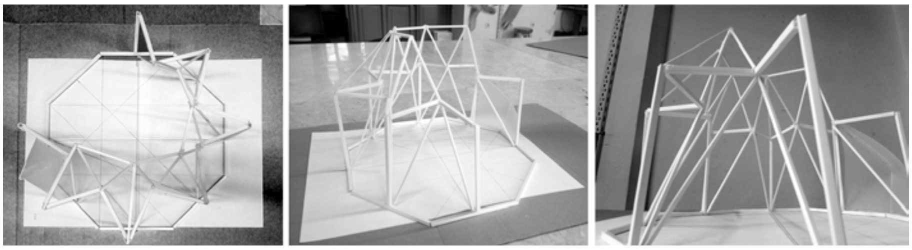

Figure 4. Contour model of a muqarnas of Friday Mosque, Isfahan.

In the translation process from physical model into digital we focused on defining parameters of a muqarnas unit (Figure 5). Instead of ‘keel’ with proper proportions, we used ‘angular’ and/or ‘continuous arc’ (See Figure 3B for the terms).

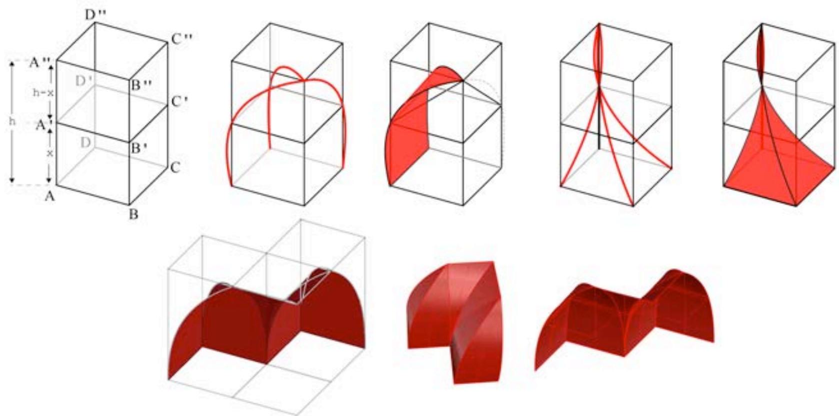

Figure 5. Modelling exercise of a muqarnas unit.

Apart from the representation of an assumptional muqarnas unit in a digital environment, the rotational and linear compositions were examined. The compositions shown in Figure 6 starts with one square. With 45 degree rotation the second square is constructed. Extracting the octagon from the two intersecting squares, adding circles to the sides of the octagon and extracting new intersection points, the following steps can be seen in Figure 6 in detail. This production was constructed manually in combination of 3D modelling and VSE. To achieve a laser cut model, the digital model was unfolded by using Pepakura interface (Figure 6).

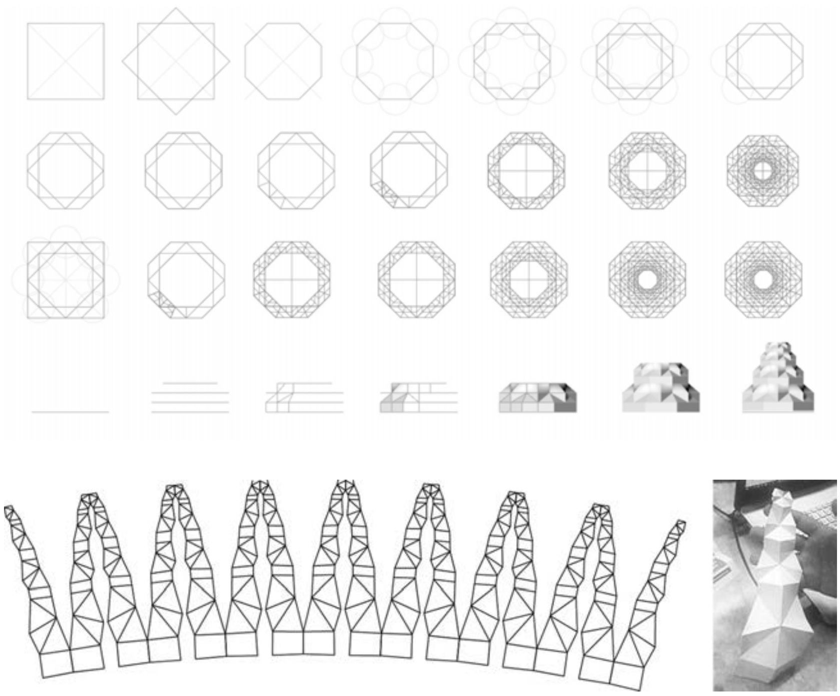

Figure 6. Modeling steps and fabrication process of a muqarnas composition.

3.2. ADAPTIBLE PATTERN MAPPING AND 3D PANELLIZATION

This approach both involves top-down (starting from a 3D surface geometry) and bottom-up (generating a unit pattern) processes. A sample generation process, in which the output was sent to the Pepakura interface and unfolded, is shown in Figure 7. This paneling method can be applied in different surface geometries. This logic of the penalization generates new sub-surfaces including new peak points, hill and valleys.

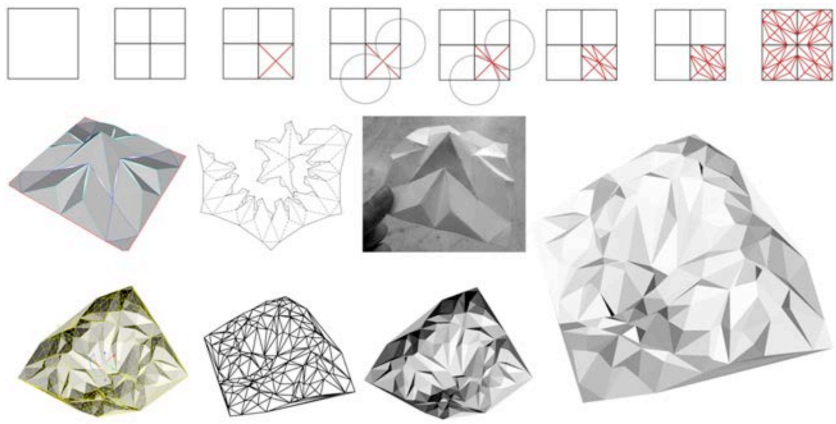

Figure 7. Mapping 2D pattern onto 3D surface.

3.3. INTERACTIVE LAYERING APPROACH BASED ON USER INTERACTION AND THE INITIAL 2D GEOMETRY

This process consists of a generation queue in which there are interactive modules taking input from users. The algorithm takes a 2D plane projection pattern as initial input. Here the critical intervention in form generation is to select points for layer lines. Here the term layer refers to contour lines in the 3D muqarnas. Height parameters for the layers can be dynamically changed from VSE. Further, the python module makes calculations for rationalization of the geometrical data and triangulates the surfaces. This algorithm can be applied to different surfaces however the boundary conditions and exception has not yet been tested.

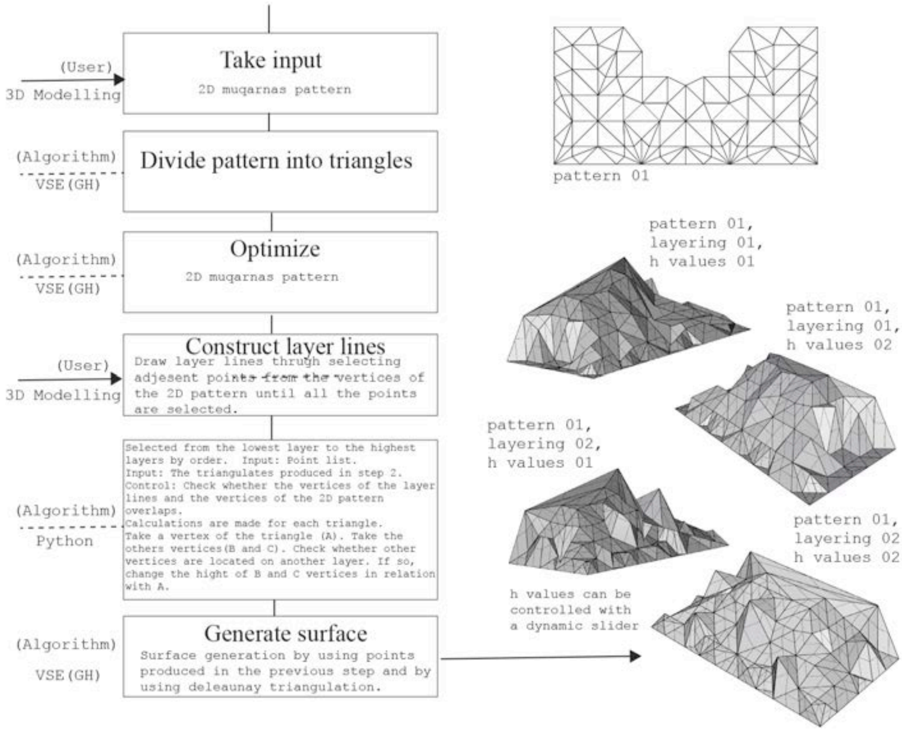

Figure 8. Algorithm schema of the 3D form generation process and samples of 3D surface geometries produced from the same 2D pattern but different layering method.

4. Concluding Remarks

Since the very beginning of its emergence, muqarnas construction had been carrying the potentials to be codified with mathematics, geometry and algorithms. Al-Kashi's (1977 [1427]) manuscripts provide core knowledge for artisans, introducing spatial and geometric relations (angles, dimensions, adjacency), rules (mathematical definitions, construction rule/order), vocabulary elements (units, cells, roofs, filling elements).

Those earliest assumptions and definitions have the potential to be encoded by algorithms. It is possible to claim that the traditional ways of constructing and the constraints derived from material properties resulted in predictable constructions and the novelty in spatial solutions or complexity of the muqarnas geometry have been limited. However, the definition of novelty is still contestable and reading a historical construction technique with today's conception, paradigms and point of view might run the risk of leading to a superficial understanding. Instead, we aim to unfold the tacit potentials of muqarnas as a foundation for today's design environment. The dominant characteristic of studies between 1995 and 2005 was approaching muqarnas with an analytic and a descriptive perspective. Component-based categorization, deterministic models and defining the parameters of the muqarnas with concrete hierarchies became common denominator for this period. After 2005, the widespread availability of digital modelling and fabrication techniques triggered the emergence of new perspectives on muqarnas (Hensel, 2008; Abbasy-Asbagh, 2013). On the other hand, we claim that the emerging digital design approaches might contribute more to exploring the unvisited potentials of muqarnas through stochastic models, topological relations, algorithmic definitions, generative design approaches, material studies or structural performance experiments. In this sense, this study can be considered as an empirical attempt to understand and interpret muqarnas through algorithmic form-finding experiments with the aim of providing a layout for the further systematic studies.

We observed that unit-based assumptions resulted in similar outcomes both in a physical and digital environment. In addition to these, the hands-on modelling and fabrication with laser cutter provided a better understanding on the geometrical organisation before/during preparation of the algorithms. The form exercises in which visual and verbal scripts were used together generated diverse results. Despite the developed algorithms being defined and closed, the selection of the initial pattern and the user decisions during the layer definition enriched the outputs with unexpected results. Moreover, the study in which only VSE was used, generated a kind of paneling method, which can be applicable to both planar and non-planar surfaces.

In our approach, it is contestable whether neglecting the historical context and the material information would still be called muqarnas studies or not. However, the focus of this study was to capture new principles and relations, which can be adapted to different contexts instead of merely developing a reconstruction, which has been achieved.

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