

## CALCULATING VISUAL COMPLEXITY IN PETER EISENMAN'S ARCHITECTURE

*A computational fractal analysis of five houses (1968-1976)*

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**Abstract.** This paper describes the results of the first computational investigation of characteristic visual complexity in the architecture of Peter Eisenman. The research uses a variation of the “box-counting” approach to determining a quantitative value of the formal complexity present in five of Eisenman’s early domestic works (Houses I, II, III, IV and VI all of which were completed between 1968 and 1976). The box-counting approach produces an approximate fractal dimension calculation for the visual complexity of an architectural elevation. This method has previously been used to analyse a range of historic and modern buildings including the works of Frank Lloyd Wright, Eileen Gray, Le Corbusier and Kazuyo Sejima. Peter Eisenman’s early house designs—important precursors to his later Deconstructivist works—are widely regarded as possessing a high degree of formal consistency and a reasonably high level of visual complexity. Through this analysis it is possible to quantify both the visual complexity and the degree of consistency present in this work for the first time.

**Keywords.** Computational analysis; fractal dimension; box-counting; Peter Eisenman.

### 1. Determining Visual Complexity

In this paper Peter Eisenman’s early domestic architecture is investigated using a variation of the box-counting method for determining fractal dimension. This research is part of a larger study using computational methods to reconsider the formal characteristics of more than fifty iconic houses of the 20th century.

Fractal geometry, emerging from Benoit Mandelbrot's mathematical proposals in the late 1970's, has evolved from its initial domain in the sciences of non-linearity and complexity to a much broader disciplinary field that now includes architectural design and urban planning. Inspired by Mandelbrot's work, Bovill (1996) developed a "box-counting" method for calculating the fractal dimensions of music, art and, significantly, architecture. Recently, this method has been developed and refined for the computational analysis of the fractal dimension of a small number of historical designs, ranging from isolated ancient structures to complex twentieth century buildings. Despite this, the method has not been comprehensively tested and without a much larger set of published results, it is difficult to validate its usefulness. This validation process is important because fractal analysis is one of only a small range of quantifiable approaches to the analysis of the visual qualities of buildings. If valid, this method could assist in the detailed analysis of historic structures or for the design of buildings which might fit into a certain location or style. Previous testing of this method indicates that Bovill's proposal has potential merit but that further analysis is required along with a much larger set of examples or case studies.

This research describes the results of the first computational investigation of the fractal dimensions of five of the house designs of Peter Eisenman. Eisenman's design approach became well known in the late 1960s with his involvement in the "New York Five" (Eisenman, Meier, Graves, Hejduk and Gwathmey) who were often known as the "Five Whites"; a reference to the Modernist aesthetic of their early designs. As part of this group, Eisenman presented his numbered series of house designs and became known for producing abstract formal compositions that refused to acknowledge site conditions or directly respond to the needs of human inhabitation. In the following two decades Eisenman was in the vanguard of the Deconstructivist movement, an approach intuitively regarded as having high visual complexity.

This paper provides an overview of the box-counting method and its application in the analysis of house designs. It then briefly introduces the five houses being considered in the present paper before discussing the results of the research. Finally it undertakes a comparative analysis between Eisenman's results and those recorded in previous research for sets of house designs by Frank Lloyd Wright, Le Corbusier, Eileen Gray and Kazuyo Sejima.

## **2. Fractal Analysis**

Fractal geometry may be used to describe irregular or complex lines, planes and volumes that exist between whole number integer dimensions. This implies that, instead of having a dimension, or  $D$ , of 1, 2, or 3, fractals might have a  $D$

of 1.51, 1.93 or 2.74 (Mandelbrot, 1982). Fractal geometry came to prominence in mathematics during the late 1970s and early 1980s. In the years that followed fractal geometry began to inform a number of approaches to measuring and understanding non-linear and complex forms. At around the same time, architectural designers adopted fractal geometry as an experimental alternative to Euclidean geometry. However, it was not until the late 1980s and the early 1990s that fractal dimensions were applied to the analysis of the built environment (Ostwald, 2001; 2003). For example, Batty and Longley (1994) and Hillier (1996) have each developed methods for using fractal geometry to understand the visual qualities of urban space. Oku (1990) and Cooper (2003; 2005) have used fractal geometry to provide a comparative basis for the analysis of urban skylines. Yamagishi, Uchida and Kuga (1988) have sought to determine geometric complexity in street vistas and others have applied fractal geometry to the analysis of historic street plans (Hidekazu and Mizuno, 1990). Significantly, the calculation of fractal dimensions allows for a quantitative comparison between formal complexity in buildings, a factor which is important in a discipline that usually relies on qualitative evaluations.

The box-counting method is one of the most common mathematical approaches for determining the approximate fractal dimension of an object. Importantly, it is the only method currently available to analyse the fractal dimension of an architectural drawing. In its architectural variant, the method commences with a drawing of, for example, an elevation of a house. A large grid is then placed over the drawing and each square in the grid is checked to determine if any lines from the façade are present in the square. Those grid boxes that have some detail in them are recorded. Next, a grid of smaller scale is placed over the same façade and the same determination is made of whether detail is present in the boxes of the grid. A comparison is then constructed between the number of boxes with detail in the first grid and the number of boxes with detail in the second grid. This comparison is made by plotting a log-log diagram for each grid size. By repeating this process over multiple grids of different scales, an estimate of the fractal dimension of the façade is produced (Bovill, 1996; Lorenz, 2003). The software programs *Benoit* and *Archimage*, the latter designed and co-authored by the Authors especially for this purpose, automate this operation.

There are several variations of the box-counting approach that respond to known deficiencies in the method. The four common variations are associated with balancing “white space” and “starting image” proportion, line width, scaling coefficient and moderating statistically divergent results. The solutions to these issues that have been previously proposed by Bovill (1996), Lorenz (2003), Foroutan-Pour, Dutilleul and Smith (1999) and Ostwald, Vaughan and Tucker (2008) are adopted in the present analysis.

### 3. Analytical Method

Five of Eisenman's house designs were selected for the present research. These are; House I (1968), House II (1970), House III (1971), House IV (1971) and House VI (1976). House V, the only one missing from this sequence, was not completed in enough detail to be used for the current research. For the rest of the designs, all of the elevations used for the analysis were redrawn to ensure consistency and were sourced from the published sets of drawings produced by the office of Peter Eisenman (Dobney, 1995).

The standard method for the fractal analysis of visual complexity in houses is as follows.

- a) The elevational views of each individual house are separately grouped together and considered as a set.
- b) Each view of the house is analysed using *Archimage* and *Benoit* programs producing, respectively, a  $D_{(Archi)}$  and a  $D_{(Benoit)}$  outcome. The settings for *Archimage* and *Benoit*, including scaling coefficient and scaling limit are preset to be consistent between the programs. The starting image size ( $IS_{(Pixels)}$ ), largest grid size ( $LB_{(Pixels)}$ ), and number of reductions of the analytical grid ( $G_{(\#)}$ ), are recorded so that the results can be tested or verified. *Archimage* results are typically slightly higher than those produced by *Benoit* although the variation is consistent.
- c) The  $D_{(Archi)}$  and  $D_{(Benoit)}$  results for the elevation views are averaged together to produce a separate  $D_{(Elev)}$  result for each program for the house. These results are a measure of the average fractal dimension of the exterior facades of the house. Past research suggests that  $D_{(Elev)}$  results tend to be relatively tightly clustered leading to a high degree of consistency.
- d) The  $D_{(Elev)}$  results produced by *Archimage* and *Benoit* are averaged together to produce a composite result,  $D_{(Comp)}$ , for the house. The composite result is a single  $D$  value that best approximates the characteristic visual complexity of the house.

This process is repeated for each house producing a set of five  $D_{(Comp)}$  values. These values are averaged together to create an aggregate result,  $D_{(Agg)}$ , which is a reflection of the typical, characteristic visual complexity of the set of the architect's works. (See Table 1)

### 4. Peter Eisenman and the Five Houses

Greg Lynn (2004) describes Peter Eisenman's early house designs as being wholly concerned with "layered traces and imprints of orthogonal movement and transformation within a turbulent but nonetheless closed system of

TABLE 1. Abbreviations and definitions.

Abbreviation	Meaning
$D$	Approximate Fractal Dimension.
$D_{(Archi)}$	$D$ calculated using Archimage software
$D_{(Benoit)}$	$D$ calculated using Benoit software.
$D_{(Elev)}$	Average $D$ for a set of elevation views of a house using a specified program.
$D_{(Comp)}$	Composite $D$ result (averaged from both Archimage and Benoit outcomes for all elevations) is a measure of the average characteristic visual complexity of the house (or average fractal dimension for the 2D visual qualities of the design).
$D_{(Agg)}$	Aggregated result of five composite values used for producing an overall $D$ for a set of architects' works.
$IS_{(Pix)}$	The size of the starting image measured in pixels.
$LB_{(Pix)}$	The size of the largest box or grid that the analysis commences with, measured in pixels.
$G_{(\#)}$	The number of scaled grids that the software overlays on the image to produce its comparative analysis.

nonfigurative cubic grids" (162). Sanford Kwinter (1995) argues that in Eisenman's design work "structure always emanates from an initial pattern that is knocked away from equilibrium. The disturbance then travels, reaches a limit, then turns back toward itself to form a self-interfering wave" (13). This tendency can be traced in Eisenman's earliest works, his numbered series of houses. The first of these, House I, is a modernist villa, with a focus on geometric form and the introduction of voids within orthogonal plans. House I, also known as the Barenholtz pavilion, was completed in Princeton, New Jersey in 1968. Designed for the Barenholtz family, Eisenman (2006) describes House I as "an attempt to conceive of and understand the physical environment in a logically consistent manner, potentially independent of its function or its meaning." (32) House I, actually a pavilion, is sited next to the original Barenholtz house. It is a small, timber-framed and timber panelled structure with some interior brick walls.

House II, in Hardwick Vermont, was designed for the Falk family and completed in 1970. The house is sited on the crest of a hill with views in three directions. It is timber-framed, clad in painted plywood panels and it "lacks [the] traditional details associated with conventional houses" (Davidson, 2006: 37). Cassara (2006) describes House II as being focused on the architectural

expression of two types of volumetric and structural relationships. “To articulate these ways of conceiving and producing [...] information in House II, certain formal means were chosen, each involving an overloading of the object with formal references” (82). In the House II, the layering of voids within the structure produces a series of perforated planes that intersect with each other leaving the relationship between exterior and interior spaces ambiguous (see figure 1).

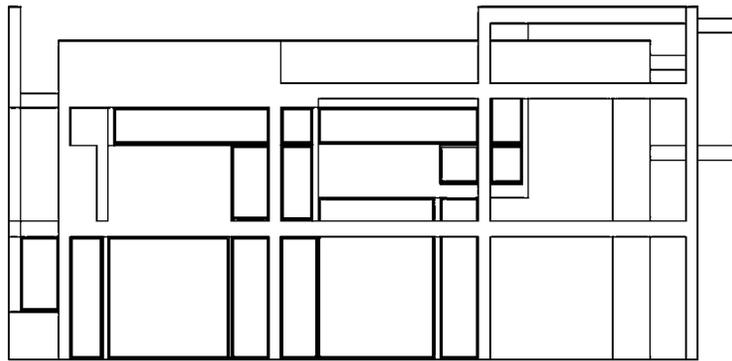


Figure 1. House II, Elevation 2 (South), Peter Eisenman [D = 1.501]  
(Elev Archi)

House III was designed for the Miller Family in Lakeville Connecticut and completed in 1971. Like Houses I and II, it is timber framed and clad, with a painted finish. The house has been described as an attempt to “produce a physical environment which could be generated by a limited set of formational and transformational rules” (Dobney 1995: 34). House III’s position in Eisenman’s formal vocabulary is associated with the introduction of the 45° angle in plan into an otherwise orthogonal 90° system.

House IV, while designed around the same time as House III, marks a return to the planning strategies of Houses I and II. Designed for a site in Falls Village Connecticut, House IV is an elaborate investigation of the process of design transformation wherein various structural systems are allowed to trace solids and voids in the overlapping multi-level plan of the house. House IV is significant because the formal transformations occur in three dimensions; prior to this, the operations were essentially planar in nature.

House VI was constructed in Cornwall, Connecticut, in 1976 for the Frank Family. Designed as a weekend house on a small rural site, it features the first clear instance in Eisenman’s architecture wherein the trace of a form (its absence represented in a void) takes precedence over its presence (the form itself). In House VI Eisenman famously divided the master bedroom, and the bed itself, in two with the trace of a missing beam; effectively cutting a void through the floor and separating the married couple.

**5. Results and Discussion**

If the original 20 views (five houses each with four elevations) are subjected to two variations of the computational method, each using 520 data points, the aggregate result for the visual complexity of Peter Eisenman's early house results is  $D_{(Agg)} = 1.425$ . (See Table 2)

TABLE 2.  $D_{(Archi)}$  and  $D_{(Benoit)}$  results for all elevations,  $D_{(Comp)}$  results for each House and  $D_{(Agg)}$  result for the complete set of works.

House	Elevations analysed with <i>Archimage</i>				Elevations analysed with <i>Benoit</i>				$D_{(comp)}$
	$D_{(Elev)1}$	$D_{(Elev)2}$	$D_{(Elev)3}$	$D_{(Elev)4}$	$D_{(Elev)1}$	$D_{(Elev)2}$	$D_{(Elev)3}$	$D_{(Elev)4}$	
I	1.337	1.348	1.450	1.290	1.330	1.362	1.399	1.302	1.352
II	1.540	1.501	1.521	1.267	1.514	1.481	1.475	1.184	1.436
III	1.525	1.534	1.579	1.611	1.481	1.492	1.546	1.577	1.528
IV	1.456	1.474	1.460	1.459	1.334	1.339	1.327	1.335	1.398
VI	1.451	1.455	1.476	1.483	1.359	1.380	1.342	1.371	1.415
$D_{(Agg)} =$									1.425

For all images:  $IS_{(Pix)} = 1200 \times 871$ ,  $LB_{(Pix)} = 300$  and  $G_{(\#)} = 13$ .

Of these five early houses of Peter Eisenman, House III had the highest average value for visual complexity with a result of  $D_{(Comp)} = 1.528$ . The lowest result is for Eisenman's first design, House I;  $D_{(Comp)} = 1.352$ . The most complex facades are typically in House III and are lead by Elevation 4 ( $D_{(Elev, Archi)} = 1.611$  and  $D_{(Elev, Benoit)} = 1.577$ ) (see figure 2). Indeed, it is relatively rare in the fractal analysis of modern architecture to produce a result which is close to or above  $D = 1.6$ . Elevations with this level of complexity have previously been found in the highly decorative designs of the Arts and Crafts movement of the late 19<sup>th</sup> and early 20<sup>th</sup> centuries and are less common in the late 20<sup>th</sup> century.

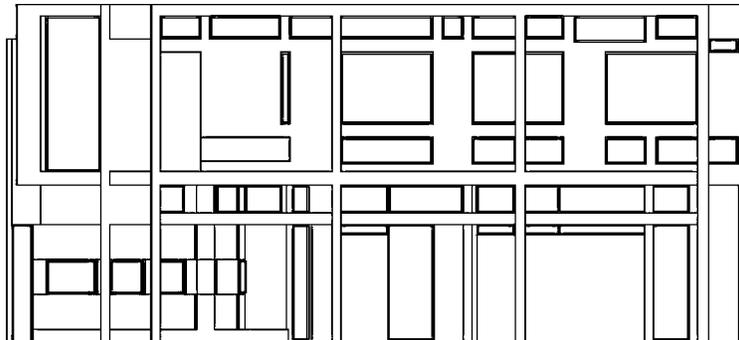


Figure 2. House III, Elevation 4 (West), Peter Eisenman [ $D_{(Elev Archi)} = 1.611$ ]

In general, the results recorded in the present paper confirm the intuitive or qualitative reading of the five houses with House 1 being relatively simple in form, Houses II, IV and VI having a similar level of complexity and House III, with its 45° angled cross structure, as the most complex. How then do these results compare with other results developed using the same method?

In five previous studies using this method, sets of houses by Wright, Le Corbusier, Gray and Sejima have all been examined. Curiously, despite having a reputation for producing architecture that is over-complex in its form, in the case of Eisenman's early houses at least, his works are less complex than those of Wright previously tested. However, there are similarities between the results for Eisenman and those for Le Corbusier's early Modernist houses and Eileen Gray's house designs of a similar period. In essence, the two sets of Modernist works and Eisenman's early houses, which were experiments with Modernism, have relatively similar results. Frank Lloyd Wright's Prairie-style architecture is more visually complex and Le Corbusier's Arts and Crafts style works are also slightly more complex. As anticipated, Kazuyo Sejima's Minimalist works produced a lower result than Eisenman's (see table 3).

TABLE 3. Comparison of results for Eisenman with those of past research.

Architect	Focus	$D_{(agg)}$	$D_{(Comp)}$ lowest	$D_{(Comp)}$ highest	$D_{(Comp - clustering)}$ closest 3 results
Frank Lloyd Wright	5 houses (1901-1910)	1.543	1.505	1.580	0.015 (<1% variation)
Le Corbusier	5 houses (1905-1912)	1.495	1.458	1.584	0.031 (~2% variation)
Le Corbusier	5 houses (1922-1928)	1.481	1.420	1.515	0.015 (<1% variation)
Eileen Gray	5 houses (1926-1934)	1.378	1.289	1.464	0.087 (~5% variation)
Peter Eisenman	5 houses (1968-1976)	1.425	1.352	1.528	0.017 (~1% variation)
Kazuyo Sejima	5 houses (1994-2003)	1.310	1.192	1.450	0.116 (~8% variation)

An additional dimension that arises from the present research concerns the clustering of results for visually similar works. When an architect has the opportunity to focus on a series of projects over a sustained period of time and then develop them to a similar level of resolution, it might be anticipated that these projects would exhibit similar levels of visual complexity. This is certainly the case for Wright's Prairie houses and for Le Corbusier's early modern houses,

both of which have a  $D$  range, for the best cluster of results, of less than 1% visual difference (around  $D = 0.015$ ). This means that, despite different sites, variations on program and levels of complexity, each of these architects' sets of works consistently featured almost identical results for their respective projects. Peter Eisenman's early works have a similar magnitude of variation. Houses II, IV and VI are all tightly clustered together with a  $D_{(Comp)}$  range of between 1.398 and 1.415 resulting in a difference in visual complexity, between these three designs, of less than  $D = 0.017$  or around 1% in comparative terms. This result is significant not only because it confirms the standard qualitative reading of each of these three sets of projects as being consistent within each architect's oeuvre, but it also affirms the usefulness of the method for such consistent works (a result with a less than 1% mathematical difference is significant). Conversely, the results for Gray and Sejima were less well clustered (5-8%), a result supported by an intuitive reading of those works as less consistent in their visual complexity (Ostwald, Vaughan, Chalup, 2008; Vaughan and Ostwald 2008).

While there are few quantitative methods available for the analysis of visual complexity in architecture, the box-counting method, and its computational variation, remains a significant approach. The method is not only repeatable, but it can produce accurate results that typically support more conventional qualitative, semantic and historical readings of the same buildings.

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### References

- Batty, M. and Longley, P.: 1994, *Fractal Cities: A Geometry of Form and Function*, Academic Press, New York.
- Bovill, C.: 1996, *Fractal Geometry in Architecture and Design*, Birkhäuser, Boston.
- Cassara, S.: 2006, *Peter Eisenman: Feints*, Thames and Hudson, London.
- Cooper, J.: 2003, Fractal Assessment of Street-Level Skylines, *Urban Morphology*, 7(2), 73-82.
- Cooper, J.: 2005, Assessing Urban Character: The Use of Fractal Analysis of Street Edges, *Urban Morphology*, 9(2), 95-107.
- Davidson, C. (ed.): 2006, *Tracing Eisenman*, Thames and Hudson, London.
- Dobney, S. (ed.): 1995, *Eisenman Architects: Selected and Current Works*, Images Publishing, Mulgrave, Victoria.
- Foroutan-Pour, K., Dutilleul, P. and Smith, D. L.: 1999, Advances in the implementation of the

- box-counting method of fractal dimension estimation, *Applied Mathematics and Computation*. **105**(2). 195-210.
- Hidekazu, K., and Mizuno, S.: 1990, Fractal Analysis of Street Forms, *Journal of Architecture, Planning and Environmental Engineering*, **8**(414). 103-108.
- Hillier, B.: 1996, *Space Is the Machine: A Configurational Theory of Architecture*. Cambridge University Press, Cambridge.
- Kwinter, S. (1995) Introduction: The Eisenman Wave in S. Dobney (ed), *Eisenman Architects: Selected and Current Works*, Images Publishing, Mulgrave, Victoria, pp. 7-15.
- Lorenz, W.: 2003, *Fractals and Fractal Architecture*, VUT, Vienna.
- Lynn, G.: 2004, Architecture Verses Sculpture in P. Noever (ed) *Barefoot on White-hot Walls*, Germany, Holzhausen Druck, pp. 160-167.
- Mandelbrot, B.: 1982, *The Fractal Geometry of Nature*, WH Freeman, San Francisco.
- Oku, T.: 1990, On Visual Complexity on the Urban Skyline, *Journal of Planning, Architecture and Environmental Engineering*, **8**(412), pp.61-71.
- Ostwald, M. J.: 2001, Fractal Architecture: Late Twentieth Century Connections Between Architecture and Fractal Geometry, *Nexus Network Journal, Architecture and Mathematics*, **3**(1), 73-84.
- Ostwald, M. J.: 2003, Fractal Architecture: The Philosophical Implications of an Iterative Design Process, *Communication and Cognition*, **36**(3/4), 263-295.
- Ostwald, M. J., Vaughan, J. and Tucker, C.: 2008, Characteristic Visual Complexity: Fractal dimensions in the architecture of Frank Lloyd Wright and Le Corbusier, in K. Williams (ed), *Nexus VII, Architecture and Mathematics*, KW Books, Turin, pp. 217-232.
- Ostwald, M. J., Vaughan, J. and Chalup, S.: 2008, A Computational Analysis of Fractal Dimensions in the Architecture of Eileen Gray, *ACADIA 08*, pp. 256-263
- Vaughan, J. and Ostwald, M. J.: 2008, Approaching Euclidean Limits: A Fractal Analysis of the Architecture of Kazuyo Sejima, *ANZASCA 08*, pp. 285-294.
- Yamagishi, R., Uchida, S., Kuga, S.: 1988, An Experimental Study of Complexity and Order of Street-Vista, *Journal of Architecture, Planning and Environmental Engineering* **2**(384), 27-35.