COMBINATORIAL PRODUCTIVITY

Knot Typologies

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Abstract. The paper investigates knotting techniques as a method for generating wall systems. The essential matter of the paper is to demonstrate the potential of knotted, algorithmic architecture through different research studies, which share the knotting of linear elements as a common methodology for design development. Combinatorial Productivity implies that by combining linear elements hidden properties of a system emerge and thereby the system becomes productive.

Keywords. Generative Design; Design methodology; Parametric Form Generation; Knot Theory; Scripting.

1. Introduction

In general we have to differentiate between two types of knots: knotting is defined as a method of fastening linear material by tying or interweaving, whereas mathematical knots are described as an embedding of a circle in the 3-dimensional **Euclidean** Space. As a commonality both types – the linear, open knot and the joined, closed knot – share the fact that they are based on rules. They have characteristics such as loop, elbow, turn etc. which define them as knots. Hence, knots are algorithmic.

The algorithm is a sequence of finite instructions, which processes abstract information. An algorithm can be seen as a method in which a list of predefined instructions for completing a task will proceed through a series of successive states, eventually terminating in an end-state. This process is based on clear rules and principles that for instance include number of steps, quantities, degree of deviations and relationships. But algorithms can also incorporate randomness. This leads sometimes to forms which are often mistaken as chaotic, natural, organic formations since they share similar characteristics with organic forms such as variation and deformation.

Generating forms that are defined by bounded and numbered steps might result in forms that are unpredictable and new. In that way the power of an algorithm as a design tool lies in its ability to infer new knowledge. The discussed projects work within the following framework:

1 Research on knot typologies.
2 Creation of a form catalogue that explores the prototype by its deviates
3 Transference of the models and principles to the context of a given project.

The paper "knot typologies" discusses the potential of the computation not just as a sophisticated drafting and representation tool, but as a potentially powerful tool in the generation of architectural design. The algorithmic potential of the computer allows creating a new language of architectural forms based on manipulation of variables. The argumentation focuses on the transformative role of digital techniques for architecture versus digital techniques/ practices with a purely auxiliary approach.

2. Knots as 3-dimensional Objects

"The knot is perhaps the oldest technical symbol and, as I have shown the expression for the earliest cosmogonic ideas that sprang up among nations. Knots serve first of all to join the ends of two threads; their strength depends principally on friction. The system that maximizes friction by lateral pressure when two threads are drawn in opposite directions produces the strongest knot.(…) A very ingenious sand ancient use of the knot led to the invention of netting.(…) The mesh of the net (…) has the advantage that damage to one mesh does not affect the whole system and is easily mended." (Semper 1860)

A knot itself is always a 3-dimensional object. Hence, a knot cannot, even by continuous deformation of the string, be undone without moving one end of the object back through the loop. The immense knotting spectrum given by weaving, knitting, braiding, crocheting and other knotting techniques goes back in history around 5000 years, when the early Maya cultures used knotted systems as an accounting system (talking knots).

In this paper we are investigating the potential of using the systemacy of knotting techniques as a method of generating form (knitted surfaces) within strict parameters given by the original knot and its thresholds as stretching, widening and tightening.

The research presented originates from a computation and design seminar at the RWTH Aachen, Germany, which researches combinatorial productivity of knot typologies and leads to the fabrication of several prototypes. It is a one-semester elective course and part of the undergraduate and graduate curriculum.

The course is the first of a series of research oriented courses, which explore ornamental and structural properties of pattern systems.

The research topics of the first course are the following:

a Celtic and Norse knot designs (infinite knots)
b sailing knots (links of one or more components)
c mathematical knots (infinite knots)
d weaving, knitting, braiding, crocheting and other handcraft knots
e Macramé
f knots in the work of Erwin Hauer

Barrionuevo, Lopez and Serrentino (2004) explore in their research "spirospaces in architectural design" the relation between spirospaces – which is defined as "geometrical entity generated from the spatial interpretation of a "Spirolateral", a well known bidimensional entity" – and knot theory. Spirospaces explores the potential of entangling a singular strand into a closed, 3-dimensional object. In addition to this research the presented work looks also at open knots next to closed, mathematical knots and introduces the aspect of multiplication of knots: from the first knot catalogue of 120 knot types, several knot categories are chosen to explore the question if the algorithm of a singular knotted object can lead to a defined surface. Each knot is a singularity in itself, but some knots have the capacity to create a multiplicity as an entity. Each knot system develops again its own surface catalogue and the behaviour of each knotted surface is tested on the same parameters as the original knot.

Next to knots the research is conducted on architecture screens of Erwin Hauer. His work makes use of repetitive complex modules in order to create perforation in a wall system. The relation between module and repetition in a larger field is highly interesting in order to study the configurations of knotted surfaces.

We will discuss in this paper several knots and their variation. The use of Grasshopper and Paracloud allows introducing parametric behavior to the knots and the knotted surface systems.

Since knotting is originally an analogue technique, the constant dialog between the digital realm and the analogue representation, e.g. physical models, is instrumentalised to create a feedback loop between the two. As figure 1 shows parametric development and prototyping are the key elements for this research. Material information, the catalogue of variants, modeling, scripting, theoretical research and optimization inform parametric development and prototyping vice versa.

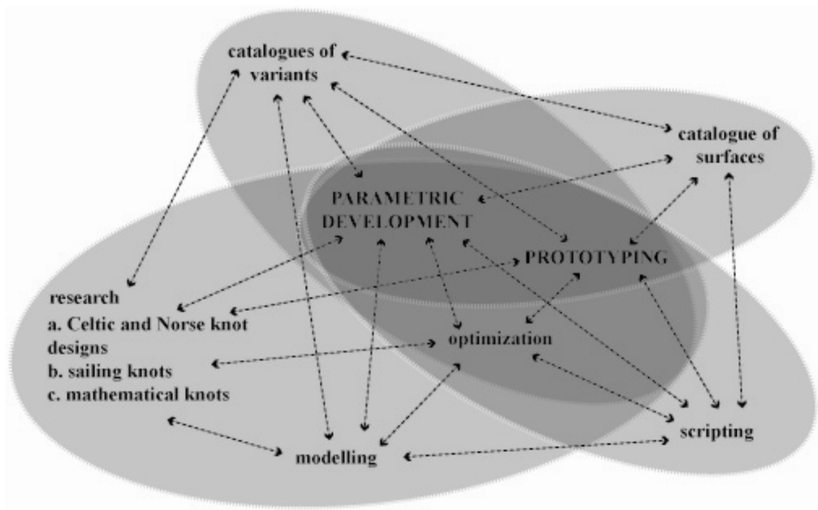

Figure 1. Strategy of combinatorial productivity.

3. Single knotted loops and links of components

The knot typology studies single knotted loops (primary knots and infinite knots). The study starts with a collection of braided string-patterns. This matrix of studies follows certain characteristics: the braiding sequence begins always by moving the first strand over and then under the other, adjacent strand. This sequence can repeat endlessly. These closed loops are then manipulated in such a way that they form one or two surfaces. The results of this operation are endless knotted fields. In the following four examples are discussed.

3.1. APPLIED RULES FOR KNOT 6

The knot 6-surface is based on mathematical knots (see figure 2). It is build upon two interleaving u-shaped loops. The following parameters are determined as applicable rules to create variants within the knot 6-surface. By adapting always one or more parameters the knotted surface will change its behavior.

a The proportion of the length of joints /sidebars defines the curvature.
b The width of bars adapts the amplitude of the cross-bending.
c The width, length and curvature of the bars define the opening.
d Rotation of the longitudinal bars and joints define the opening size.
e The length of the bars varies the curvature of the overall surface.

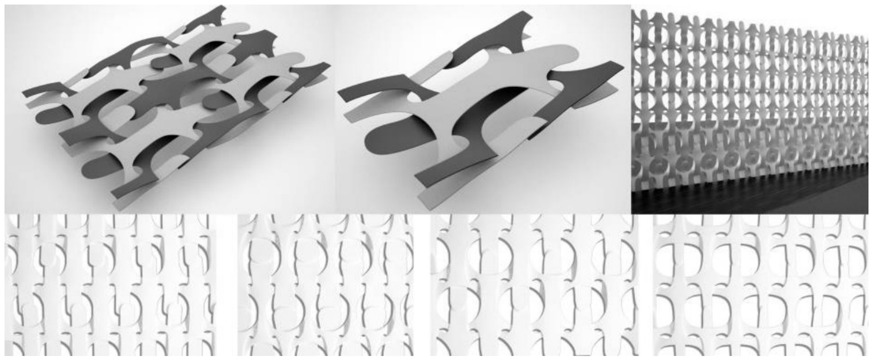

Figure 2. Single knotted loops and links of components.

The variation of the described parameters leads to a catalogue with four slightly different types of knot 6- surfaces. Each of the developed surfaces derives from the original prototype and undergoes slight changes according to parametric variations. For further research the structural characteristics (tensile and compression forces) will be explored and different materials will be tested.

3.2. BROKEN REGULARITIES AND FIELD BEHAVIOR

While investigating basic open knots, the focus lies on links. A link is made by more than one component. Using knot 19/ Josephine knot as example for basic open knots, a versatile catalogue of knotted carpets is developed. These carpets result from the entanglement of longitudinal strands, the wrap yarn. The patterns - shown in figure 3 - demonstrate different conditions of tightening and loosing. These operations allow for openings in the carpet, zones of higher and lower density. In combination of these diagrams a complex surface behaviour can be developed.

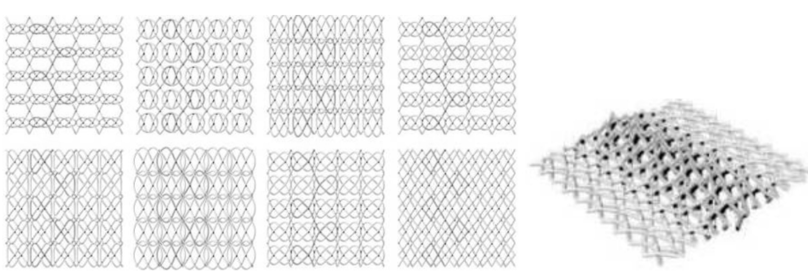

Figure 3. Projection of Josephine knot: catalogue of knitted surfaces.

The last pattern (figure 3, bottom right) transfers the curvilinear diagram into a straight edge diagram. From this transformation a hidden property of the pattern emerges: the curvilinear state gives the impression of symmetry, whereas the linear diagramming reveals that there are a series of non parallel angles that constitute the overall pattern.

3.3. LINKS OF TWO COMPONENTS AND WEAVING TECHNIQUES

The reef knot (see figure 4) is a link made by two components, which joins two separate strings. Reef knots are made by interweaving two overhand knots, while both knots have an alternating orientation and while the ends of each string stay parallel.

In a next step the model is extended in the transverse direction by entangling the loops. These two operations create a spatial framework which is studied in the next design iteration more closely. As you can see in the bottom row of figure 4 two different approaches of filling the wireframe are studied. The first study fills all elements in such a way that a continuous surface is created. The second approach fills all overlapping elements, basically the areas where the reef knots interweave. This approach results in discontinuous surfaces: isolated patches.

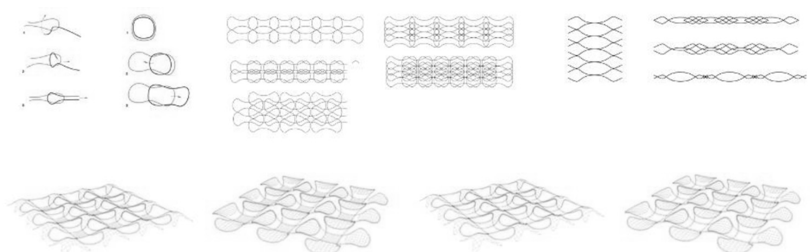

Figure 4. Reef knots and weaving techniques.

3.4. INFINTE SYSTEMS AND SUBSEQUENT SCULPTURES

(…) out of the matrix with which I had been working, an infinite surface evolves… One cannot physically expand their structure into infinity but at some point must stop adding modules, or subtracting from the infinite matrix. Where, when and why there? Like crystalline minerals, the infinite web contains countless planes of cleavage, or planes of symmetry, along which it can be split, exposing its microstructure, which consists of an outstanding variety of shapes and configurations.(…) (Hauer, 2004)

The third research study (figure 5/6) looks at screens from Erwin Hauer (Continua 2004: Design 1, Design 4 and Design 5). The analysed screens from Erwin Hauer are based on the interweaving of two planes.

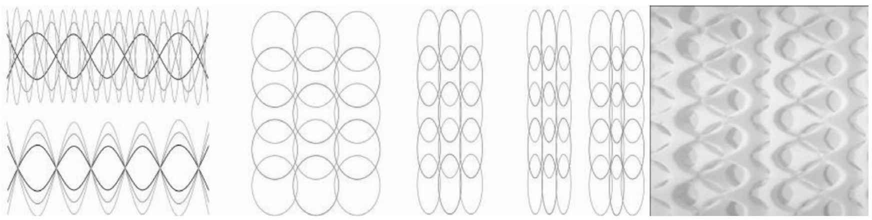

Figure 5. Screen studies - interweaving of two planes.

In a sequence of studies, which interweave two surfaces, a system is developed that is based on two identical surfaces with oval shaped perforations. These perforations are offset from each other so that the surfaces can interweave through the holes by creating an undulating section for each surface. The system is able to vary the size of the oval shaped perforation.

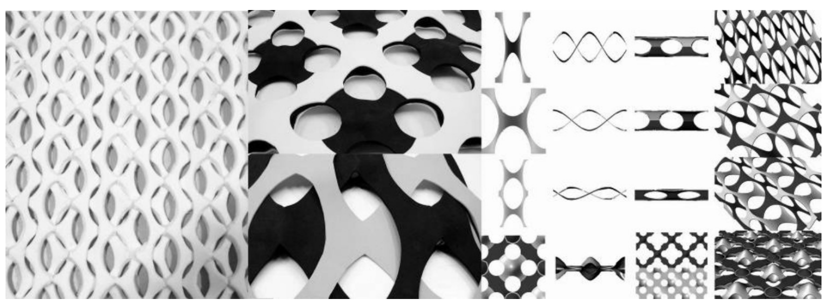

Figure 6. Screen studies – shaped perforations.

4. Grasshopper and Paracloud

In a next step the research on knots is transformed into a parametric model using mainly two different software applications: the first is Rhinoceros in combination with Grasshopper and the second is Paracloud.

Grasshopper models allow introducing variation to the repetitive system based on mathematically defined variables. Figure 7 shows an example of a braided wall which is able to vary its density and pattern within the defined framework: A logarithmic function causes a compression in the vertical direction. The given example demonstrates a system which generates itself without a guiding geometry. The parts constitute the whole.

The second method utilizes Paracloud – shown in figure 8 - and starts with a pre-existing mesh body linking the rhino surface to excel and importing geometry, the parts, back to rhino. Here the variation is mainly deformation resulting from the mesh matrix.

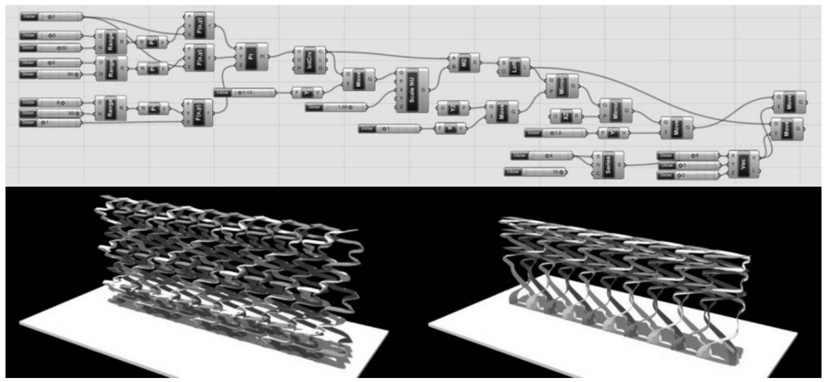

Figure 7. Grasshopper definition and the resultant model.

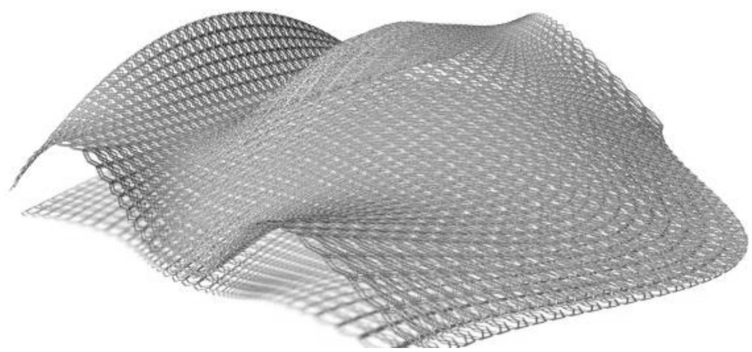

Figure 8. Paracloud model/mathematical knots.

5. Fabrication:

In a last step the digital prototypes will be used to create physical prototypes (figure 9). This step requires a close investigation of material properties and of the fabrication method. Mainly CNC milling and laser cutting techniques are used for the final prototype. The consequence of these fabrication methods is that the flatness of the material has to be considered, since 2d-milling and laser cutting are restricted to a plane: not any geometry can be fabricated.

The fabrication also allows introducing another aspect: structural properties become inherently more relevant. Basically the knotted skin, the ornament, has to function as a structural system, which is highly interesting since structure and ornament usually are thought as different systems: in a classic sense the bold structure just becomes beautiful by applying ornament. But here the two merge and become one.

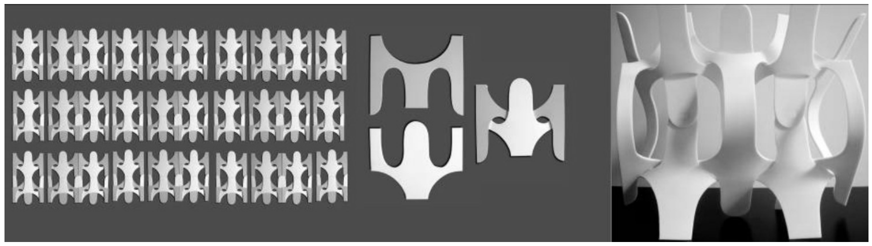

Figure 9. Combinatorial productivity.

6. Conclusion: combinatorial productivity and knot typologies

The research on knots as algorithmic systems has proven to be highly productive on two different levels: first of all the knots provide a systematic machinery to explore rule based systems. Design is almost generated on its own, just by following the properties of each system.

Secondly the employed machinery works as a pedagogic instrument to guide the design process of students from research to design. The close analogy between physical knots, CV curves and surfaces allows for a constant feedback between analog and digital.

Further research could investigate structural properties and material properties deeper than this is possible within the framework of one seminar. Ideally these properties would be explored by a series of full-scale models that allow actual testing.

As Kozlow (2007) states in his research about "topological methods of construction of point surfaces" that a knot is in its natural appearance highly flexible and complex. Kozlows' (2007) polymorphous nodus structure (Nodus translates from Latin: a knot) understands the knotted surface as a basis of real-size kinematical architectural structure. This understanding of the knot system as loose and kinetic opens interesting aspects for further research.

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