DYNAMIC RELAXATION OF TENSEGRITY STRUCTURES

A Computational Approach to Form Finding of Double-layer Tensegrity Grids

GUSTAV FAGERSTRÖM
London Metropolitan University
Department of Architecture and Spatial Design
40-44 Holloway Road London N78JL
United Kingdom
g.Fagerstrom@londonmet.ac.uk

Abstract. The structural hierarchy inherent to tensegrities enables a building skin that performs on multiple levels simultaneously. While having one function in the global building mechanics, its individual components can work as self-contained systems balancing tensile and compressive forces locally within them. The behavior of elements under load is linear and thus describable analytically. When these are aggregated in a tensegrity however, the performance of the assembly as a whole is non-linear. In order to investigate further these relationships a method of dynamic relaxation will be developed. This tool allows for simulation and load analysis of a complex tensegrous network, based on the relationships between force, stiffness and dimension formulated by Young and the computational means provided by a parametric/associative modeling environment. This research investigates the possible form-finding through computational means of a double-layer tensegrity grid.

Keywords. Dynamic; relaxation; tensegrity; form finding.

1. Introduction

1.1. POSITIONING OF RESEARCH

The main positioning of this study in contemporary discourse is the creation of a form-finding technique for double-layer tensegrity grids, the potential of which has largely yet to be explored within architectural space-making. The collected study content is organized around a design research which aims at an easily deployable form-finding technique while dealing with relationships within mathematics, physics and mechanics. Requiring a certain–but not extensive–knowledge of computer programming and mathematics, the resulting computational process is possible to use for architects, designers and other non-engineers.

The study carried out in 1984 by René Motro in which he proposed to apply the dynamic relaxation method to tensegrities marked an important step in form-finding of tensegrity structures (Motro, 1984). Motro and his team of researchers have remained at the forefront of developments within the field, proposing a number of form-finding methods alongside an astonishing array of studies and full-scale prototypes of planar tensegrity grids (Motro, 2003). Other important research is that of Tibert and Pellegrino, of the Royal Institute of Technology of Stockholm and University of California at Los Angeles respectively, on kinematic and static form-finding methods (Tibert and Pellegrino, 2003). Y. Kono at the Center for Space Structures Research, Taiyo Kogyo Corp., has produced studies which add equally important knowledge to the discourse (Kono et al, 2000).

A collaborative approach to design where architects and engineers team up permanently is becoming ubiquitous in contemporary practice, however this model is still mainly to be found within environments where engineering is the specialty. For this reason the discourse, research and case studies around more technically intensive areas of design, such as that of tensegrity, tend to be led by engineers and mathematicians. As an example hereof an overwhelmingly large portion of the literature consulted in this study originates in various structural and mechanical engineering faculties around the world; very few come from architectural academia or practice. The ambition of this study is to join the discussion on tensegrity initiated by among others White (2004) and Gómez Jáuregui (2004). Thus tensegrity structures are considered in relation to elasticity theory and mechanics, but in addition to this also to the actual notion of architectural design.

1.2. THE CONCEPT OF TENSEGRITY

Tensegrity structures are a class of structures whose primary structural action derives from tension rather than compression. They are inherently material efficient and resilient, with tension and compression members sustaining each other in an equilibrated, rather than hierarchal, system.

R. Buckminster Fuller defines tensegrity as a structure whose shape stability is due to continuous tension–‘tensional integrity’ (Fuller, 1975)–as opposed to the continuous compression used in construction in stone, masonry or other ‘bearing-borne’ static principles.

1.3. FORM-FINDING AND DYNAMIC RELAXATION

1.3.1 Definitions

Form-finding is a kind of–usually–structural optimization by which a final deformed shape is determined through a given stress pattern applied to the structure. The form-finding process can be performed with physical prototypes, or in some cases even full-scale constructs, and with a variety of digital techniques. One of these is the dynamic relaxation method, so called due to its iterative search for a structural convergence and stability.

1.3.2 Historical context and contemporary techniques

Dynamic relaxation is an established procedure for pure tensile constructs such as cable nets. It is one of the so-called kinematical form-finding methods for tensegrities. From an initial layout member lengths are gradually altered until an equilibrium condition is reached (Tibert and Pellegrino, 2003).

The use of digitally aided relaxation methods in this instance is preceded by the physical modelling techniques deployed most notably by Antoni Gaudi in the late 19th century for his cathedral La Sagrada Familia (1882-ongoing) and his Church of Colonia Guell (1898, 1908-15, unfinished)–the famous Gaudi chain models. This work is carried forward with digital techniques by among others Axel Kilian of MIT Media Lab/TU Delft. Using the particle-spring principle several form-finding tools for funicular structures have been developed (Witkin, 1997; Kilian and Ochsendorf, 2005).

Tensegrity pioneer Kenneth Snelson has for nearly 50 years been realising tensegrity sculptures using a hands-on iterative approach. Slender struts that buckle easily are used in the form-finding stage, subsequently they are replaced by stiffer ones with appropriate lengths (White, 2004). Such “manual dynamic relaxation” has been successfully deployed by the artist throughout his career, however when considering tensegrity on a building system scale this method could possibly become too exhaustive, not to mention likely to conflict with modern demands on production and delivery times.

2. Dynamic Relaxation Study

2.1 SUMMARY

A comparison is made of two configurations of the form-finding routine: at the base of both sits a laid-out flat assembly of the hexagonal network as visualized by Kono et al (2000). The network is created from sub-systems that each is a self-stressed unit with its own structural integrity.

2.2 NETWORK MAPPING

2.2.1 Node allocation

One way of approaching the mapping of components is projective geometry–laying the 3D construct out as if flat on a 2D plane. The node network is established by an algorithm, which steps through a sideways and height wise count resulting in an alternating zigzag pattern. Row and column indices are evaluated together; if odd then an upward pointing triad of points replaces the single original point; if instead even then the triad points downward.

2.2.2 Compression member layout

Compression elements are mapped in a triad based logic. Each point in a triad pointing upward becomes a node in the upper cable layer and vice versa. The study deals with an assembly whose projected image resembles a hexagonal grid shell. In reality it is a double-layer grid based on what's known as an independently self-stressed 3-6-9 module (3 struts-6 nodes-9 cables). A network of these is set up in such a way that each module is flanked by exactly 3 other ones, with which it shares exactly one compression member and three tension members each.

2.2.3 Tension member layout

The tensile component consists of three different cable typologies–long and short "planar" network cables, and inter-layer cables. These are laid out in what essentially is a set of triangles oriented in different planes. Planar cables

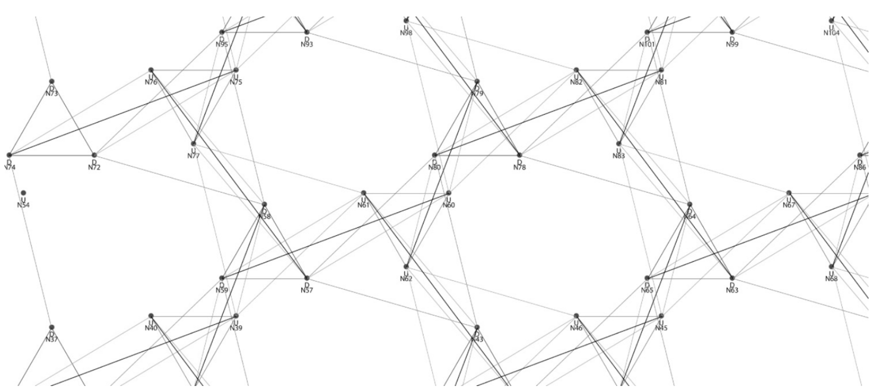

Figure 1. Mapped node network with triangulated members.

triangulate the top and bottom tensile layers; these triangles are intrinsic to the original 3-6-9 units but get merged together at the edges. Inter-layer cables triangulate in different planes and together with compression members where 2 cables and 1 strut make up each triangle. DOF are equal to number of connections at each node (Kono et al, 2000).

2.2.4 Network layout pseudo code

TABLE 1. Pseudo code for node and element mapping.

```
FOR-loop running left to right
   FOR-loop running bottom to top
       CONDITIONAL specifying a combination of rows and columns
       IF TRUE instantiate node point in list NODES
   NEXT
NEXT

FOR-loop stepping through NODES
   CONDITIONAL specifying node indices of boundary conditions
   IF TRUE store "1" in corresponding index of list NODEFIX
   IF FALSE store "0"

   CONDITIONAL specifying node indices according to rule for nodes to
   be linked by linear elements
   IF TRUE put node in list START and node [+/- the step in the above
   rule] in list END, instantiate position in list ELEMENTS, store
   corresponding position in stiffness parameter list EA with either
   pre-set value for EA[tensile] or EA[compressive]
NEXT
```

2.3. COMPUTATIONAL RELAXATION PROCEDURE

2.3.1 Brief mathematics

At the base of dynamic relaxation sits Newton's second law of motion and the iteration of it over many time steps, at each step:

$$F = ma \rightarrow F = mv \qquad (1),(2)$$

Due to the derivative relationship of velocity and time, (2) is rewritten thus:

$$F\mathit{d} / m = \Delta d / \Delta t \leftrightarrow \Delta d = F\mathit{d}\, \Delta t / m \rightarrow \Delta d = F\mathit{d} / m \qquad (3)$$

when considered inside each individual time step. The three Cartesian space complimentary equations then become

$$\Delta x = F\mathit{x} / m\ ,\ \Delta y = F\mathit{y} / m\ ,\ \Delta z = F\mathit{z} / m \qquad (4)$$

Using this in conjunction with Young's modulus for material elasticity,

$$E\ (= \sigma/\varepsilon\ ) = FL0 / A0\Delta L \qquad (5)$$

(where L0 is original/unstressed length, A0 or A is the cross-section area and

ÄL is the deformed length) we obtain a formula for considering the relative displacement of a particle, e.g. a node, in 3D-space as a function of the accumulated force acting on it as well as the relative stiffness/elasticity properties of incising members. EA-values are known provided that a specific material has been chosen, the resulting equation to be used in the relaxation process becomes:

$$\Delta d = EA\Delta L / mL0 \qquad (6)$$

For each time step this is re-applied to all nodes whereby the relative displacement is calculated. The more the structure is out of balance the more steps this will require, as each step needs to "reassemble" all residual forces at each node and then reiterate the relative displacement. If relationships of m and EA are chosen appropriately the accumulated residual nodal force will converge to zero and the structure will find equilibrium. Some sort of damping entity is also required as the nodes otherwise would travel to infinity. This is what in the context of the related particle spring systems is defined as Viscous Drag, which in its ideal form is expressed

$$f = -kdv \qquad (7)$$

where kd is a constant whose effect is to resist motion as to gradually allow the body to come to rest (Witkin, 1997; Tibert and Pellegrino, 2003; White, 2004).

2.3.2 Double-curvature configuration

The main feature in this variant of the routine lies in how the elastic stiffness of each node is calculated. For all nodes, each connecting compression or tension element is registered and each individual EA/L*0* is calculated for the initial state. Depending on how this calculation is performed the form-finding produces different results. With the method described here below the nodal stiffness becomes such that the construct converges toward a doubly curved shape. This is due to the nodal stiffness in the script being calculated on the actual initial lengths of unrelaxed members, as opposed to assigning to them a "forced", ideal slack length.

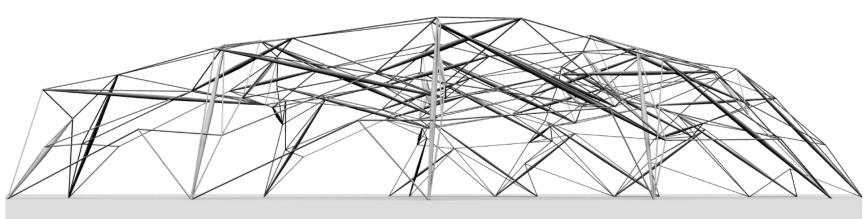

Figure 2. Double-curvature configuration at 100 cycles of relaxation.

2.3.3 Flat configuration

The flat convergence is achieved through an identical procedure as the one used in 2.3.2, with the one difference that slack lengths (L) are predefined. This means that whichever the initial geometry and hence member lengths, the slack length (and therefore elastic stiffness) will relate to the same value, that defined as *length ratio*.

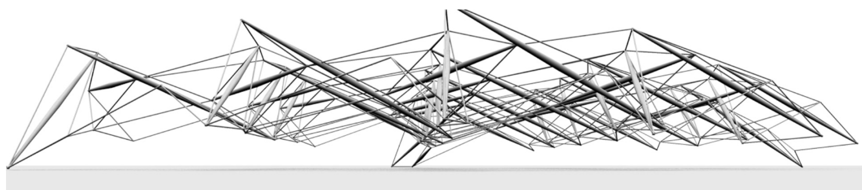

Figure 3. Flat configuration at 100 cycles of relaxation

2.3.4 Relaxation Procedure Pseudo Code

TABLE 2. Pseudo code for relaxation procedure

```
FOR-loop stepping through ELEMENTS
   For 2.3.2: COMPUTE initial lengths by subtracting list END from list
START
   For 2.3.3: ASSIGN initial lengths by variable SLACKLENGTH
   COMPUTE nodal stiffness by dividing EA with initial length/SLACKLENGTH
NEXT

FOR-loop stepping through number of relaxation cycles
   FOR-loop stepping through NODES
      FOR-loop stepping through ELEMENTS
         COMPUTE actual element lengths
         COMPUTE tension coefficient - TC -as per Young's modulus
         COMPUTE node displacement as per TC and damping variable
         FEED BACK resulting coordinates into NODES
      NEXT
   NEXT
NEXT
```

3. Conclusions

In essence the outcome of form-finding through relaxation depends on three main parameters: member stiffness, damping and length ratio. Wrongful calibration of any of these manifests itself in either an “imploding” or "exploding" assembly; in the former case the reason is insufficient damping, in the latter the culprit is excessive member stiffness.

Stiffness parameters: tensegrities inherently consist of both rigid and non-rigid members. At the beginning of the procedure two member stiffness parameters are defined: EA(tensile) and EA(compressive). At the force application stage in the procedure, this is what will ultimately decide the element's deformation under the gravity load and hence the displacement of the nodes they connect to.

Damping: as outlined in 2.3.1 the damping factor is what renders possible geometric convergence of the structure and therefore the relaxation in itself.

Length ratio: as mentioned previously this has been experimented in two different ways, one of which declares one or a few global length ratio-s initially as the slack length (unrelaxed length, L) against which all element deformation is calculated. The other variant is to register the initial length of each member as laid out in their unrelaxed state.

The stiffness property assignment is crucial to the relaxation procedure. Experiments show that the least successful procedures are those in which the relative difference in stiffness between the members is the greatest. Among the trials executed, the most grossly inaccurate end geometry is obtained when the ratio of stiffness between taut and compressed members (EA(tensile) to EA(compressive)) is 1/1000 and the two values are in fact 1 and 1000, respectively. A number of relatively successful cases however also have the same EA ratio of 1/1000–for example EA(tensile) = 0.1, EA(compressive) = 100. This suggests that not only relative stiffness differences between members but also absolute stiffness in relation to overall structure dimension and topology govern the properties of a tensegrity assembly.

In this instance it becomes central to create the appropriate interrelation between what is to be taut and what is to be compressed in the assembly. Two main differences therefore exist between purely tensile form finding processes and the one at hand. Aside from individual stiffness values, it is necessary to specify that elements are to be in pure compression and pure tension respectively. To achieve this, for each relaxation cycle if a compression member were to be going into tension or a tension member into compression, the forces acting on that member are re-set to zero. This encourages the procedure to work around solutions where such unwanted forces are present, rather than propagating them further downstream as was the case initially.

Nodal stiffness is composed by not only elastic stiffness as per Young's modulus, but a sum of this and the geometric stiffness (White, 2004),

$$F(t)/L(t) \tag{8}$$

where (t) is the current time step, F(t) is the force acting on the member at each step and L(t) is the member's length at that step. Thus:

$$[\text{nodal stiffness}] = EA/L0 + F(t)/L(t) \quad (9)$$

4. Discussion

The dynamic relaxation method is applicable as a form-finding tool for the investigated types of tensegrity structures. It however is most successful when initial geometry is reasonably well defined; it loses track when initial geometry is too inaccurate. Within the scope of this research it was not possible to fine-tune the experiments as to find out more in detail at what level of inaccuracy problems start occurring. Parallel to this study I have performed some rudimentary but relatively successful trials to this end. From these it seems that it would be useful to further study the possibilities of altering the form-finding algorithm for greater tolerances in initial geometry. Ideally a relationship between topology, connection mapping and slack lengths could be such that initial geometry can be made to converge geometrically to whichever relaxed geometry. Succeeding in this would create a powerful tool for form-finding and subsequent manufacturing and assembly, literally capable of taking an arbitrary collection of members and deploying them into a useful, stable 3D-construct. The investigated assemblies can assume both single and double curvatures without any single component actually being curved or going into bending moment. This is a promising feature in terms of manufacturing and assembly of straight and planar components.

In the pseudo code under 2.2.3 one example is given as to how the network's layout can be designed. By altering the variables controlling the stepping […], density […] and relative distance […] of the node points any network topology can be experimented. If an associative-parametric software application is used that offers a mouse-controlled GUI, as is the case in the present study, node points can simply be drawn by hand and fed into the node mapping algorithm. This also offers the opportunity of choosing the number of nodes that will be static, i.e. locked in space, and those that will be subject to the dynamic movement. In this way a high degree of design input is possible, whereby malleability is created of design intentions as well as formal outcomes.

References

Fuller, R. B.: 1975, *Synergetics: Explorations in the Geometry of Thinking,* MacMillan Publishing Co., Inc., New York.

Gómez Jáuregui, V.: 2004, *Tensegrity Structures and their Application to Architecture,* MSc Thesis, School of Architecture, Queen's University Belfast.

Kilian, A. and Ochsendorf, J.: 2005, Particle-Spring Systems for Structural Form-Finding*, Journal of IASS,* **46**(147).

Kono, Y., Choong, K.K., Shimada, T. and Kunieda, H.; 2000, An experimental investigation of a type of double layer tensegrity grids*, Journal of IASS*, **41**(131).

Motro, R.: 1984, Forms and Forces in tensegrity systems, in *Proceedings of 3rd Int. Conf. on Space Structures*, Amsterdam, pp. 180-185.

Motro, R.: 2003, *Tensegrity: Structural Systems for the Future,* Kogan Page Science, London and Sterling, VA.

Tibert, A.G. and Pellegrino, S.; 2003, Review of Form-Finding Methods for Tensegrity Structures*, International Journal of Space Structures*, **18**(4).

White, J.: 2004, *Form-finding and Load Analysis of Tensegrity Structures,* MEng Dissertation in Civil and Architectural Engineering, Department of Architecture and Civil Engineering, University of Bath.

Witkin, A.: 1997, *Physically Based Modeling: Principles and Practice. Particle System Dynamics,* Robotics Institute, Carnegie Mellon University, Pittsburgh.