DIGITAL METHODS OF CONSTRUCTION (3A)

PARAMETRICS OF MOVABLE POLYHEDRAL MODELS IN PERFORMATIVE ARCHITECTURE 185
HEIMO SCHIMEK, MILENA STAVRIC, ALBERT WILTSCH, OTTO ROESCHEL

NOVEL TRANSFORMATIONS OF FOLDABLE STRUCTURES 195
DANIEL ROSENBERG

CRAFTING NEW ARTEFACTS 205
GONÇALO CASTRO HENRIQUES

UBICOMP-KAIZEN 215
ODILO SCHOCH

DIGITAL CAD/CAM MEDIA REALIZE CHINESE CALLIGRAPHY AESTHETICS IN ARCHITECTURAL DESIGN 225
CHOR-KHENG LIM

THE STUDY OF BUILDING MANAGEMENT BY USING 3D DIGITAL MODELING AND DATABASE: ABFM 235
PIYABOON NILKAEW

STATICS AND DYNAMICS IN THE PROCESS OF CAD/CAM FABRICATION 245
CHUNG-YANG WANG
PARAMETRICS OF MOVABLE POLYHEDRAL MODELS IN PERFORMATIVE ARCHITECTURE

Applications for moveable shading systems for freeform façades

H. SCHIMEK, M. STAVRIC, A. WILTSCHE, O. ROESCHEL
Institute of Architecture and Media, Institute of Geometry, Graz University of Technology, Austria
{schimek / wiltsche} @tugraz.at, {mstavric / roeschel}@tugraz.at

Abstract. We present a parametrical approach to movable polyhedral models. Based on polyhedral geometry the whole structure consisting of an interconnected series of prisms (with dual spherical joints) can move 3-dimensionally. The principles of polyhedral geometry allow constraint movements of the prisms with a certain degree of freedom. We use these geometrical principles to open and close façades for ventilation or structures for shading control superimposed on building envelopes. The different groups of regular polyhedra in the Euclidean 3-space and their specific topological types will be discussed in order to choose the appropriate model and show the geometrical theory of movable polyhedral models can be successfully applied to performative architecture.

Key words. Moveable polyhedral models; kinematic architecture; parametric design; geometry of joints; performative architecture.

1. Motivation

Moveable, adaptive architecture has always been a dream and a challenge for architects. New (digital) technologies made this dream come true – virtual that is. Software designed for the film animation industry makes it possible to simulate almost any motion or transformation, but real projects often fail, apart from some rare examples (figure 1).
Bigger entities of buildings come to life in a superimposition of a building envelope with shading devices or roofs. These devices are mostly made of linear elements, horizontally or vertically arranged, moveable and/or rotatable. These systems can be applied to any planar façade. Jean Nouvell used a different but rather costly technique. Using a diaphragm, light incidence is assessable through a mechanical Mashrabiya (a traditional, artistic and heavily ornamented Arabic shading screen), computer controlled by sensors. The variable opening size of each single element produces light and shadow effects in the interior of the building (Moussavi, 2006).

Horizontally curved façades often use fin like elements or panels which can be rotated or moved horizontally or both. But thinking of a shading device for a (double) curved building envelope or shading screens consisting of non-linear elements, how would a moveable/rotatable system be like applied to any three dimensional surface?

2. Geometrical Basics

In the three dimensional Euclidian space an object can be translated to any position in space by a congruence transformation. This can be described
mathematically by means of six independent parameters. These parameters may be interpreted as the three coordinates \((x, y, z)\) of a translational vector \(v\) which moves an object parallel from the first position to a second one and three angles \(\phi_x, \phi_y, \phi_z\) which rotate the object around the axes \(x, y, z\) of an appropriate local coordinate system (see figure 1).

![Figure 3. Congruence transformation in the Euclidian three space by translation along the vector \(v\) and rotations around the axes \(x, y, z\).](image)

If one object is connected to another with a joint then its mobility is restricted. Depending on the joint there are less than six independent parameters left to describe the object’s motion. The sum of the remaining parameters is called “degree of freedom” of the motion. If we link several objects of number \(n\) and if we use \(r\) joints we get a mechanism with the theoretical degree of freedom \(F\)

\[
F = 6 \times (n - 1 - r) + \sum_{i=1}^{f_i}
\]

(1)

Equation (1) is called “Gruebler Equation” (Beyer, 1963). If \(F = 0\) our mechanism is theoretically rigid. But if we use specific dimensions and configurations our system of linked objects can be movable although the number \(F = 0\). Figure 6 shows such a physical model. Such a mechanism is called over-constrained. In Roeschel (1995, 1996, 2000, 2001, 2002) several cases are discussed and investigated. The main construction idea uses the theory of plan aequiform motions (Stachel 1992, 1992). We can define such a motion \(M_1\) in a plane \(T_1\) by a global fixed point \(A_1\) and a point \(P_{12}\) which is moved on a straight line \(s_{12}\) (with \(A_1\) not on \(s\)). Then every point in \(T_1\) is moved on a straight line in as well (Bottema and Roth, 1979). Such an aequiform motion can be obtained by a combination of a rotation around \(A_1\) (with angle \(\alpha\)) and a similarity (centre \(A_1\) and scale factor \(1/\cos \alpha\)).

If we rotate the plane \(T_1\) around the straight line \(s_{12}\) in a new position \(T_2\) the point \(A_1\) moves along an arc \(r\) into the new position \(A_2\) and our aequiform motion \(M_1\) becomes an aequiform motion \(M_2\) with the global fixed point \(A_2\).
M₁₂ is uniquely defined by the point P₁₂ which is of course coincident with the rotation axis s₁₂. Via the common point P₁₂ we can achieve a linkage between the motions M₁ and M₂ (figure 4).

Furthermore we can connect several motions Mᵢ in different planes Tᵢ. All motions are congruent in an Euclidian sense and they are derived from the first one M₁. If we act in an appropriate way we can get a closed configuration of this system (figure 6). In Roeschel (1996, 2001) a characterization of closed configurations is given.

With the scale factor $\cos \alpha$ (centre C) the polygons T become polygons U which move towards C and retain their dimensions.
If we take a system of linked aequiform motions with common scaling factor $1/\cos \alpha$ and follow an idea of Stachel (1991) the given system can be superposed by an additional dilatation $D$ with an arbitrary center $C$ and the scaling factor $\cos \alpha$. Center $C$ can be placed anywhere in space even coincident with one of the centers $A_i$. In so doing the planar aequiform motions become spatial congruent Euclidian motions. This means that the dimensions of the transformed objects do not change any more and additionally the linkage between two (neighboring) systems remains. Figure 5 shows five quadrilateral shaped polygons $T_i$ performing aequiform motions and five objects $U_i$ obtained by the mentioned dilatation $D$.

Changing the angle $\alpha$, the quadrilaterals $T_i$ are rotated around their fixed points $A_i$ and scaled by $1/\cos \alpha$. The linking points $P_{i,i+1}$ move on the intersection lines $s_{i,i+1}$ between $T_i$ and $T_{i+1}$ (the lines $s_{i,i+1}$ are not displayed in figure 5). Simultaneously the objects $T_i$ are scaled by $\cos \alpha$ and so they generate new objects $U_i$. They are moved parallel on straight lines towards the center $C$ and rotated around these lines. The quadrilaterals $U_i$ do not change their size and so we can use them for our implementation.

Due to the fact that a dilatation does not change the angle between two planes a linkage between two adjacent ones can achieved by a spherical dual joint. A spherical joint consists of two rotation axes meeting in a common point (see figure 9, right). The implementation of the shown configuration can be easily obtained by prisms or polyhedrons. That is why we speak about moveable polyhedrons (figure 6).

![Figure 6. A physical model of a closed moveable polyhedron system (torus’ type).](image)

In literature four different topological types of moveable polyhedrons are investigated. These are the Sphere’s type, Cylinder’s type (cylinder of revolution), Torus’ type (see figure 4) and Möbius’ type.

In our work we took the cylinder’s type (figure 9, left) and “opened” it. This means that we didn’t take a circle as basis of the shape but a plane circle spline in order to get more design possibilities. In figure 4 an arc $c_{12}$ is shown.
It is tangent to the planes $T_1$ and $T_2$. A combination of such arcs between consecutive planes $T_i$ generates our circle spline. In figure 5 the spline of the aequiform part is indicated by $c$ and its transformed Euclidian part by $d$. We chose a circle spline because it ideally shows the gradient of the shape, can be constructed tangential to the planes which carry the aequiform motions and therefore they are tangent to the transformed planes and can be easily inserted between the rotation centers $A_i$ or $B_i$.

Our transformed objects are plane polygons orthogonal to the plane which contains $c$. Figure 5 shows the aequiform part with the polygons $T_i$ and the spline $c$ and the important Euclidian part with the polygons $U_i$ and its circle spline $d$. In every position there exists a tangent circle spline $d$ which is similar to the initial spline $c$.

3. Application of the polyhedron concept for building envelopes

Based on the theory of section 2 about moveable polyhedrons we explore possible applications for architecture. We use the topic of a light fixture to explore variations of different geometries and ornamentations applied to a parametric model (Schimek, 2008). This digital model (build in SolidWorks) serves both as design and simulation model. This way we can work on the design performing optimization loops never loosing contact to the fabrications process’ necessities. This method leads to a formalization of the design and construction process and allows us to perform simulations of the kinematics of the opening and closing mechanisms. This way we create complex, geometrically fully defined structures reminding in their motions of the opening and closing of a flower and lets the building seemingly move as a whole.

Employing a laser cutter we build plywood models after defining the parametric model. With these models the theoretical, virtual simulated concept of the moveable polyhedral structures will be confirmed and the assembly and a smooth motion sequence will be tested. The substitution of the scale factor $1/\cos \alpha$ by the factor $\cos \alpha$, mentioned in section 2, results in a motion of the linked solids towards each other achieved through the use of spherical dual joints. By opening up the closed polyhedron structure based on a cylinder of revolution it is possible to transfer the theory of the moveable polyhedral motion to any cylindrical surface based on a plane circle-spline and consequently we can apply moveable systems onto any cylindrical curved façade form. The curvature of the spline is a crucial factor for the angle of the spherical dual joints and vice versa. Figures 7 shows the transition of the structure superimposed on a building envelope (the building envelope is not shown on the figure) from closed to open position. The spline curve transforms as the panels are moving.
In closed position the curve of the moveable structure follows the curve of the static building envelope. We furthermore observed that the movements of the panel centers (e.g. B21, B22) towards center C on their dash dotted motion axes are defined by the geometry. These motion axes are the possible location to mount the moving parts on the support structure using sliding brackets. C can be located anywhere in space, that gives the needed flexibility to meet constructive needs for any kind of single curved façade. If C is incident with one panel center then this panel has to be fixed in its position (mounted on the building envelope), only turnable around a perpendicular axis, the remaining panels (turnable on their own axis as well) are slideable mounted on the geometrically defined motion axes through C (dependent on the load condition not each panel has to be mounted) with a specific spacing – theoretically just a single drive is needed to propagate the rotation in the whole structure.

![Figure 7. Curved façade in closed and opened position – simplified scheme without substructure. The circle spline in closed position follows the form of the building envelope. As the system is moving the spline transforms. The panels are moving towards and away from C keeping their centers (e.g. B21, B22) on the dash dotted motion axes.](image)

**4. Design parameters**

The 3D-model’s structure is hierarchical, the control structure of the shape and all building parts are stored in separate files (Part-files), and then assembled in an assembly-file where external references define their relationship to each other and turn the single components into intelligent components. Following the rules there is one component of the structure, which has to be mounted on the supporting structure keeping just one degree of freedom – the rotation on its own axis. Depending on the rigidity of the joints and the panels it is necessary to mount the moveable structure on the supporting beams with a specific spacing.
How can it be achieved to connect a building part which is both rotating and moving to a rather static structure?

To accomplish this task we have to figure out the geometry of the motion paths of all components involved. In our case (cylindrical polyhedron) we identified a motion of the panel’s center on a straight line targeting to the center of the fixed panel (Figure 7). Exactly on these axes we may mount the panels on the supporting structure using sliding brackets. Once these constraints are set the parameters of the panels are defined based on the geometry of the system and the design of the panels (Killian, 2007). There are two conditions we have to consider in the virtual model: static condition ruling the design and construction process, and dynamic condition defined by the motion of the system.

The motion of the panels – rotation and linear motion on a straight line – which has its foundation in the theory of the moveable polyhedral structures results in a volumetrically alteration of the whole structure. The parameterics of such a structure can only be found using steps. Step one is to define the layout (plan) of the facade in xy, which can be a straight line or a more complex form like a spline and likewise in z-axis direction. If the structure follows a double curved surface (with planar panels) the best way for implementation is to use 4 neighboring panels which are tangent a common sphere or cone. Such structures are called conical meshes (Pottmann, 2007).

Based on the geometry of the facade we establish relations and constraints between panels and joints - single joints with a connecting angle of 180° and dual joints (spherical joint) with angles < or > 180° between neighboring panels, see figure 8.

Figure 8. Left: Simplified Aluminum substructure of panels applied to closed test polyhedron model (cylindrical type). Right: Suggested detail of dual and single joints.

As supporting structure we suggest to use aluminum components carrying the panels which allows a high degree of standardisation of the moving structural parts and eventually leaves a higher degree of freedom to the design of the
panels (to a certain degree they are independent of the support structure) which eventually are responsible for the formal composition of the façade. They can be mounted later in the assembly process what makes them exchangeable in the case of damage (a simplified example is shown in Figure 8).

We use two types of pivots, single joints allow a rotation of two parallel neighbouring panels on an axis whereas dual joints permit rotations on two axes what we need to employ for (double) curved structures. Since dual joints are the key elements for moveable facade systems we build a 3D-model in a MCAD software program to perform all simulation and optimization work virtually. We do not only test fluent kinematics of the system but also identify weak spots or oversizing of the joints and the aluminum substructure. The variation of the panel geometry can be also carried out parametrically dependent on the geometry of the substructure, which is again dependent on the surface geometry of the building envelope. Possible ornamental variations are defined (Figure 9) and limited by the overall form. In addition perforation of the panel material controls light intensity of the closed shading system depending on the screen ruling.

![Figure 9. Prototypes made of plywood to test variable ornamentations and movement](image)

For the Motion-simulation we cannot sustain the dependencies established for the structure’s design process since these constraints would ultimately lock the rotation of the panels and consequently prevent the desired shift of the structure as a whole. The chosen type of polyhedral model determinates the horizontal and vertical gradient of the movement. For that reason we design the superior skin in closed condition where it has the smallest volume expansion (closest distance to the support structure). For the dynamic conditions of the system defined by the motion of the system we establish rules and constraints given by the geometrical basics of section 2 what mainly serves one goal, namely the aequiform motion of the panels towards a defined center on a straight line.

SolidWorks tests the virtual model using FEM (Finite element method) to optimize all components of the system in real load conditions. Tested load conditions involve net weight of the components, bending moment in pivots as a result of motion and moments due to wind loads. From the optimization
results we build an 1:10 mock-up to test frictionless motions of the system’s pivots and substructure with a physical model. In a second step a full-scale model will be build to test the load presumptions of the FEM simulation.

5. Conclusion

Non-Standard architecture with freeform building envelops are becoming more mainstream thanks to digital design and fabrication methods making non-standard affordable (Sakamoto, 2007). Now we need alterable shading systems for those forms that current systems cannot provide. By applying the theory of moving polyhedral models on architectural elements we can provide moveable skins for every cylindrical curved surface. Architecture can be dramatically aesthetically enriched with moving building envelopes capable of changing their entire volume. For future investigations we will face the challenge on applying our system on double curved surfaces.

6. Acknowledgements

We would like to thank Paul Randig for providing the digital 3D model of the substructure. We deeply appreciate Urs Hirschberg’s consistent encouragement.

7. References