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ONE-PIECE WEAVING

Reconfiguring folding and knotting algorithm in computational design

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Abstract. A beneficial symbiotic relationship between traditional crafts and new technologies may be achieved when computational designers view the existing traditional art and craft as partners to collaborate with and when traditional cultures are willing to accept new technologies in an enthusiastic yet critical manner. This research aims to reconfigure computational design paradigm at the intersection of traditional and digital technology by evaluating a series of relatively recent computational design experiments aimed at reconceptualizing weaving as a combination of folding algorithm and knot theory with respect to the apparent dialectical tension between traditional context and computational theories in architectural design.

Keywords. Weaving; folding; knot

1. Introduction

This research evaluates weaving mechanism as a rational artefact. In particular, we consider weaving as an assemblage of folding creases and knot's crossing points. To weave means to create a knot, and to make a knot, we need to fold. Since weaving has a rich cultural aspect in its abstraction and rationality, folding and knotting history has a strong cultural and computational background.

1.1. FOLDING

Folding ubiquity exists both in nature and culture. In nature, folding can be found in leaf mechanism, protein folding, insect wing, and many others

(Forbes, 2006; Couturier et al, 2009). In traditional culture, folding has been widely used since 794-1185 A.D. (17^{th} century) primarily as a form of art, e.g., origami in Japan during the Heian Period (Demaine and O'Rourke, 2007). Moreover, folding is also utilized in art and technology. In contemporary design, folding has been pervasively used in architecture (Vyzoviti, 2006), fashion (Miyake, 2008), and many industrial products (Yigit, 2004). In engineering, folding is used to fold the solar sail for the satellite and safety bag for cars for its compactness as well as in shell structure for its rigidity (Candela, 1964; Engel, 1968).

1.2. KNOTTING

The ever-present knot has existed since humans first used a knot for tying a rope in hunting and clothing. In nature, birds use a knot to fasten their nests and spiders knit their webs with knots. In ancient culture, knots were physically used in numerous functions, such as textiles, huts, bridge construction, and accounting devices (khipu). Additionally, the knot is also visually depicted in many primitive symbols associated with folklore, magic, and mythology. In science and engineering, the knot has been widely scrutinized by the physicist, (Kauffman, 1987) biotechnologist, transportation engineer, electrical engineer, and many others. In today's contemporary design, the knot (like the fold) has been utilized as a main circulation concept in architectural design (e.g., Mercedez-Museum in Stuttgart by UN Studio) and tessellation in fashion design (e.g., Emily Hiller Silk Knotted Garment). Thus, folding and knotting are not unique in either traditional or computational design

1.3. PAPER CUT (KIRIGAMI)

To combine folding and knotting, we look at the *kirigami* method, a variation of origami techniques that allows paper cutting ("*kiru*" = to cut, "*kami*" = paper) (Temko, 1962). Kirigami has been commonly used for pop-ups artwork in children's books, gift wrapping, and greeting-cards. Some recent attempts to enlarge the scale of kirigami shows its promising use in architectural design, (Vyzoviti, 2010) fashion design, (Ryan, 2009) and computational biology (Harth, 2010). In Poland, the paper-cutting art has become part of their folk-art (Wycinanki) and national identity, as shown in the Poland pavilion in Shanghai 2010. Kirigami's artists have been exploring paper cutting art in both two-dimensional works such as Gjerde's Yaguchi and in three-dimensional works of art (Chatani, 1985; Badalucco, 2001). While most of these designers are using a continuous surface, kirigami can also be assembled by discrete modular elements (Hart, 2007). George Hart's *modular kirigami* is one of the first attempts to combine paper cutting and knotting, and shall further be elaborated in this research.

Given the versatility of these three techniques in natural, cultural and computational designs, the following experiments attempt to elaborate the virtue of folding, knotting and paper cutting as parts of the weaving techniques within a mathematical model.

2. One-piece weaving

Here, we would like to compare the knots theory and folding algorithm underlying the weaving configuration. In mathematics, the knot theory studies the complexity of knots indicated by the number of crossings (Tait, 1860) and the different projections of the knot in a two-dimensional diagram (Reidemeister, 1920). With this complexity, one of the main questions in the knot theory is whether certain projected knots are equivalents (isotopic) or not. On the other hand, the folding algorithm starts by understanding the linkage behavior on a surface. Folding complexity emerged from its two simple rules, mountain and valley crease assignment, to create various linkage configurations, two-dimensional surfaces or three-dimensional polyhedral. Some of the main questions in the folding algorithm are whether the crease pattern can create a flat-foldable surface, folding without collision or generate many forms from the same shape with different crease patterns. Following are the comparison between knot and folding.

2.2. SIMILARITY: JOINTS

We first share the compatibility of the folding and knotting definition within a paper strip and its interlaced configuration. In a folding algorithm, the paper strips are considered one-dimensional (1D) *linkages,* while the segments are called *links* and the shared endpoints are called *joints*. Linkages can be open as one straight chain or closed as a convex chain where the start and end are the same (Demaine and O'rourke, 2007). In the knot theory, the convex chain paper strips are understood as unknotted links that are homeomorphic to a circle in a 3D-space (Kauffman, 1987). When we twist a rubber band for example, a knot can be seen as a crossing point resulting from the intersection of the circle perimeters where the crossing creates an over and under link condition. In other words, the knot is topologically the same as convex linkages in folding, where the *crossing* is equivalent with *joints*.

2.3. DIFFERENCE: FLEXIBILITY

Secondly, we look at two fundamental differences between folding and knotting: the creases and the crossing point. Knot theory assumes the link as a flexible surface and focuses more on its crossing rather than the motion mechanism of how the flexible surface turns and bends to create the knot. In contrast, folding algorithm constrains the link mostly to a rigid surface and crossing is forbidden in folding algorithm. Therefore, folding algorithm is highly concern with the movement behavior and the directionality of the links and vertices, in which the valley and mountain crease's patterns act as an axis to control the rotational degrees of freedom of the embedded linkages on a surface.

3. Interwoven paper strip

Based on the above compatibility and differences, we use long narrow paper strips to explore folding algorithm and knot theory. Paper strips have both flexibility for knotting and surface rigidity for folding that allows us to use principles from the one-dimensional folding motion mechanism, the straight cut method, and interlocked chains. To perform a rigid interwoven joint on the paper strips, we need notches where the surface can be interlocked. Here, the notch is achieved by simply making a half-cut on the paper strip from its edge perpendicular to its axis so that two surfaces will share the half-cut as tab and notch. The main constraints in this project are the continuity of the single paper strip, which means all notches and tabs are laid out along the same surface (Figure 1).

We assume that all notches and tabs can be made in one single cut. In particular, we are interested to find out how to fold the paper strip into the shortest segment possible but long enough to create the least minimum half-cut pattern required to assemble the overall knitting pattern as described in the following two sub-problems:

- Given a 2D weaving pattern design, segmentize the pattern to define the knotting unit, which has at least one (+) or (–) skein notation.
- Characterize the crease patterns to fold the whole paper strip into one segment so that all the half-cut line would mingle in one point for a single cut.

Based on this problem, the following three studies look at how this interwoven paper strip related to the assembly, form-finding, and structural issues in design.

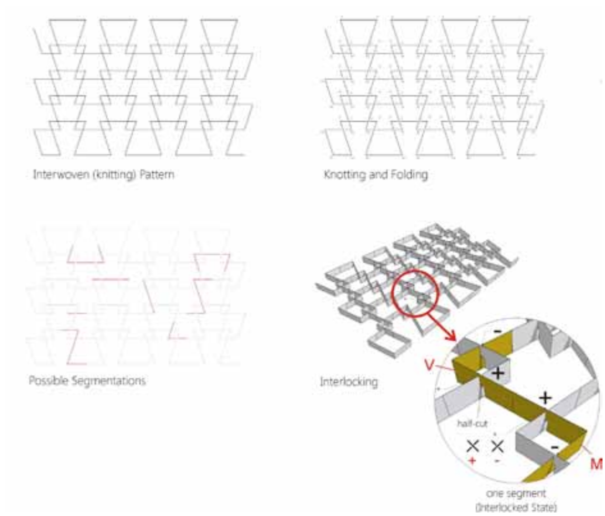

Figure 1. Interwoven surface as an interlocked, one continuous paper strip

3.1. ASSEMBLY: STRAIGHT SKELETON METHOD

In pursuing the half-cut pattern in the paper strip, we adopt the straight skeleton method and the one-dimensional flat-folding principle. In the straight-skeleton method, the skeleton is basically a folding crease that resulted from offsetting the edge of a particular graph to the inside until all the offset-edges collapse into a point (Aichholzer, 1995) and also as a reflection axis resulting from the bisection of two edges (Demaine, 2006). The trajectories between this point, bisector axis and all graph vertices, become the mountain-valley creases to fold the paper so that all original graph edges will meet in one straight skeleton line to be cut. In this case, the half-cuts are considered as the graph edge (Figure 2). When we position two identical half-cuts face to face, for instance, a neighbor of a notch is also a notch, and both of them are parallel. We may use only the offset method where the half-cut line between the half-cut will collapse right at the midpoint between the two and create just one M–V crease (Figure 4).

Otherwise, if the neighbor is different (i.e. the neighbor of notch is a parallel tab), we need a bisector method to first de-parallelize the two half-cuts from the diagonal axis between them, and then find the reflection axis between them, and create four M-V creases. With this method, we may be able to cut all random-distance, half-cut patterns in a single cut (Figure 3).

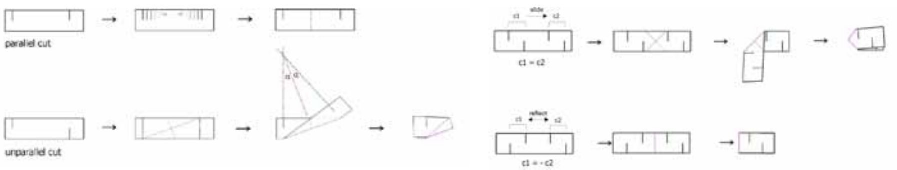

Figure 2. Half-cut distribution along the paper strip

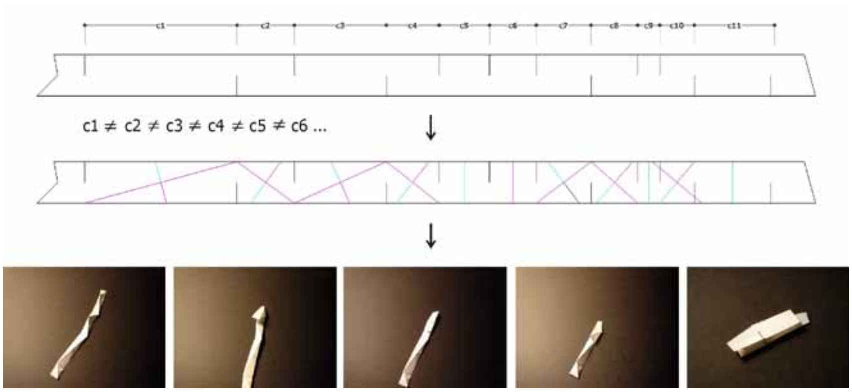

Figure 3. Random half-cut pattern in folded states

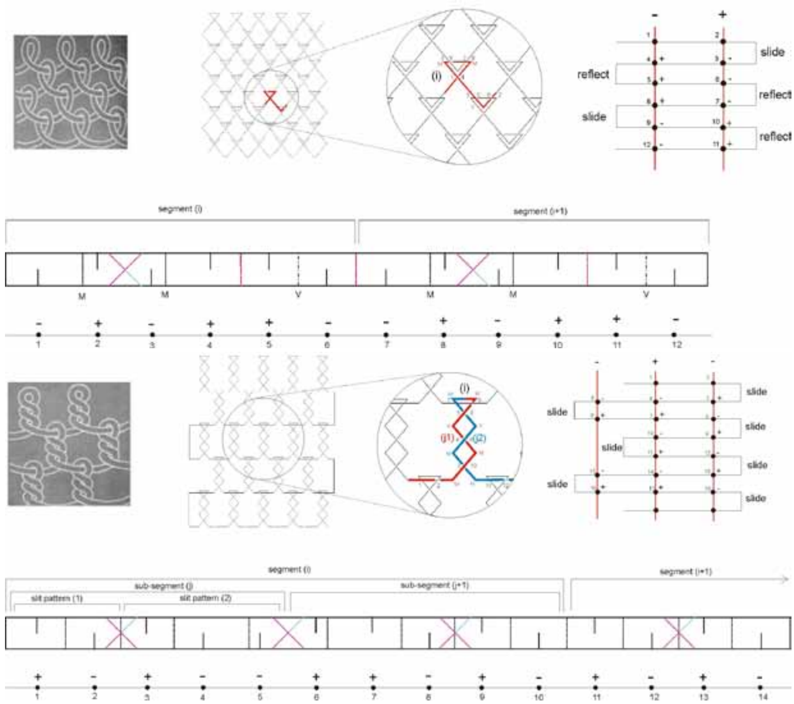

Figure 4. Converting an ancient looping knot into a paper strip pattern diagram.

3.2. FORM-FINDING: GLUING METHOD

Because a knot is considered homeomorphic to a circle in the algebraic topology, it can have numerous variants in its planar projection. A ten-crossing knot, for example, can have over 160 different projections on the knot table (Tait, 1860). Similarly, in folding algorithms, one Latin-cross polygon can be folded into 23 different convex polyhedral shapes by assigning different valley-mountain crease patterns and applying Alexandrov's theorem to join the edges and vertices (Demaine and O'Rourke, 2007). For this form-finding process, we exploit this ambiguity by combining these two types of representations. Alexandrov's gluing method works by looking at the possible paths that one vertex or edge might take to travel to other vertices and edges in the same polygon; these paths would then create mountain or valley crease patterns. Thus, in adapting Alexandrov's theorem in the paper strip activity, we essentially glue the half-cut vertices as a constant variable, and change the mountain-valley assignment as a parameter to generate different compositions. Figure 5 shows the basic rules for how to glue two vertices together as a knot's crossing point, as well as how to fold the paper strips to create convex or concave shapes within the mountain-valley assignment.

As a case study, we considered paper strips as the representation of the edges of three-dimensional objects (Figure 6). The generated designs in Figure 7 shows how we might have a different design by swapping the mountain-valley assignment and the crossing-point connection in a knot, each of which can potentially be transformed back to its three-dimensional shape to create a different architectural design.

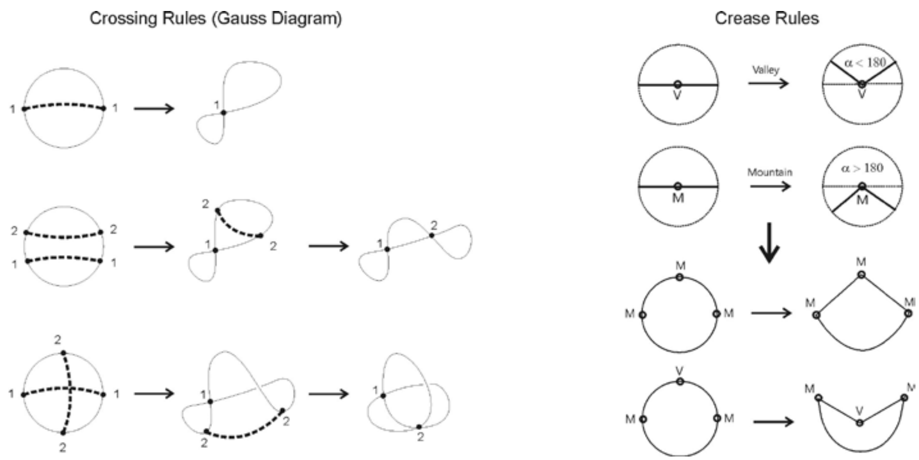

Figure 5. Knotting and Folding Rules

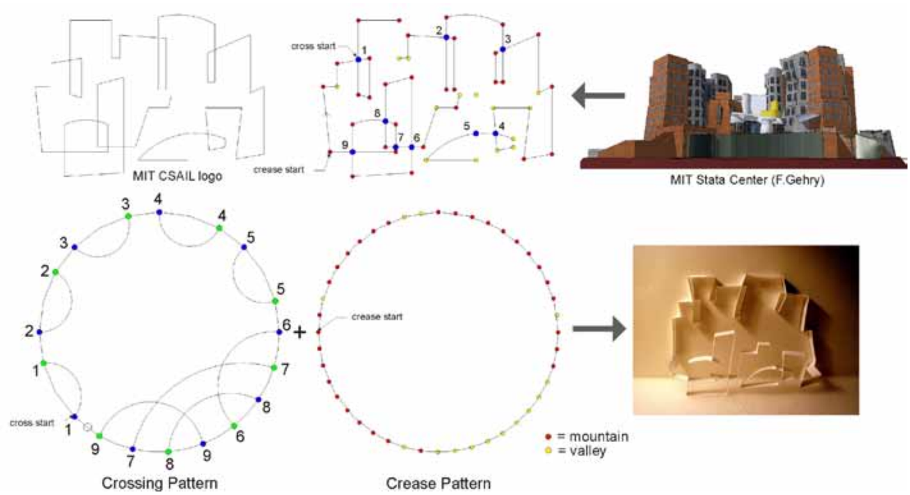

Figure 6. Knot as 2D Representation of Architectural Design

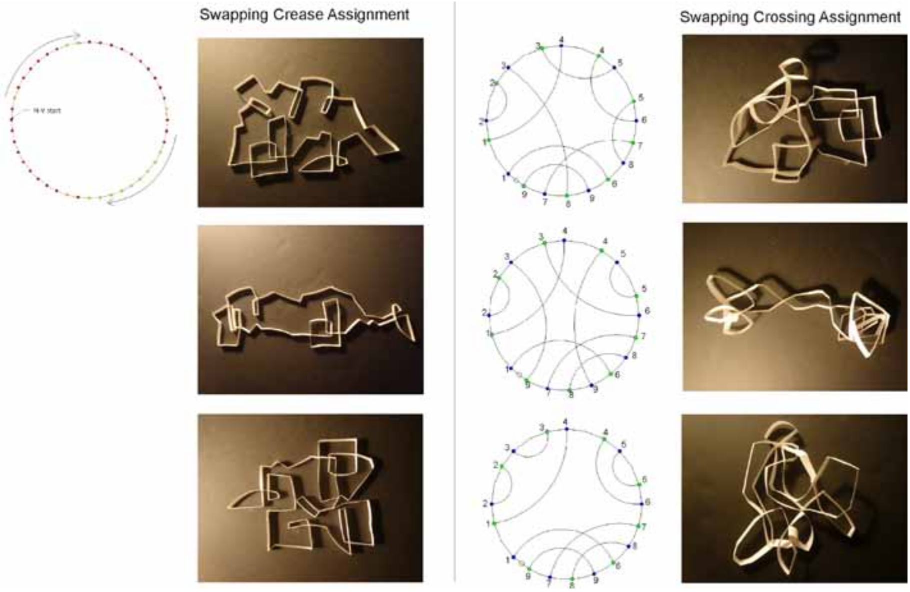

Figure 7. Modifying Crease and Knot Assignment to Generate Different Designs

3.3. STRUCTURE: LINKAGE RIGIDITY

The interwoven surface is rigid when a yarn segment performs a fixed-support structural joint by having an over-and-under configuration with other nonparallel yarns. As the number of crossings may be increased once a knot is projected in its planar projection, we need to investigate the looseness/tightness of a knot by reducing all the unknotted crossings into the knot's invariant; i.e., the real over-under configuration that tightens the knot. For this investigation, we use two approaches. First, we use three Reidemeister moves to undo the knot and find the *minimum* invariant of the real number of crossing point in 3D space (Reidemeister, 1926). Second, we use the Alexander-Conway Polynomial to assign a traverse direction to a knot (*oriented links)* and calculating the *linking number*—the number of invariants of a knot (K) resulting from the sum of the number of over (+1) and under (–1) at each traversed crossing point (P) [link (K,L) = ∑P, such that if ∑P/2 = 0 then K = [unknot] (Alexander, 1928; Conway, 1970). This knot algorithm corresponds to the interlocked-chain problem in the folding algorithm. A tangled linkage is considered rigid if it is interlocked and cannot be separated, which would require at least a flexible three-chain with closed-quadrilateral linkage and a flexible four-chain with closed triangle linkage (Demaine et al, 2007).

Folding rigidity is also maintained by keeping the linkage consisting of exactly *n* joints and *2n - 3* or more links to avoid the nontrivial motion of the joints and links (if the graph has fewer than 2n-3 links, then it is never rigid) (Lamans, 1970) (Figure 8).

Thus, Alexander-Conway notation is useful to perform knot rigidity on the interwoven surface while Laman Theorems can evaluate the rigidity of the folded structure. In addition, the interweaving process also reveals that an area where the crossing points are very close to each other can be rather difficult to assemble even with a flexible paper strip. In this situation, it is necessary to loosen the knot by avoiding a crisscross over/under knot (e.g. + - + - + - + -) and instead, do an interval crossing (e.g. + + + - - - + + + - - -).

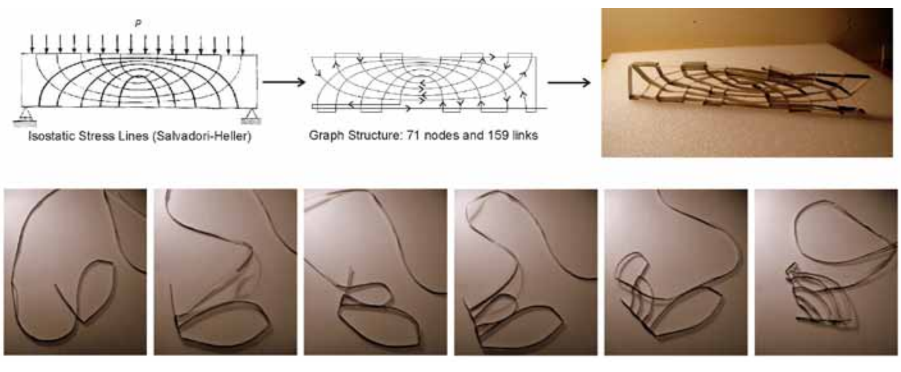

Figure 8. Stress lines as interwoven surface and assembly process

Figure 9. Load testing on the interwoven stress lines

4. Discussion

This research shows a novel way to generate a new kind of architecture based on traditional production technique. In particular, the experiments have demonstrated the role of one-dimensional paper strips in three domains:

- **Generative design**. The application of Alexandrov's Gluing Algorithm in Gauss's diagram shows an intrinsic feature of folding and knot theory to generate various computational designs by manipulating the crossing points and folding the creases physically. The use of paper strip in the form-finding experiments allows any type of one dimensional representation to be generated within this algorithm.
- **Fabrication**. Both folding and knotting can work seamlessly in the fabrication process to assemble interwoven surfaces with just one continuous straight paper strip. This process works well in both traditional and computational manner. The experiments were held with lesser digital computational devices yet the processes are combinatorial.
- **Structure**. The interwoven paper strips configuration exhibits some degree of rigidity that can support a uniform load distribution. This finding not only opens a new path for expressing ornamental structure in a more affordable way, but also allows an alternative view in assessing structural performance with knot and folding algorithm.

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References

Buck, D. and Flapan, E. (ed.): 2009, *Applications of Knot Theory: American Mathematical Society, Short Course, January 4-5, 2008, San Diego, California*, American Mathematical Society, Providence, Rhode Island.

Ascher, M.: 2002, *Mathematics elsewhere: an exploration of ideas across cultures*, Princeton University Press, Princeton, New Jersey.

Demaine, E. D. and O'Rourke, J.: 2008, *Geometric folding algorithms: linkages, origami, polyhedra*, Cambridge University Press, New York.

Engel, H.: 1968, *Structure systems*, Praeger, New York.

Sossinsky, A.: 2004, *Knots: mathematics with a twist*, Harvard University Press, Cambridge, Massachusetts.