T. Fischer, K. De Biswas, J.J. Ham, R. Naka, W.X. Huang, *Beyond Codes and Pixels: Proceedings of the 17th International Conference on Computer-Aided Architectural Design Research in Asia,* 7–16.

RATIONALISATION OF COMPLEX BUILDING ENVELOPES

STYLIANOS DRITSAS
Singapore University of Technology and Design
stylianosdritsas@sutd.edu.sg

Abstract. Rationalisation of architectural geometry is paramount to a design's manufacturing and construction. This paper presents a methodology of pre/post rationalisation of building envelope geometry using statistical computation.

Keywords. Rationalisation; optimisation; geometry; computation.

1. Introduction

Rationalisation is the process of information reduction or compression of an architectural design with the goal of simplifying and optimising its geometry towards manufacturing and construction. Building rationalisation shares common goals with value engineering. However, rationalisation is both a qualitative in as much as quantitative process. Thus, it is a core architectural responsibility rather than a cost management exercise, and as such, a process of design in as much as a process of analysis. Architectural rationalisation aims at the control of quality from detail design to a building's construction phase as well as gains in building performance in term of material resources use, manufacturing time and labour expenditures.

This paper studies the process of building envelope rationalisation using statistical computation methodology of optimisation as means for addressing the problem of varying geometry in contemporary free-form design. The aim of this paper is to: (a) Discuss methods for understanding, determining and addressing design complexity (b) Review two theoretical frameworks for design rationalisation: the pre-rational and post-rational design principles, illustrate their benefits and limitations and demonstrate their meeting point and (c) Propose an integrated performance-oriented model for analysis and design for building envelopes, using digital design technology.

2. Design complexity

Design complexity is a difficult notion to discuss in absolute terms as there is no single metric that entirely captures it. We can employ for example a design's inputs or outp uts as complexity indicators; for instance, geometric complexity or the absolute size of parameter or solution space. In this paper we use a functional definition of complexity composed of three indicators: (a) Predictability: the insight a design system may offer as its capability of revealing the critical points of complexity escalation (b) Compactness: the degree by which a system can capture design rules and exceptions with a limited number of parameters. (c) Reliability: a system's capability to ease the verification and communication of its state. We feel these indicators are expressive of the procedural qualities of a system rather than merely its quantitative characteristics.

While a rectangular-grid curtain wall system implicitly ensures all of the above criteria, a free-form envelope does not offer the same guaranties. The only certainty is that there will be ranges within building dimensions vary. This becomes a significant problem for contemporary architecture and its construction as it raises questions about a responsible and cost effective use of natural and human resources. The aim of our methodology is to assist in assessing complexity, rationally evaluating the costs and benefits of a design and taking positive steps in optimising its construction.

3. Related work

There are two contemporary methodologies of architectural rationalisation within the context of digital design: the principle of (a) pre-rational and (b) post-rational design. Both have been extensively discussed by Shelden (2002), Glymph et al. (2004), Whitehead (2004), Hesselgren et al. (2007), Ceccato et al. (2010). The pre-rational method is founded on the proactive application of logical constraints and the expression of design as a process of interpolating composition of geometric primitives, such as lines and arcs, quadratic and developable surfaces. Those are known to exhibit desirable properties (Leopoldseder and Pottmann 2003, Hoffmann and Kocacs, 2004, Pottman 2007) which may simplify construction. The post-rational method is a functional approach, in the engineering systems notion, towards architectural rationalisation. It is heavily based on the use of design metric analysis and iterative optimisation processes of design and computation. The critical assumption made is that design is an input and it is retrospectively approximated to a rational model up to a notional design intent error tolerance. Optimisation strategies for building structures and envelopes have been demonstrated in the past by Williams (2001) as well as by Luebkeman and Shea (2005).

4. Geometric description

We investigate a prototypical rationalisation exercise of a complex building envelope optimising its geometric properties towards visual, manufacturing and construction constraints. Our system is composed of three components: (a) Geometric description using parametric design principles (b) Analysis using metric performance criteria (c) Optimisation using statistical methods. Our goal is to design a curved wall in plan and section; a generic description for a free-form envelope. We express this geometrically by employing a pre-rational strategy of a piece-wise circular tangent arc sweep surface (Figure 1), also known as translational surface (Schober 1999). The result envelope belongs to a family of surfaces exhibiting certain desirable characteristics: (a) Components are linear and circular arc segments which are simple and offset without degeneration (b) Discrete surface patches along the surface's primary directions are planar quadrilaterals and the envelope is developable (c) Visual constraint of tangent continuity is ensured by definition.

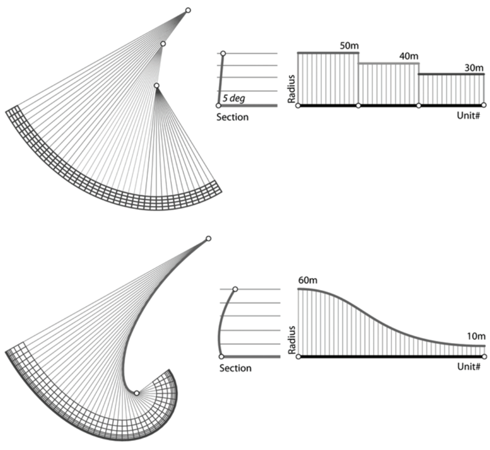

Figure 1. Rational envelope definition as a function of radius.

Parametric design with pre-rational envelopes entails defining a chain of explicit geometric relationships between the radii of consecutive arcs such that at the transition point both arcs share the radial vector but not the same magnitude (Figure 1). A sectional profile may be swept along the base path to generate the envelope. Our implementation expresses planar geometry as a function of curvature, instead of individual arcs, such that we can span a wide range of free-form geometry. We achieve this generalisation by a mathematical composition of the basis rational function with a NURBS curve geometry graph. The planar section may be of arbitrary complexity without violation of the said geometric properties. In similar fashion we may also parameterise the spanning angles/chords such that we can increase resolution density at façade regions of high curvature.

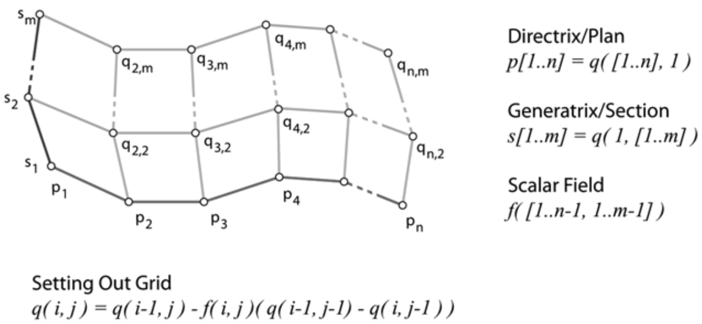

Figure 2. Recursive definition of rational envelope determined by linear displacements.

Radius/chord parameterisation is capable of spanning a large range of pre-rational envelopes containing both discrete and continuous curvature characteristics. An important limitation of radius-driven geometry is that it becomes numerically unstable at planar or near planar wall regions as radial vectors become parallel and radii approach infinity. This shortcoming may be mitigated by employing an alternative rational basis function using linear combinations of displacements instead of radii (Figure 2). The definition is comprised of a planar scalar field, a directrix curve in plan and a generatrix curve in section (Dritsas and Becker 2007). Both curves may be of arbitrary geometric complexity without violation of the desirable rational properties. Displacements are directly related to the physical dimensions of envelope's bay sizes which are guaranteed to remain within numerically stable limits. A typical cladding bay size for instance does not exceed a few meters due to architectural and manufacturing constraints. While this definition is more versatile it is also more complex as the scalar field, a 2D matrix of bay sizes, is more cumbersome interacting with compared to the radius/chord graph.

5. Metric analysis

Setting out geometry spans large regions of a building's envelope and requires subdivision into smaller components which can be manufactured using standard shapes. If the arcs' radii are large enough they may be approximated by linear segments without visual distortions (Figure 3) such that manufacturing costs of bending can be eliminated. There exists no objective metric determining when this simplification is acceptable, as it is bound to visual preferences, fabrication constraints and cost control considerations. However, as a general rule of the thumb shallow arcs of sagittal lengths of a few millimetres may be linearised with nominal errors.

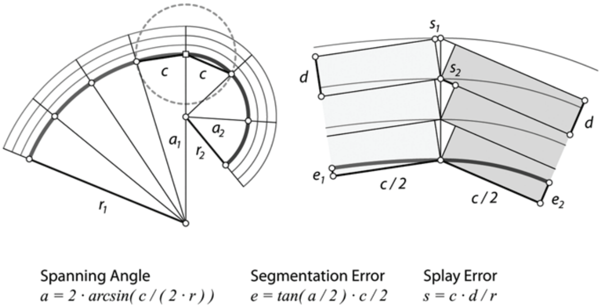

Figure 3. Geometric principle and expressions of approximation errors.

An additional constraint at the phase of segmentation is often imposed by the desire to control the total number of dimensionally unique parts; a logistical and cost control consideration as large volumes of varying parts increase manufacturing and assembly time and also encumber the process of building information management. It is thus reasonable to attempt reducing variations by using a single or small number of notional cladding unit sizes. There exists two common segmentation strategies: (a) Using a single chord length to greedily subdivide every arc of every level and accumulate errors at the start/end of each (Hesselgren et al. 2007) (b) Using a single chord length at the base of the setting-out and allowing each level a unique type per arc such that errors are distributed between units.

Our system employs the later strategy and in specific we use a fixed chord length for determining each base arc's total spanning angle. In this fashion transitions between arcs occur exactly at the end/beginning of a unit and every segment belongs exclusively to one particular arc instead of spanning across two. The aim of this constraint is to eliminate angular errors induced per level and special unit types required to span transitions had we allowed segments to span across arcs.

We further observe that a chain of linear elements along an arc suggests that each joint between segments is mitred at the bisecting angle a/2 which indicates that for each arc there is a unique type. For transitional segments between tangent arcs of different radii there are additional types as the bisecting angle varies laterally. For mitigating manufacturing complexity these angles are averaged or squared to 90 degrees, such that additional time and cost is reduced. This approximation induces a particular visual artefact, commonly known as the splay error; whereby small gaps can be occur between consecutive segments (Figure 3). There is no general consensus as per when splay may be ignored or introduces significant visual nuances in a building's appearance. In the case of a tall building's envelope for instance, splay artefacts become of lesser concern proportionally to an observer's distance from the ground level. In practice, a common rule of the thumb is to ignore splay errors when they remain below typical architectural detailing and/or manufacturing tolerance. For example 10 mm splay error is within an acceptable range since even planar walls exhibit this dimensional range of detailing between joints.

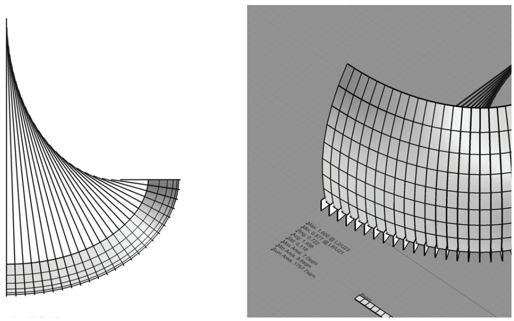

Figure 4. Interactive visualisation of envelope performance metrics.

Our design and computation system provides interactive performance feedback via direct visualisation of dimensional as well as error metrics onto the design envelope (Figure 4) such that design and analysis become unified rather than sequential and disjoint tasks. Schematic design and construction implications thus merge intuitively as bridges across three levels of detail, namely setting-out, zoning and detailing, are established. Free-form design becomes creative while informed.

6. Design optimisation

Our final step in rationalising the envelope's geometry attempts to further reduce dimensional variation. We note that we cannot improve on the total unit number without drastically simplifying geometry to a smaller number of radii in plan and/or a simpler section or alternatively without introducing new approximation errors. A degree of freedom we may exploit is to attempt compounding additional errors within existing. For instance, it may be more desirable to constraint a unit's size rather than its splay error. Allowing errors to vary may permit clustering units by length increments and in return reduce the total type number required to span the entire envelope.

Identifying a discrete number of unique types that best approximate variation among a set of samples while bound by an error metric can be seen as a data clustering problem. In particular it is an NP-hard problem that can be solved via numerical approximation (Jain et al. 1999). As a benchmark we employ a basic fixed-increment distribution strategy in order to understand the qualitative aspects of our problem but also compare and contrast alternative clustering strategies. We define a unit increment step-size of for example a few millimetres, we then round elements down to their nearest cluster, measure the average and maximum errors and count the number of non-empty or unique types (Figure 5). The maximum rounding error in this scenario is bound by the predetermined cluster's size, while the average error is approximately half of that. The process is efficient to implement yet far from optimal in finding the best cluster partitions as it disregards the density characteristics of the original probability distribution.

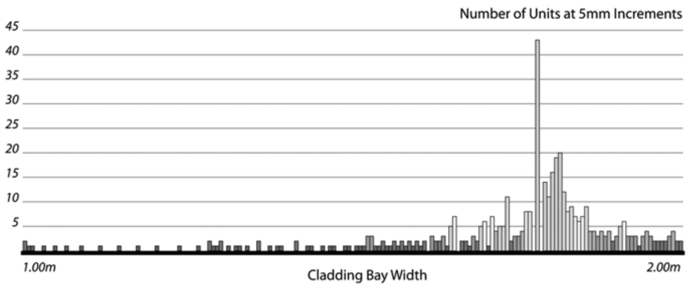

Figure 5. Unit type distribution of continuous radius function envelope.

For selecting better cluster pivots we developed a constrained k-means algorithm (MacQueen 1967). K-means clustering is a statistical method for minimising intercluster variance. The process selects randomly a number of k-cluster partitions, associates each element with its closest cluster and repositions the partition centres at the mean of each cluster's elements. The algorithm converges after a few iterations and it is sensitive to the initial seed-cluster locations. Our application employs a modified k-means method because we cannot allow units to increase dimensions up to their nearest cluster; expanding units would result to geometric clashes on the envelope. Instead, we pin one cluster to the lowest bound of the initial distribution and skew the final cluster's positions to their individual lowest bound. While the modification challenges the distance metric of the original algorithm it still performs on average 11% and 35% better than the benchmark strategy in terms of maximum and average errors, respectively (Figure 6).

While k-means improves intercluster error it cannot address the problem of minimising the maximum error which is visually more pronounced. A qualitative interpretation may suggest that minimising the maximum error expresses the desire of controlling the largest visible gap across an envelope while minimising the average error implies that rounding transitions need to be as smooth as possible. In order to minimise of the maximum intercluster error we employ the k-tMM strategy (Gonzalez 1985). The algorithm splits an initial cluster containing every envelope unit into sub-clusters while maintaining the said criterion. Each unit is rounded to the lowest bound of its cluster. The algorithm deterministically converges within two times the optimal solution (Gonzalez 1985). Our implementation achieves on average 16% and 50% better than the benchmark strategy in terms of maximum and average errors, respectively (Figure 6).

Overall, we achieve improved results with both alternative min-average and min-max strategies compared to our initial benchmark strategy. In terms of complexity k-tMM is simpler to implement, it runs faster, it is much more predictable and yields better results for large cluster sizes in terms of both maximum and average errors perhaps due to k-means modifications that skew what should have been a symmetric metric. However for small cluster sizes the randomised nature of k-means achieves better results than k-tMM.

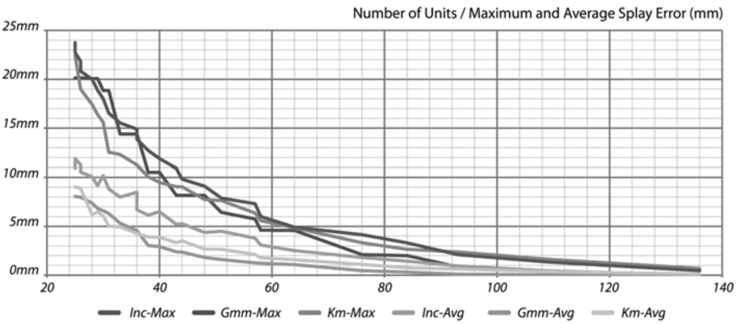

Figure 6. Maximum and average errors of clustering strategies.

7. Discussion

In conclusion, our case study makes a series of assumptions and follows through one of the multiple decision making trajectories available during the process of envelope rationalisation. As we cannot exhaust the entire range of design permutations we offer instead a strategic thought map comprised of three domains: (a) Schematic: geometric definition of the design envelope (b) Detailing: design decomposition of an assembly into components and identification of performance metrics and (c) Construction: mediation of visual and manufacturing considerations via optimisation methodology.

Our notion of design rationalisation as a process of design information compression touches the topics of: (a) Operations: the reduction of design variance in free-form building envelope and as a result achieve gains in the ability to control of quality over cost, and (b) Design thinking: structuring design relationships via pre and post rational modalities in an expressive yet compact process that assists gaining insight in critical design factors and in turns leverage overall design complexity. (c) Design computation: present an integrated design and analysis method where schematic design bridges across to construction considerations; thus transforming those traditionally sequential and remote phases into an expressive while informed process.

On the domain of operations we presented a study of creating typology clustering of variable building parts via statistical method. The contributions of this study are in (a) Defining relationships between quantitative metrics, approximation errors, with qualitative assessments, visual appearance (b) Offer tools for cost/benefit analysis by expressing typology as a function of error; or in other words cost as a function of visual performance. Using this approach we may select a limiting visual threshold and derive the number of units and cost, and vice-versa, or seek equilibrium optimisation whereby the marginal error equals the marginal cost.

In design thinking we selected an indicative path of employing pre/post rational methodology according to aforementioned complexity assessment criteria of predictability, compactness and reliability. We chose pre-rational approach to schematic and detailing analysis while in construction we engaged into post-rational methodology. Our principle was to select which ever strategy was most effective towards said criteria per task identified and addressed. Pre-rational analysis allows us to decompose design into simpler parts and nest considerations in linear manner via simple geometric concepts which are familiar to the architecture language. Those are relatively easier to mentally trace, gain insight, make educated guesses and communicate. Post-rational analysis while capable in addressing far more complex problems it has a steeper learning curve as it requires abstract mathematical, statistical and computation concepts which have very different rules of composition than traditional architectural methods. We conclude that both strategies are inherently inter-dependent even though they appear approaching the same subject from opposite directions. We could have not solved the typological clustering problem directly without approximation for instance. Meanwhile, we may replicate the descriptive methodology to approximate an existing spline design envelope in addition to measuring deviations and/or clustering components towards both dimensions and curvature metrics.

In the future we will explore a rationalisation process with bias on post-rational design principle to assess the process pros and cons in greater detail. We would also expand on meta-heuristic analysis of cost/benefit of design to construction optimisation via economic equilibrium theory.

Acknowledgements

The author would like to thank the International Design Centre at the Singapore University of Technology and Design for supporting this research.

References

Ceccato, C., Hesselgren, L., Pauly, M., Pottman, H., and Wallner, J. (eds.): 2010, *Advances in Architectural Geometry 2010*, Springer-Verlag, Vienna.

Dritsas, S. and Becker, M.: 2007, *Research & Design in Shifting from Analog to Digital*, Association for Computer Aided Design in Architecture, Halifax, Canada.

Shelden, R. D.: 2002, *Digital Surface Representation and the Constructability of Gehry's Srchitecture*. PhD thesis, Massachusetts Institute of Technology, Cambridge.

Glymph, J., Shelden, D., Ceccato, C., Mussel, J. and Schober, H.: 2004, A parametric strategy for free-form glass structures using quadrilateral planar facets, *Automation in Construction*, **13**, 187–202.

Gonzalez, T. F.: 1985, Clustering to minimize the maximum intercluster distance, *Theoretical Computer Science*, **38**(2–3), 293–306.

Hesselgren, L, Charitou, R. and Dritsas, S.: 2007, The Bishopsgate Tower case study, *International Journal of Architectural Computing*, **1**(5), 61–81.

Hoffmann, M. and Kocacs, E.: 2003, Developable surface modelling by neural network, *Mathematical and Computer Modelling*, **38**(7–9), 849–853.

Jain, A. K., Murty, M. N. and Flynn, P. J.: 1999, Data clustering: a review, *ACM Computing Surveys*, **31**(3), 264–323.

Leopoldseder, S. and Pottmann, H.: 2003, *Approximation of Developable Surfaces with Cone Spline Surfaces*, Institut fur Geometrie, Technische Universitat Wien.

Luebkeman, C. and Shea, K.: 2005, CDO: computational design + optimisation in building practice, *The ARUP Journal*, **3**, 17–21.

MacQueen, J. B.: 1967, Some methods for classification and analysis of multivariate observations. *Fifth Symposium on Math, Statistics and Probability*, Berkeley, 281–297.

Pottmann, H. and Wallner. J: 2006, *The Focal Geometry of Circular and Conical Meshes*, Technische Universitat Wien, Vienna.

Schober, H.: 1999, Glass roofs and glass façades, *in* Behling S. (ed.), *Glass: Structure and Technology in Architecture*, Prestel Verlag, Munich.

Whitehead, H.: 2004, Laws of form, *in* Kolarevic, B. (ed.), *Architecture in the Digital Age: Design and Manufacturing*, Spon Press, New York.

Williams, C. J. K.: 2001, The analytic and numerical definition of the geometry of the British Museum Great Court Roof, *in* Burry, M. et al. (eds.), *Mathematics & Design 2001*, Geelong, 434–440.