RATIONALISATION OF FREEFORM FAÇADES

A technique for uniform hexagonal panelling

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Abstract. This paper is an account of the façade rationalisation strategy developed at Gehry Technologies for mitigating the tradeoffs between project constructability and the aesthetic implications of arranging flat panels over a complex surface. The strategy outlined was developed as the result of the digital building delivery of the Museo Soumaya façade in Mexico City designed by Fernando Romero LAR. This report documents the development process of a method for obtaining relevant construction information flows enabling the assembly of the façade system by a third-party sub-contractor in support of the façade system coordination. This report concludes on a method using bespoke software for extending the parametric modelling system Digital Project™ in the design support role of obtaining an aesthetically pleasing arrangement of flat hexagonal panels over a freeform surface.

Keywords. Sphere packing; façade rationalisation; hexagonal tiling; parametric design; k-means clustering.

1. Rationalisation of unconventional forms

The execution of a conventional construction project is a reasonably low risk endeavour with readily accessible formulas for project cost control and time estimation, while on the contrary an unconventional construction project is subject to a high level of risk associated with the lack of well-known rules to support cost estimation and the challenges of quantifying non-Euclidean geometry (Shelden 2002).

Using a rationalisation strategy early in the planning of fabrication and assembly phases can potentially have a dramatic impact in reducing the increased cost and risks introduced with the use of unconventional geometry.
Fischer suggests the availability of different design rationalisation strategies for taking advantage of the fabrication economies gained with mass-production, as for example the rationalisation strategy employed in the *Beijing National Swimming Centre* allowed a reduction of the number of prefabricated unique building components in the assembly and simultaneously met the aesthetic goals of apparent irregularity (Fischer 2007). Fischer resurfaces three strategic opportunities in the rationalisation of unconventional forms originally presented by Hugh Whitehead, of the foster and partners *Specialist Modelling Group*; the three strategies deal with firstly the opportunities in decision making (pre-rationalisation), secondly how to cope with design consequences (post-rationalisation) and thirdly a hybrid strategy namely co-rationalisation where parallel decisions affecting the rationalisation of form can be made alongside the process of design.  

This paper will demonstrate a façade rationalisation strategy developed at *Gehry Technologies* for mitigating the tradeoffs between project constructability and the aesthetic implications of arranging flat hexagonal panels over the unconventional geometry of the *Museo Soumaya* in visually pleasing arrangements. The rationalisation strategy had to manage the tradeoffs between a low number of unique building components and the effects these have over the perceivable irregularities of the panel-to-panel gaps (Fischer 2007).

### 2. Initial assumptions and rules

The façade rationalisation strategy of the *Museo Soumaya* was guided by a finite set of rules imposed by the design team as follows: 1) maintain a uniform gap between all six sides of the hexagon panel system; 2) begin with a standard hexagon size of dimension 63 cm diameter; 3) force the hexagon system to grow in scale (from a diameter range of 63 cm to 175 cm) as the pattern turns the corners of the façade; 4) group the panels into families, by minimising the amount of unique panels that need to be fabricated.  

Parting from these rules, an addition of two important tactical design decisions guided the process of rationalisation, the first of these decisions opted in selecting early in the project a pre-rationalised turn-key solution patented by *Geometr Erica* for the fabrication of freeform space frame structures, the system would serve both as the panel positioning device and the support structure for both the aluminium panels and the waterproofing panels. The second decision was to freeze the MDS Master Design Surface as a fixed design component in the project; this is due to concurrent assembly processes within the interior of the project completing at a faster pace than that of the exterior façade engineering exercises, in order to avoid errors in misalignments further down the
construction process, the MDS was frozen, dimensions can be extracted from “slicing and dicing” the surface, although the MDS cannot be edited further.

As a design consequence of selecting Geometrica’s space frame strut and node system, the rationalisation strategy was reduced substantially, triangular configurations of same size circles produce hexagonal intersection patterns, each node in Geometrica’s space frame holds the centre of gravity of a single hexagonal panel and three incoming struts, forming a secondary triangular aluminium structure over the primary steel of the project providing support for hanging the outer aluminium panels and the inner waterproofing panels also developed by Geometrica.

The information requirements needed by Geometrica for the assembly of the secondary aluminium space frame using their proprietary system needed only the XYZ position and XYZ normals of each node in the assembly, as a result of a one to one relationship between the node and centre of gravity of the panels, obtaining a packing of panels over the surface and extracting the centre of gravity of the packing, provides sufficient information for coordinating the overall panel, strut and node assemblies.

In the following sections two unsuccessful rationalisation methods for obtaining the coordination data are examined, while the methods arrive at hexagonal packing’s of panels over the MDS, they fail to address the design rules outlined in Section 2.

3. Conformal mappings

An often employed rationalisation strategy for obtaining panels over a doubly curved MDS is to roughly speaking wrap a 2D pattern drawing over the surface. In mathematical terms this operation is known as a conformal mapping. Depending on the results obtained, a mapping could exhibit two useful properties, firstly if the lengths in the original 2D drawing are preserved after transformation, the mapping is said to be isometric, and secondly if the angles between lines in the original 2D drawing are preserved after transformation, the mapping is said to be conformal.

The optimal solution to the rationalisation strategy would be a mapping that exhibits both isometric and conformal properties. In the case of the Museo Soumaya the MDS was defined as a NURBS surface digitised from a physical model. When wrapping 2D drawings over a NURBS surface, the mapping is neither length preserving nor angle preserving, unless the MDS is either a cylinder or a plane, in which case there is still the possibility of deformations in the conformal mappings due to the spacing of surface knots not being uniformly distributed over the surface.
4. Curvature gradients

Another traditional useful rationalisation strategy is to map the local Gaussian curvature properties onto the MDS, and from the mapping extracting the areas in the surface with relatively low curvature conditions, developable areas. Even though this approach allows wrapping 2D patterns over the Developable regions of the surface, the method cannot handle easily the transition between one area and its adjacent neighbour. Subdivision of the MDS by the Gaussian Curvature coefficient can prove to be useful in other panelling situations where a seamless panel distribution is not such a high aesthetical priority.

5. Circle hex meshes/sphere packing

This section presents the final successful rationalisation strategy employed for obtaining the panel, strut and node assembly coordination data for manufacturing and assembly. Intersecting circles produce the best hexagonal distributions in two dimensional spaces as shown in Figure 1, there is a one to one relation between the hexagon diameters and the circle diameters that produce them.

To control the diameter and the gap of the hexagon, we build a circle mesh of intersected circles, constructing the diameters of the circles from the values of the desired diameter plus the value of the desired gap, we suspend the actual hexagon by inscribing it on a circle offset inward using 1/2 of the desired gap as the offset value, a diagram of the extra wide circle and the suspended hexagon is shown on Figure 2.
Inspired from research on cellular aggregate structures, the intersection of circles and their inscribed hexagonal patterns resemble closely the formation of tissues or cellular aggregates in nature. Thompson (1961) modelled the morphological growth of cellular aggregates as the boundary conditions of intersecting spheres; Thompson (1961) identifies a “geometrical strategy” (Glyph et al. 2002) for reconstructing the emerging hexagonal patterns found in natural forms: “the four cells do not meet in a common centre, but each cell is in contact with two others, a so-called polar furrow, the visible edge of a vertical partition-wall, (joins or separates) the 2 triple contacts and so gives rise to a diamond shaped figure, identified more than a hundred years ago by Rusconi in a salamander and called by him a tetracitula.” (Thompson 1961) In other words when two circles intersect they produce two intersection points as shown on the left in Figure 3, if we draw a sphere on each of those two points we should arrive to the polar furrow configuration as indicated on the right in Figure 3.

Following Thompson’s insights of the polar furrow, we can construct any semi-hexagon as a local property of the intersection of two spheres with a doubly curved surface and then intersecting the two resultant curved circles supported on the surface and subsequently getting the north and south point, if we choose only the south point to locate another sphere and repeat this process until we have reached the end of the row, and then repeat the same process over and over again obtaining a new row for each pass, we can obtain a circle mesh grid over the surface as many rows and columns as needed see Figure 4 for the algorithmic process that creates the mesh, see Figure 4 for the result of the algorithm.
From the circle mesh grid as pointed in Figure 4, the hexagonal pattern is extracted by hanging hexagons inscribed in an offset circle 1/2 of the desired gap.

6. Correcting stretching artefacts

Although the mesh is close to the desired result, the hexagonal mesh contracted vertically on the regions of high curvature and as a result the outer edge rose and retracted from the initial position due to the reduction in the height of the panels. In order to correct these artefacts, a post-production process must be executed over the resultant mesh to correct the height reduction; this process corrects the height of each panel from its current reduced height to the height of an ideal hexagon, which is a hexagon where each of its vertices has an interior angle of 60 degrees. The reduced height panels selected are shown on the left on Figure 5, and the result of correcting the panel heights is shown on the right in Figure 5.

![Figure 5. Custom panel skirt stretching process.](image)

The process of correcting the heights of the panels requires first to identify all the panels that require correction. Passing a curve vertically through all the points in a column of panels, these curves are placed on top of the surface of the envelope and extended past the boundary edge of the surface, effectively these curves where used as rails to scroll the surface downwards until the vertical dimension of all hexagons corresponds to the horizontal dimension, there is a height to width proportion on a 60 degree hexagon, this ratio can be used to calculate the percentage needed to correct the vertical dimension of each hexagon. Before this process is executed, the panels that exhibit an abnormal squashing are filtered and selected as shown in Figure 6, the panels with abnormalities are referred to as custom panels and the ones with acceptable conditions are referred to as standard, Figure 7 shows the process of extracting rails from the panels and then stretching the panels over the extracted rails to correct their abnormality.
The sphere packing and the hexagonal mesh extracted from it are sensitive to initial conditions, after conducting many trial and error studies, the pattern with the best aesthetics results were achieved by starting the sphere packing horizontally in the middle of the envelope. The location of the first sphere in the sphere packing algorithm has a high impact on the aesthetic output of the overall mesh, we found the most homogenous solutions where achieved by starting the sphere packing at a location on the surface with the least Gaussian curvature, to identify this location, an optimisation process was executed to identify this location, as shown in Figure 8.
7. Clustering: reduction of unique building components

The client wanted to have only 7 or 24 unique type of panels within the overall mesh, this is due to the cost of producing a single mould, and a personal fixation with the numbers above, the challenge was to maintain a pattern that would keep the family as strands from the middle of the façade waving outward in both directions.

By executing a statistical k-means clustering analysis over the panel population using 21 parameters as attributes and weighing the Area of the panel as the key attribute, the clustering analysis yielded the result wanted.

Given the area in the middle zone is the closest to a cylinder and the panels seem to stretch as they move away from the middle band, the striation effect in the resultant families was achieved almost naturally as shown on Figure 9. The inclusion of the area in the calculation of the clustering analysis is crucial to achieving the banding effect over the distribution of the families. If instead the area is omitted in the calculation, a more patchwork distribution is achieved, the patchwork distribution is more faithful to the original geometry, although it increases the logistics of fabrication, as there would be more fragmentation over the distribution requiring the use of schedules for the location of each panel, in contrast with the striation, only the boundaries of the families need to be identified and all the panels inside the boundary can be safely placed knowing they correspond to that particular family, this is convenient to employ when the assembly is carried out by unskilled labour, while minimising construction errors.

The k-means clustering process is outsourced to excel using a statistical plug-in for the k-means clustering function, each panel was exported as an instance with 21 parameters measuring intrinsic properties of each panel, these properties are internally compared to one another, in order to determine how to sort the panels into the fix number of families.

The K-Means cluster analysis starts with a setup function that is executed once at the beginning to seed the process, in this setup function the algorithm creates k number of families, and then iterates over the list of panels assigning randomly a family to each panel.

The algorithm then recursively runs the following pseudo code until it converges. Convergence in this case means until no panel needs to be reassigned to its nearest family:

1 – For each family in a list of families with k number of families,
1a – Find the Centroid of each family.
Centroid: defines the best panel representing the family.
2 – For each panel reassign to the family with the closest Centroid.
3 – Repeat Step 1 and Step 2 until convergence.
Depending on which parameters are given as attributes of the panels for cluster analysis the results of the clustering produce different results, Figure 9 shows the differences between omission or inclusion of the panel area, as well as increasing the number of families, both affecting the fragmentation of the resultant clusters as well as the pattern continuity.

![Figure 9. Left: 7 families with area, right: 49 families without area.](image)

8. Conclusions

This report outlines various rationalisation strategies for negotiating tradeoffs between geometrical and aesthetical implications.

The most successful rationalisation strategy found the most aesthetically pleasing distribution of panels by using a stable sphere packing method, and a self-adjusting pattern stretching algorithm, by extending Digital Project™ through bespoke software solutions.

A solution to cataloguing the resultant panels was found using a straightforward approach involving the use of a k-means algorithm for the clustering process in Excel™ and the subsequent mapping process for returning the result into Digital Project™, by using a k-means algorithm as opposed to a hierarchical agglomerative clustering algorithm, an approximate solution was found at a faster time, while giving the user control to specify directly the exact number of families desired.
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