SYMMETRY-BASED GENERATIVE DESIGN: A TEACHING EXPERIMENT

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Abstract. Throughout history, symmetry has been widely explored as a geometric strategy to conceive architectural forms and spaces. Nonetheless, its concept has changed and expanded overtime, and its design exploration does not mean anymore the generation of simple and predictable solutions. By framing in history this idea, the present paper discusses the relevance of exploring symmetry in architectural design today, by means of computational design and fabrication processes. It confirms the emergence of a renewed interest in the topic based on two main ideas: On the one hand, symmetry-based design supports the generation of unique and apparent complex solutions out of simple geometric rules, in a bottom-up fashion. On the other hand, despite this intricacy, it assures modularity in the design components, which can bring benefits at the construction level. As the background for testing and illustrating its theoretical arguments, this paper describes the work produced in the Constructive Geometry course at FAUP.

Keywords. Geometry; symmetry; computational design; digital manufacturing; education.

1. Introduction

Symmetry\(^1\) is an old ordering principle that has inspired and explained the developments in many different scientific and artistic fields, like physics, biology or architecture. Nonetheless, its meaning has changed and expanded overtime, moving from the classic concern with prescribing ideals of harmonious proportion and beauty, to the modern sense of order resulting from the application of geometric transformations to an object, since they keep it invariant. While the former meaning is permeable to the aesthetic and cultural values of each place and time, the later is a precise and stable concept to describe the structure of formal systems (i.e. natural, built, abstract...), unfolding either in the (two dimensional) plane or in the (three dimensional) space.
Implying the application of isometric transformations, the list of basic symmetries are: Rotational symmetry, reflection symmetry (bi-lateral), Rotoreflection symmetry, Central inversion symmetry, Translation symmetry, Glide reflection symmetry, Helical symmetry. In any possible combination of those geometric transformations, both the shape and the measures of a transformed object are kept invariant.

However, if one accepts other possibilities for variation, then it is possible to distinguish non-isometric symmetries in other geometries, as it was systematized in the Erlangen program, by Felix Klein in 1872 (Devlin, 2003, p. 152). These variations can thus unfold through more elaborated symmetric transformations like similarity, affine, projective and topological ones (March and Steadman, 1971, p. 25).

In this context, this paper argues that there could be two modes of approaching symmetry: an analytical and a generative. When studying an existing object, like a platonic solid, its shape can be described from the symmetry point of view. In a top-down manner, one can explain a solid by identifying the set of its constituents (parts) and subsequent geometric transformations that originated it. For instance, the cube can be described by a group of rotations of a square in space around a common central point. This kind of analysis is thus essentially descriptive, and it is intimately linked to the mathematical concept of symmetry groups, proposed by Evariste Galois in the early XIXth century (Devlin, 2003, p. 146). In a different way, when performing a creative task, symmetry can be explored in a bottom-up manner. Instead of looking at a pre-conceived form and decomposing it, the attention is now focused on the application of geometric transformations to an initial form as a way to conceive an original one. Here, symmetric transformations are used as a generative engine to discover of new design possibilities. In other words, the shape is not itself the search space, but the starting point to entail a morphogenetic evolution that can give rise to other configurations and combinations. The application of spatial rotations to a square can generate not only the cube, but also other solids, like the triangular prism, the cuboctahedron or the rhombicuboctahedron, among others (Figure 1). This structured generative aspect is of critical importance for a designer, as acknowledge by Stiny, when outlining a constructive approach to the definition of languages of design (1980), or by Knight’s works and overview on the shape grammars field (1999).

2. Symmetry in Architecture

Throughout the history of architecture, symmetry has been widely explored as a geometric concept to describe and conceive architectural forms and spaces, following the two meanings described earlier.
Back to the ancient Greece, symmetry meant the definition of a harmonious system of proportions, or ratios, between the whole and its parts of an object. Following this understanding, Vitruvius recommended the architectural importance of symmetry for the design of a Temple, and triggered a long tradition in the discipline where the ideal of beauty and perfection were assured by means of symmetric structures and compositions. Despite some minor different conceptual motivations (e.g., religious, philosophical, psychological...), this notion prevailed in the theory and practice of architecture, through the Renaissance and until the XVIIIth century (Von Meiss, 2004, p. 65). It was clearly expressed by influent architects like Alberti or Palladio, and can be read in the dominance of reflection and rotational symmetry in the design of buildings, especially in those with public importance.

When theorizing about the impact of modern science in architecture during the XVIIIth century, Pérez-Gómez (1983) describes a set of cultural and technological facts that contributed to a paradigm shift in the way man saw and comprehended the world. With particular relevance for this paper, the appearance and systematization of non-Euclidian geometries in mathematics prompted the development of a general and rational theory about symmetry, based on groups of geometric transformations. The modern scientific advancements of this period brought cultural and technological changes, which naturally affected the discipline of architecture and its values. Like Perrault, several influent architects reacted to tradition, detaching symmetry from proportion, and subjective ideals of beauty and harmony.
For instance, while J.N.L. Durand recommended the use of symmetry, together with regularity and simplicity, just as a way to achieve economy in building construction, others, like Viollet-le-Duc, argued that balance and proportion could be achieved by means of asymmetric formal and spatial compositions.

Since then, the architectural evolution during the last two centuries has revealed an expansion of formal languages towards an increasing degree of geometric freedom. Conventional reflective or rotational symmetry as privileged compositional rules gave place to more elaborated geometric arrangements, either of symmetric (isometric and non-isometric) or asymmetric order. The modern movement, the deconstructivism trend in postmodern architecture, or the recent exploration of free-form buildings enabled by computer technologies, are just three of the tendencies that have challenged the traditional notions of balance, order and proportion. Many theorists and architects since Viollet-le-Duc have pointed several contradictions in symmetry, which lead to a progressive decrease of interest in the discipline. Klaus Mainzer resumes the current tendencies happening not only in architecture but in the modern globalized world as a move from symmetry to complexity, where the platonic union of truth and beauty is broken and transformed into diversity and heterogeneity (Mainzer, 2005, p. 329).

3. Symmetry as a Generative Design Strategy

In the last thirty years, the world has witnessed the rise and influence of computer technologies in almost all aspects of our life. In architecture, the use of digital design (CAD) and manufacturing (CAM) technologies have dramatically expanded the creative space of design and construction possibilities. While the use of advanced 3D modelling allows for the easy and rigorous representation in the screen of 3D complex forms, the use of computational design processes allows for the interactive calculation of geometric variation and evolution. Facing this design flexibility, the employment of CAM technologies offer solutions for materializing complex geometries and exploring mass production logics based on customization. This digital continuum from design to production has drew a new paradigm in architecture, where design representation and construction are no more limited by formal complexity and repetition, as it happened until then. Thus, in a moment when contemporary architecture experiences an increasing geometric freedom at many levels, why does a traditional and, apparently, old-fashion subject like symmetry could be of interest for architects?

Considering the mathematical notion of symmetry, its actual relevance in architecture seems to reside in its generative potential for design rather than on its descriptive. If one agrees that its creative dimension is tied to the capacity of rep-
resentational tools to describe and calculate transformations over a form then, the impact of current digital design processes on symmetry should not be ignored. The move from explicit design tools, either analogical or digital, to computational design ones has allowed the exploration of symmetry in ways that would be difficult in the past. By stating that "computers discover symmetry of invariant structures hidden behind the appearance of chaotic attractors" Mayer (2005, p. 385) establishes a direct and inspiring relation between symmetry and complexity. Thus, when architects employ parametric and algorithmic strategies to describe iterative and recursive sequences of symmetric transformations in space, the evolution of an initial geometry can lead to unpredictable solutions, in a constant dialogue with digital interactive models. The idea of symmetry as a generative principle leading to monotonous and obvious configurations, may be surpassed by taking advantage of the calculation power embedded in computational design processes, either through software-based or programming approaches. Spanning over isometric and non-isometric invariants, it seems also reasonable to question if symmetry is not laying at the core of most parametric design approaches in contemporary architecture.

4. The Teaching Experiment (Isometric Symmetry)

To investigate and test the arguments described above, this paper is now focused in isometric symmetry, which is the most regular and predictable scenario. To illustrate the contribution of computation to embrace it as a generative strategy, it will be presented a teaching experiment developed during the Constructive Geometry course at the Faculty of Architecture of the University of Porto. Named as "Parametric Symmetry", this assignment looked for generating reconfigurable symmetrical forms using Grasshopper, the parametric design plug-in for Rhinoceros software.

Implying an important shift in representation and thinking processes, it was important to introduce the students to computational design thinking prior to the design techniques. Among several theoretical and practical examples, D’Arcy Thompson still offers today a clear source of reference about the creative link between mathematics and design, when he mentions that “we are apt to think of mathematical definitions as too strict and rigid for common use, but their rigour is combined with all but endless freedom. The precise definition of an ellipse introduces us to all the ellipses in the world” (Thompson, 2000, p. 269). By centring their focus on the implicit geometric definition of form(s) instead of on the explicit features, the students were ready to develop the assignment and embrace the technique in a proper way. As an additional note, given the nature of the course, the exercise rejected the involvement of any programmatic design concerns. The goal
was to approach the subject on an abstract and mind-opened way, which could further serve to guide its multi-scalar application to address architectural or even urban planning problems.

In this context, each student had to select an initial object (a group of surfaces or a solid) and program, in Grasshopper, the parametric application of a set of symmetric transformations over it (Figure 2).

To guide this investigation, two premises were given:

- the initial object should be simple;
- the symmetry explored should be isometric.

Both of these conditions were important to facilitate, in the end, the perception of the degree of complexity that was introduced by the (generative) process rather than by the specificities of the initial form. To prove their control over the design exploration, the students had to make a physical model of one possible solution generated by their parametric definitions (Figure 3).

The “Parametric Symmetry” experience resulted in a diversity of symmetric models emerging from the application of different sequences geometric transformations, both in the plane and in space. Although just one solution was materialized, the students had to explain the parameters controlling their generative model and map, with the help of tables, the spectrum of variation of possible solutions. During the work, there was a common sense that dealing with parametric models was an interactive experience where each one could express and evolve
their design intuitions (Figure 4). The visual and interactive programming with Grasshopper revealed to be very helpful to support design thinking and decision making along the process. The behaviour and results obtained out of simple rules was frequently surprising enough to motivate design changes towards novel and unexpected directions.

It was also interesting to notice that, at a certain point, almost every student claimed for the introduction of metric variation on the model. Nonetheless, although it was not difficult to start adding non-isometric transformations to the model, they were forced to resist such intentions in the assignment to really test the generative potential of symmetry only by using isometric transformations. In the end, independently of their apparent complexity, all parametric models and constructions developed can be described and physically made out of the repetition of
a modular (standard) element. The geometric exploration of symmetry can thus benefit a lot from design computational processes, which can foster a renewed interest in contemporary architecture (Figure 5).

5. Conclusion

Besides exploring new opportunities, the current evolution of architecture by means of digital technologies is also occurring by looking at the past. Motivated by the use of computers in practice, we have assisted to a renewed interest in architects from the past, like Vitruvius, Alberti, Palladio, Gaudi, Eladio Dieste, Buckminster Fuller or Frei Otto, just to name a few of them. Hence, it is aligned with this kind of interest that this paper looked at the topic of symmetry with the goal of contributing today for rethinking such a classic topic of the architectural history.

As it was described and illustrated with the teaching experiment, the use of computational design tools to investigate symmetry has proved to be a creative window to achieve geometric complexity out of a finite set of very simple transformation rules. By proceeding this way, although the design process becomes structured and easily traceable, its outcomes are not easily imaginable \textit{a priori}. This generative ability of simple rules producing rich results suggests an idea of productive economy in the design process, which can be expanded to the construction level. In fact, after an industrialization ruled by mass production logics

Figure 5. Physical models produced out of isometric transformations.
based on standardization, current digital design and manufacturing technologies are supporting the current move towards the production of singular objects with unique forms and customized building solutions. Thus, in a moment where contemporary architectural discourses balance between doing everything the same or everything different, exploring symmetry through digital means seems a powerful design strategy to negotiate the best of both tendencies.

While we have focused in the teaching experiment on isometric symmetry, a complementary paper is under preparation to extend the illustration of this subject into the world of non-isometric symmetries.

Endnotes

1. Symmetry has its origins from the Latin term *symmetria* and the Greek *symmetros*, and generally means “having a common measure”.
2. Vitruvius recommends that “the design of a Temple depends on symmetry, the principle of which must be most carefully observed by the architect”, and makes evident the classic notion of symmetry by saying that “proportion is a correspondence among the measures of the members of an entire work, and the whole to a certain part selected as standard. From this result the principles of symmetry” (1998, p. 72).
3. A detailed account on the contributions of classic and renaissance architects on the subject of symmetry can be read on Wittkower (1971) and Pérez-Gómez (1983).
4. It is significant that, according to Mitchell, “when Claude Perrault published his French translation of Vitruvius (1673), he rendered *symmetria* as ‘proportion’” (Mitchell, 1998, p. 30).
5. Jean-Nicolas-Louis Durand insisted about the importance of geometry to achieve economy in building construction which was, together with fitness, the two guiding concepts for the discipline (Durand, 1999, p. 88)
6. Referring that “Viollet-le-Duc was constantly inveighing against the ‘banal rules of symmetry’”, R. Etlin explains how he argued that, in an asymmetrical design, “through the skillful (...) ponderation of masses the architect could achieve a ‘subtle system of harmony’” (Etlin, 1994, p. 82). Asymmetry became thus understood as another possible principle to achieve order and balance.
7. P. Von Meiss refers to the devaluation of symmetry through its popularization during the XIXth century, which led, during the XXth century, to the emergence of asymmetry as a valid way to “achieve buildings representative of authority or religion” (Von Meiss, 2004, p. 65). In science, the interaction of intrinsic and extrinsic forces started to explain the generation of a diversity of forms out of common formal principles (Pearce, 1990, p. xiv). The acknowledgement of symmetry breaking processes occurring in nature fostered the acceptance of asymmetrical balance. On this respect, G. Lynn argues that “Symmetry is not a sign of underlying order but an indication of a lack of order due to an absence of interaction with larger external forces and environments” (Lynn, 1998, p.70). Thus, besides losing its symbolic authority, symmetry became unsatisfactory to negotiate the design of buildings with the heterogeneity of the contextual environment. R. Venturi expands this idea in its *Complexity and Contradiction in Architecture* work.
8. This observation is extensively discussed and illustrated in B. Kolarevic book (2003).
9. Computational design implies the use of digital design processes that merges calculation in representation by means of parametric and algorithmic processes. Terzidis (2006) describes and opposes this concept to that of computerization.
10. The Constructive Geometry course is dedicated to investigate the impact of computational design and digital fabrication processes in architectural geometry.
11. One could mention here William J. Mitchell, George Stiny or Larry Sass works on Palladio, Mario Carpo review of Alberti, Bernard Cache focus on Vitruvius, Mark Burry work in-practice about Gaudi’s legacy at the Sagrada Familia, or Achim Menges and Lars Spuybroek inspiration on Frei Otto work. In Portugal, a current work about Alberti has been conducted by Mário Krüger and José Pinto Duarte.

12. When designing and building variation is possible through digital processes, the building logistics may be taken into account to prevent the project failure. Therefore, given that isometric symmetry still keeps modularity in the design of components, it can bring benefits at the construction level (e.g., making easier storage, transportation or assembly processes).

Acknowledgments
The authors would like to thank the students of the Constructive Geometry Class (2010/11) for their motivation and tireless efforts.

References