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INVESTIGATIONS ON QUADRILATERAL PATTERNS FOR RIGID FOLDING STRUCTURES

Folding strategies – rigid and curved folding

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Abstract. A rigid spatial structure represents a three-dimensional structural system in which the size of the singular planar elements is very small related to the whole construction. In this paper we will do investigations of quadrilateral patterns and we will propose an analytical method for designing structural rigid folding with quadrilateral patterns following geometrical surfaces of different topology. Our method offers folding structures with four fold lines meeting in one node which allows a simpler solution of join connections and assembling of the whole spatial and structural system. As the physical characteristics of paper can lead to all kinds of wrong conclusions it is necessary to use CAD tools in addition to scale models, where the entire folding element is reconstructed and its geometric characteristics are controlled. This kind of control reflects on the scale model. Models are then adjusted, examined and built to reach certain conclusions that are once more tested in CAD software.

Keywords. Rigid and curved folding; quadrilateral folding pattern.

1. Introduction

In the field of origami it is investigated how to design different shapes only from one sheet of paper using folding techniques. In the folding technique single planar material is folded without stretching, tearing or cutting. This theoretical definition of folding technique exists only in geometrical and mathematical context using ideal zero-thickness. If we consider material thickness and stiffness for large scale space structures, stretching and tearing inevitably appear in the construction. We start our research with the three basic origami patterns Yoshimura Pattern, Diagonal pattern and Muira Ori pattern in order to find geometrical properties according to the load, stiffness and stability that we can use as a pattern in architectural context. Our approach is not making classical kinetic foldable structures but to use geometrical principles and static advantages of modular and tessellated origami in order to classify quadrilateral folding structures and develop interactive methods for designing structural rigid folding with a load-bearing effect for topologically different geometrical structures with quadrilateral patterns.

The theoretical basis for our research is funded in geometrical and mathematical properties of classical origami (Belcastro and Hull, 2002a, 2002b; Nojima, 2002; Tachi, 2009) and in applied research of folding plate structures made out of cross-laminated timber panels (Buri, 2010; Haasis and Weinand, 2008). Our research differs from previous research in that way that it is based on tessellated origami. We use analytical methods to define quadrilateral patterns in rigid folding structures for topologically different surfaces.

2. Practice and Application

In architectural terminology, the term folding structures stands for structures consisting of polygonal elements. Their main characteristic is that individual polygonal elements are very small in size compared to the scale of the entire structure's bearing capacity.

Folding structures are found in many fields, such as industrial design, fashion, interior design, architecture, textile industry and jewellery. They can be made out of different types of material, such as paper, textiles, cardboard, wood or metal. Literature dealing with this topic can be divided into three groups. The first group is written for designers (Jackson, 2011). The second group analyses folding structures and principles from the mathematical point of view (Kawasaki, 1989; Mitani, 2009; Miyazaki et al., 1996; Nojima, 2002; Zimmer et al., 2012). The third group deals with practical applications. It analyses the static and dynamic characteristics of folding structures, taking into account the type and thickness of materials of which they are made (Tachi, 2011). There also exists a software package offering an origami simulator (Tachi, 2012).

3. Folding Structures

Rigid folding structures are three-dimensional structures. They are built by folding planar materials while keeping all the elements planar after folding. Curved folding structures are built by folding a single curved surface so that the resulting structures consists of single curved elements as well (Figure 1).

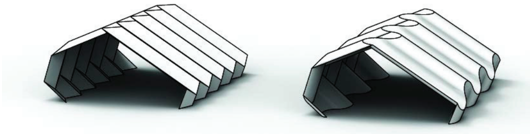

Figure 1. Rigid and curved folding structures.

4. Intuitive Folding

Folding techniques are a field often explored with "learning by doing" methods. The work on these structures begins with simple paper folding, playing with paper and using the simplest drawing materials. Many different forms are produced in a short time, intuitively and easily. If these structures are made of paper, which allows further bending, then the simplest folding patterns can produce many different forms. Figure 2 shows a cross-section of a folding structure and the analysis of possible folding positions within a flexible system of boundary curves. Since it is made of paper, its extreme flexibility is more than obvious.

The physical characteristics of paper can lead to all kinds of wrong conclusions. For example, paper is easy to twist out of its plane, thus changing the position of the entire three-dimensional structure. Hence, the folding structure surfaces shown in Figure 2 do not consist of planar elements, that is, they are not rigid structures. It is therefore necessary to use CAD tools in addition to scale models, where the entire folding element is reconstructed and its geometric characteristics are controlled.

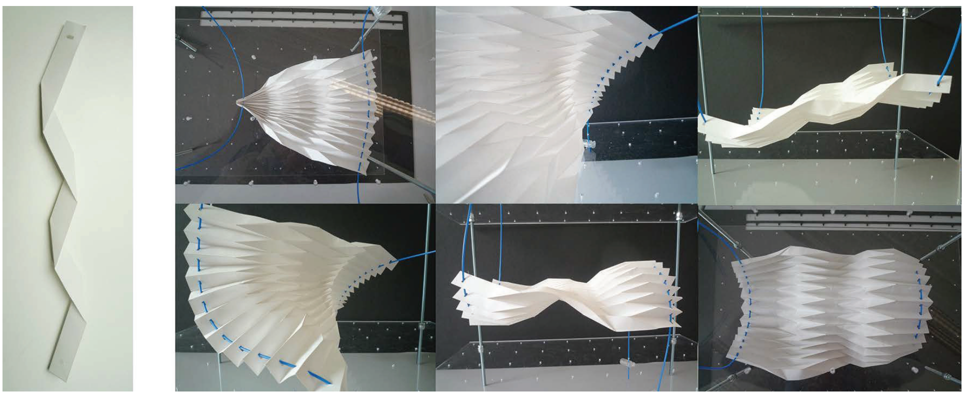

Figure 2. A cross section (left), its folding and numerous analyses of the form.

This kind of control reflects on the scale model. Models are then adjusted, examined and built to reach certain conclusions that are once more tested in CAD software.

One of the most useful methods is defining parametric models. The biggest advantage of parametric models is that, depending on the form of objects, they provide a direct connection to the folding pattern. Yet another advantage is the possibility to simulate dynamic models by opening the structure, starting from the open position (sheet of paper) to the complete folding into a single strip.

To be able to model complex structures and generate patterns, one needs to understand the theory of folding. This includes knowledge of basic patterns, folding methods and understanding of spatial transformations in the folding process.

5. Basic Folding Patterns

Folding patterns are based on lines (rigid folding) or curves (curved folding) and on a two-dimensional material which is folded to make a three-dimensional structure. Folding produces lines or curves which alternate in mountain and valley folds.

Among many types of folding patterns, architects are particularly interested in the Diamond Pattern, Diagonal Pattern and Miura Ori Pattern, especially their design and structural aspects. Rigid folding structures are based on these patterns. Their significance is reflected in the following:

- They give structural stability to three-dimensional forms;
- Basic pattern can be modified, achieving remarkable variability;
- Different patterns can be combined;
- They are the basis for the development of curved folding patterns.

5.1. DIAMOND PATTERN (YOSHIMURA PATTERN)

This pattern was named after the Japanese scientist Yoshimura who observed the behaviour of thin cylinders folded under axial compression force (Miura, 1969). He found out that the surface folds of a folded cylinder follow a specific pattern which is similar to a diamond (Figure 3). The base for this pattern is a deltoid that

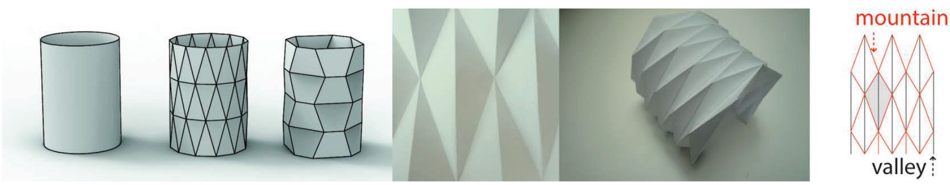

Figure 3. A cylinder and its folding patterns consisting of triangles and trapezoids.

Figure 4. Diagonal Pattern as a result of cylinder torsion.

is folded along a diagonal. Deltoid edges are folded as a mountain fold, while one diagonal is a valley fold.

A variant of this pattern can be produced if the deltoid structure is stretched following one diagonal, thus becoming a hexagonal form (Figure 4 center). In this case, instead of getting triangles we get two symmetrical trapezoids. With diamond pattern folding, all diagonals define polylines which approximate the cross sections of the cylinders.

5.2. DIAGONAL PATTERN

Diagonal Pattern is very similar to the Diamond Pattern. This pattern is achieved when torsion is applied to a folded rotational cylinder (Figure 4).

The basis of this pattern is a parallelogram, folded along its diagonal. All diagonals define the valley fold, while all parallels define the mountain fold. With the Diagonal Pattern, in contrast to the Diamond Pattern, diagonal lines define helical polygonal lines, used to define helical folding structures.

5.3. MIURA-ORI PATTERN (HERRINGBONE PATTERN)

This pattern was named after the Japanese scientist Miura who used this spatial structure system to make kinetic solar systems in space. This pattern consists of symmetric parallelograms forming in two directions a zigzag configuration (Figure 5).

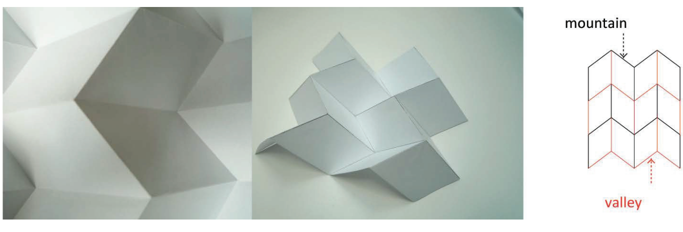

Figure 5. Miura-Ori pattern.

This configuration allows the opening of patterns in two directions. A variation of this pattern is reflected in the transformation of the parallelograms into trapezoids, which enables fabrication of concave or convex folding structures.

In architectural terms, it is very important to continue with the development of these three basic patterns to produce new variations of the form. This kind of research can be divided into two trends. The first one involves the study of variations in relation to basic 2D patterns which in return generates new patterns. The second trend starts from an already known 3D shape, as a basis for the construction of the folding structure. The first step to understand pattern generation is to study the basic folding techniques.

6. Basic Techniques

Fabrication of three-dimensional folding structures involves two dominant techniques: parallel folds and reverse folds.

With parallel fold technique paper is folded along the lines generating alternating valley and mountain lines (for example Figure 3 and 4 right). Reverse folding technique changes the direction of basic folding (Figure 6).

After folding around the diagonal d, the mountain fold a turns into the valley fold b. In this way two planes become four, meeting in point A. The position of the diagonal d affects the change of folding direction.

Figure 6b shows the reverse folding as a geometric transformation of a 3D reflection about a plane α. The plane α is defined by the diagonal d and it is perpendicular to the vertical plane β which is symmetric to the parallel folding. Figure 6c shows a part of the structure with the belonging folding pattern (Figure 6d).

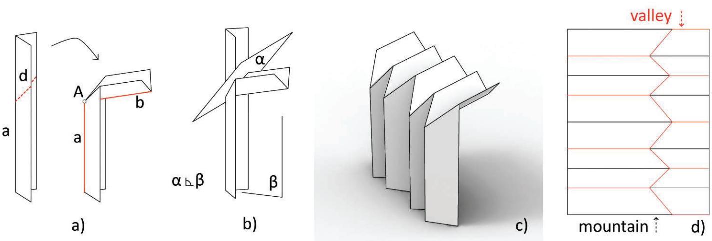

Figure 6. Reverse folding with reflection on the plane α.

7. Grid Generation Analyses

In our analysis we consider cylindrical shapes, based on a modified diamond pattern. As we have seen from basic patterns, the folding structure pattern is composed of a series of polygonal or triangular elements. When these elements are folded, they take specific positions in space so that two adjacent elements meet along a single line, while several elements meet in a common point. If the folding structure is composed of triangular elements, then a single point can be the meeting point of six or more adjacent polygons. If we assume that such structures are made in real scale (with appropriate material thickness) it can be very complex to calculate and implement the intersection point of a number of different polygons. That is why we have chosen pattern configurations consisting of trapezoids and representing a modified diamond pattern. The advantage of this pattern, compared to the triangular structure, is that at one common point "only" four planes meet.

Figure 7 shows the whole process of the folding structure generation, from the original cross-section to the final grid. The convex example shows all the measures used to build the scale model, while the concave example shows the values in general. The starting point for defining a folding structure is its cross-section. It is given in the form of a spline curve. To approximate this curve, we used the curve's control polygon.

For further calculation of folds (Figure 7c) it is necessary to measure the angles formed by each of the lines with the adjacent line of polygons (angle β) and measure the length of segments (a). Based on these angles, it is possible to calculate the folding angle of paper at each point in the polygon:

$$\alpha = (180 - \beta)/2$$

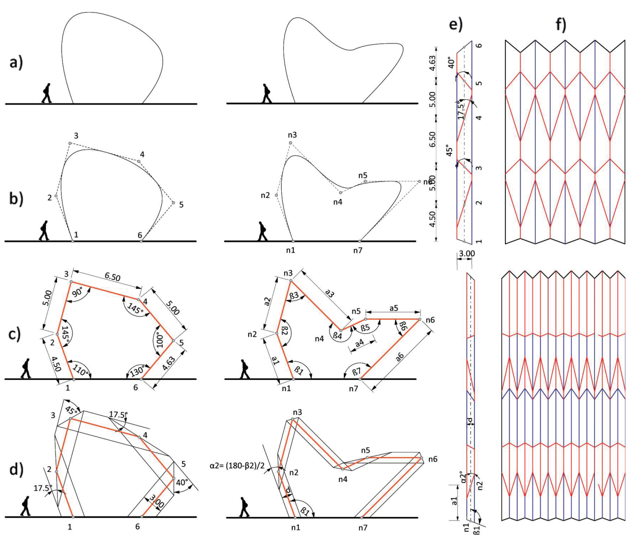

Figure 7. Folding construction and calculation of the folding structure grid.

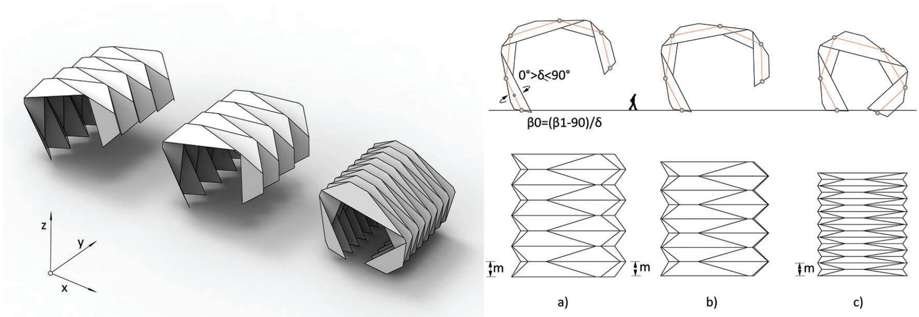

Figure 8. Different opening shapes and folding structure forms.

The distances between the vertices 1–6 of the polygon are distances representing trapezoids centre lines. They define individual structural elements. Parallel to these edges are lines that define the thickness of the folds (Figure 7d). The angle α defines the directions of the two unparallel trapezoid edges.

In convex grids all unparallel trapezoid edges are valley folds. The concave structure grid shows that in the concave part (vertex n4) the trapezoid edge is a mountain fold and it is precisely this point where the structure changes direction from the convex to concave position.

Simulation of paper folding can be done after the grid has been drawn. The entire structure can be folded starting from the flat plane of paper to the maximum folding position, in which the entire structure looks like a folded strip. The structure goes through various stages of folding in this process, thus enabling the generation of many different forms. Figure 8 shows three arbitrary positions of the folding structure.

8. Curved Folding

Reverse folding is a technique that can also be applied to generate curved folding. If one selects a cylindrical surface and then apply reverse folding technique, the result is going to be a curved folding structure (Figure 9). These structures have patterns composed exclusively of curves. Since curved folding structures result from the folding of two-dimensional paper, the resulting surfaces can only be single curved surfaces – cylinders, cones or generalized developable surfaces.

More complex forms of these structures can be achieved if we repeat the primary folding several times. Figure 10 shows some complex cylindrical and conical structures.

If 3D models of rigid and curved folding are generated with 3D software, it is possible to develop a folding pattern of the entire structure directly in software

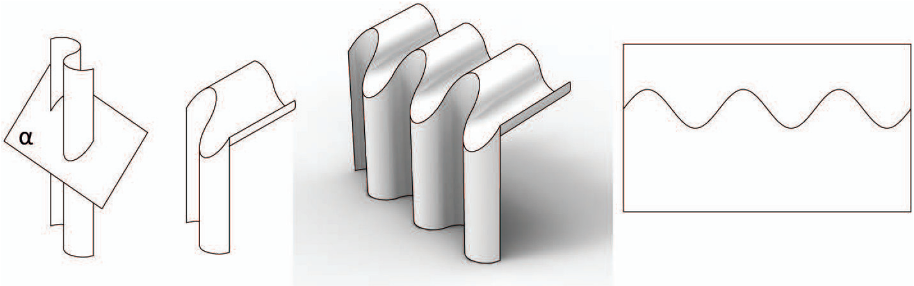

Figure 9. Application of reverse folding on a cylindrical surface.

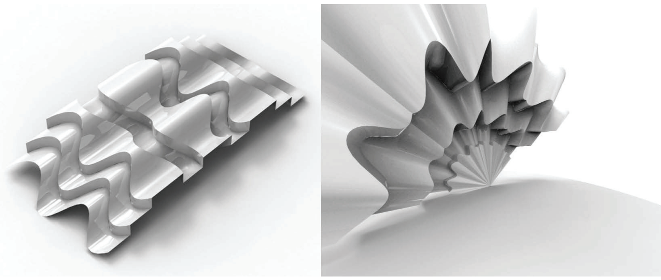

Figure 10. Cylindrical and conical structures – curved folding.

(f. i. Rhinoceros). This is possible because each individual component is a developable surface. Naturally, it must be kept in mind that automated software solutions sometimes contain errors that need to be corrected.

When patterns are first drawn, then 2D drawings are most commonly made in CAD programs. In the process of folding, paper is always folded along two different sides, which results in mountain and valley folds. While drawing, these lines can be marked in different colours, labelling the folding side of paper. Since folding can be complex, engraving the lines on paper (for instance with a laser cutter) helps to accurately and precisely fold the paper along the folding lines. The preparation of such patterns sometimes requires that all the mountain lines are engraved on one side of the material and all the valley lines on the other side.

9. Conclusion

In this paper we investigated geometrical properties of rigid and curved folding. With the help of basic rules of parallel and reverse folding we established a parametric model for computer aided supported design to generate various steps from pattern generation to folding simulation. The geometrical rules of rigid folding can also be extended to curved folding which we showed on the examples of cylinders and cones.

Future work will be concentrated on parametrically supported folding of various types of surfaces with the aim to generate dynamic architectural elements like shadowing or interactive facade systems using smart materials.

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