MODELLING, SIMULATION AND VERIFICATION OF PNEUMATICALLY ACTUATED AUXETIC SYSTEMS

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Abstract. This paper presents the development of an SLS 3D printed auxetic structure actuated to a predefined form by an embedded pneumatic network through an iterative process of feedback between digital simulation and physical testing. This feedback process is critical to the development of a more accurate predictive model, and to compose the geometry of the suggested structure. An approach based on the emergence of the final structure from the convergence of the behaviour of sub-structures and a methodology based on the analysis and synthesis of the simplest sub-system is the core of this research. The results indicate a promising simulation environment and a novel methodology for the design and fabrication of auxetic structures with embedded pneumatic actuation. This exploratory research suggests a fertile space for investigation within the field of adaptive architecture and soft kinetic design.

Keywords. Auxetic; fabrication; simulation; pneumatic; kinetic.

1. Introduction

Today kinetic architecture is understood as a response to technological, economical and social change (Kronenburg, 2007) and its role is extended to the creation of an architectural artefact able to adapt to human needs and environmental changes. Although the theoretical approach to kinetic architecture promotes the benefits of dynamic environments, its physical application is still less developed (Hoberman, 2011) due to the limited options for kinetic systems. For the purpose of this research, it is suggested that the categorization of kinetic systems in relation to their technological and behavioural characteristics be made in two groups: hard and soft systems. The first group, the hard system, which is more developed, refers to structures of which the function and design relies on rotational nodes and rigid links (trusses, pin joined, tensegrity), and they are made of many components. They have many disadvantages like heavy weight, a large number of components, many assemblies, rigid movements, high-energy consumption, and they need very complex computation to
control their movements (Pfeifer et al., 2012). On the other hand, the less developed soft kinetic structures with soft behaviour are promising as they use smartness in their composition to simplify movements, and eliminate computation processes during control. Furthermore soft kinetic systems, due to their intrinsic compliance, can safely interact with humans as it is stated in “Festo Bionic Handling Assistant” (Pfeifer et al., 2012). In addition with digital-driven fabrication and design technologies they can be optimised, simulated and manufactured to adapt to specific conditions.

Accordingly the motivation for this research lies in designing and creating a morphing compliant auxetic structure, actuated from an embedded pneumatic network, and fabricated with additive layer manufacturing as a design opportunity for use in soft kinetic architecture. Therefore it is suggested to synthesize the geometry of the structure using a re-entrant unit cell, and subsequently to define in what extends the geometry of the cell, that it is related to the behaviour of the structure. Finally the proposed simulation environment is able to predict the overall behaviour in relation to the geometry of the unit cell, which is based on Direct Stiffness Method and on an algorithm that creates and solves hierarchies between the sub-systems.

2. Background

Auxetic structures have a negative Poisson ratio and an inverse elastic behaviour (Figure 1), which exhibit interesting mechanical properties of which the more intrinsic for an architect is their ability to form synclastic surfaces. Evans argues that by varying the cell geometry of auxetics it is possible to produce different combinations of curvatures (Evans, 1991). This argument forms the basis of the research held in this paper as it explores how the variations of cell geometry affect the behaviour of the suggested structure, of which the geometry is based on the basic unit cell as introduced by Masters and Evans (1996).

The geometry of the unit cell (Figure 1) leads to the hypothesis that its repetition could form bellows. Through the studies of bellows, it has been suggested that the optimisation of the behaviour of the structure could be done by changing the length of the incident edges of the two vertices pointing ‘inward,’ or towards the interior of the polygon of the unit cell, as it is described in Figures 2–3. By adjusting the length of the edges L₁ and L₂ (Figure 2) it will differentiate the compliance behaviour between the upper and lower sides of the structure, and in this way compelling the prototype to bend by preventing its uniform linear elongation.

![Figure 1. Re-entrant cell proposed by Masters and Evans (1996).](image-url)
While the internal pressure is known and is homogeneous, it is proved from the equation of pressure/thrust that in order to create eccentricity it should be changed the $A_{\text{eff}}$ (Figure 2), and therefore the ratio of $h_1$ divided by $h_2$ should not be equal to one, as it is explained in the diagram below (Figure 3).

There are many methods to digitally study the deformation of a structure (Kaminakis & Stavroulakis, 2012), and most of them are related to numerical methods and non-linear analysis (Kirk, 2001), particularly in the case of Real Time Deformations (RTD). However in this research it is suggested that RTD can be solved by analytical methods, specifically using the Direct Stiffness Method based on Felippa’s book (2004).

3. **Research Method**

To begin, 3D printed (SLS) bellows—which are known examples of compliant mechanisms based on the selected auxetic cell, and which are actuated from internal pneumatic forces—were tested to determine what extent those structures can be formed and still function. Having achieved satisfactory results during the initial tests, the next step was to develop the simulation environment using the finite element method, informed from observations gained after the juxtaposition of the 3D scanned (Xbox Kinect Camera) physical prototype to the digital one.

3.1. **FIRST MATHEMATICAL MODEL OF BELLOW STRUCTURE**

The first mathematical model derived from the studies of the auxetic cell. It was suggested that it could be translated into a mathematical representation based on pin-joined trusses, while the internal pressure could be represented as forces — vectors acting on nodes (Figure 4), and that it could be solved by an algorithm based on the Direct Stiffness Method. Therefore the first simulation algorithm has been developed as a pin-joined continuous mathematical model, and after an optimisation of its stiffness by adjusting the elasticity of its trusses, its adequacy has been tested.
The test required a manual alteration to the mathematical model and the geometry of the auxetic unit in order to achieve an anisotropic deformation (Figure 5). Having achieved the proposed shape shift in the digital model it has been 3D printed and its physical artefact tested. The high correlation of their behaviours (juxtaposition of the 3D scanned model with the digital) lead to the conclusion that by changing the geometry of every unit cell it was possible to force a bellow to bend, while the mean curvature of bending is proportional to the ratio of the heights of the auxetic unit (Figure 5). Furthermore it is also concluded that it is possible to simulate the behaviour of a bellow using a continuous pin-joined mathematical model and DSM.

3.2. MATHEMATICAL MODEL AND FIRST ALGORITHM OF AUXETIC STRUCTURES

Having achieved adequate results in the previous experiments, the next step was to test its performance in auxetic based structures following an inverse procedure. Using the knowledge gained in previous experiments, digital analysis and synthesis of the geometry prior to the fabrication and test of the physical artefacts was attempted.
Specifically it had been suggested that the unit cell of the tested bellows could form auxetic structures in a certain geometrical composition. Thus the following experimental procedure was an attempt to define and simulate a possible digital auxetic geometry based on the properties of bellows. The auxetic unit cell proposed (Figure 6) was a polar array of four bellow auxetic units with 90° angle offset between them. The final auxetic structure was a rectangular array of those unit cells.

The research led to the development of a three-dimensional digital auxetic model having all possible triangulations in every unit. The model worked in small dimensions and with a small number of cells during the digital tests. When the number of cells increased, the simulation became very slow due to the huge number of equations it had to resolve. The above method of simulation proved insufficient to be used with complicated auxetic structures due to the large size of the resultant stiffness matrix. The fact that a digital simulation environment should be fast and flexible stimulated the introduction of a new hypothesis for the simulation of these kind of structures, which stated that “an algorithmic analysis and synthesis of sub-systems could be the base for the simulation of a complex system using linear analysis”.

3.3. REFORMULATION: ALGORITHM BASED ON SUB-SYSTEMS ANALYSIS

According to the discretization hypothesis, the structure should be separated into sub-systems, and the structural behavior of each sub-system should be solved separately following a hierarchical procedure. The direct stiffness method calculates the deformation of a pin-joined structure, which always has some nodes fixed in 2 or 3 directions. The fixed nodes could have prescribed displacements (boundary conditions), which will be calculated while the method is solving the deformation of the structure. The idea behind the discretization, solution, and hierarchical connection relies on this part of the method. It is argued that if the algorithm begins the simulation from one single cell with fixed nodes, other than where the nodes are connected to other elements, and a constant force, the DSM is able to calculate the displacements of the remaining nodes. The second element will have as fixed nodes the nodes
where they are connected to the first element. The known displacements of those nodes from the simulation of the first element will be the prescribed displacements (boundary conditions) for the simulation of the second element. Following this procedure, the algorithm will be able to solve the structural behavior of all the elements. Then applying a transformation matrix to combine the elements together, the algorithm will result in the simulation of the behavior of the whole system (Figure 7 and 8). To test the new hypothesis of the mathematical model it was suggested to test it first in a simpler model (bellow) and in a more complex auxetic structure.

During the second phase of the algorithmic procedure, a rotation matrix $R$ of unit vector $\mathbf{w}$ and a transformation $T$ is applied to each element after structural analysis $S$, in order to combine the digital model in its final shape (connect elements).

The proposed algorithmic method has been successful in simulating the bending. However from the comparison of the curvatures of the two models, the conclusion is derived that the physical bellow has non-linear viscoelastic behaviour, which increases in an amplitude of a sine wave until the middle of the structure as it decreases in amplitude of a cosine wave from the middle to the end of the structure.

From the above observations it was suggested to add viscoelasticity to the mathematical model by changing the magnitude of force in relation to the length of the structure. The proposed force acted along the structure from the beginning to its centre in a sine function, while it decreased from the centre to the end in a cosine function.
After several tests it was determined that the correlated mathematical model was successful in simulating fabricated bellows. From the results, it has been proven that by simplifying the mathematical model, it is possible to simulate quickly and accurately the behavior of a fabricated pneumatically actuated bellows.

3.4. AUXETIC STRUCTURE: FINAL DIGITAL MODEL AND ALGORITHM

To test the algorithmic process with an auxetic structure, it was suggested that a subsystem geometry derived from the diagonal separation of four sub-elements (Figure 9), which has four auxetic units would be able to transform the component in four directions. The proposed algorithm (Figure 10) for this approach used three types

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\sum_{j=1}^{n} j = S(C1) + \sum_{j=1}^{2^{2-n}} (S(C2_{j} + \left|u_{j}\right|).TR_{i}(\overline{w}_{j}) + \sum_{j=3}^{4} S(C3_{j} + \left|u_{j-1}\right| + \left|u_{j+1}\right|).TR_{i}\left(\frac{w_{j-1} + w_{j+1}}{2}\right)
\]

\[S = \text{Structural analysis}, \quad C_{k} = \text{Component } k, \quad u_{k} = \text{prescribed displacements derived from component } k, \quad TR(\overline{w}) = (\text{Transformation}) + (\text{Rotation matrix}) \text{ of unit vector } \overline{w}\]
of components in relation to the hierarchy of connections with their neighbours. The first component C1 had eight rectangular connection-faces (CF) with fixed lower nodes. The second component C2 had four fixed nodes at the upper and lower corners of its rectangular CF, and the third component C3 had two rectangular CF with eight fixed nodes at the upper and lower corners of each face. The first component C1 was positioned in the centre, and was constructed only one time at the beginning. After its structural analysis the displacements of the edges of its CF have been applied as prescribed displacements to the fixed nodes of the CF of the components C2, which had been connected to the four CFs of the component C1. During the third iteration, a component C3 had been positioned between two components C2 after the displacements of the two CF of neighbouring C2s applied to the edges of CFs of C3. The last two steps had been repeated proportional to the size of the auxetic structure.

3.5. RESULTS: DIGITAL MODEL SIMULATION AND PHYSICAL ARTEFACT ACTUATION

According to the measurements the simulation algorithm, and the geometry of structure, proved able to transform the structure and to form synclastic bending (Figure 11) when the ratio of the heights of auxetic units was not equal to one (Figure 3). Therefore it was concluded that the curvature of the structure could be optimised according to the ratio of auxetic units.

Having achieved adequate results with the digital model, its physical artefact was then fabricated. During the tests, the physical prototype exhibited an auxetic behaviour and formed synclastic geometries (Figure 10). The forces acted internally from the pneumatic actuation and produced the expansion of the structure in
two perpendicular axles, and therefore have created auxetic behaviour (Figure 12). The fabricated auxetic structure had similar viscoelastic behaviour along its axis with fabricated bellow structures. The application of “viscoelastic force” to the mathematical model also created viscoelastic behaviour and a high degree of correlation (Figure 13).

4. Discussion

The combination of physical and digital models, and the iterative experimenting process, proved adequate in the creation of the hypothetical system, and to the acquisition of its properties in a digital environment. Additionally the discretization of the geometry in simpler units made possible the use of linear analysis for the simulation of the geometry. The final algorithm was responsible for creating hierarchies between unit cells. Starting from a centre cell, which had four links, the geometry expanded by connecting them to four other cells. By applying boundary conditions to the connection nodes, and using translation matrices, the simulation of the behaviour of the structure was achieved.

This research in soft kinetic systems through the optimisation of the geometry of a re-entrant unit cell suggests a further exploration in possible forms and behaviours using other re-entrant geometries. Even though this paper has analysed only a simulation algorithm, it is suggested that it could be the basis of future research into a more sophisticated optimisation algorithm able to prescribe behaviours in fabricated soft kinetic systems. The development of a smart CAD environment able to prescribe behaviours could invigorate the research in methodologies for behavioural design, and a theoretical discussion on form finding through these methods.

Figure 13. Comparison of curvatures of mathematical and fabricated models with ratio 0.29 along their latitudinal axis.
5. Conclusion

The methodology based on an iterative process (hypothesis, test and feedback), succeeded to define an adequate method of creating, simulating and optimising the behaviour of an auxetic structure. Following this procedure, both the digital and physical systems were constructed. By understanding and manipulating the micro-behaviours of the auxetic sub-components it was possible to design, optimize and simulate the emergent macro-behaviour of the whole.

The implementation of digital techniques in design, simulation and fabrication of auxetic based pneumatically actuated structures breaks new ground for research in soft kinetic systems in dynamic architecture. Consequently it proposes a research space for fabricated soft kinetic systems with prescribed behaviours able to interact softly with humans and to adapt smartly to the environment.

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