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MATHS & CRAFTS

Fractals for small-scale design

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Abstract. This paper discusses the potential of the application of principles of fractal geometry in small scale design, using the design of parametric furniture as an example. It begins by summarizing the fundamental mathematical principles of fractals and the, relatively limited, application they have had so far in design. A theoretical stance is taken, viewing fractals as form-generators with significant potential. From this, the main principles of a computational design methodology are established, drawing on parallels from other design periods. As proof-of-concept, the paper presents the application of the methodology in a specific brief for the design of a piece of furniture, accompanied with images that document and illustrates the process. A parallel is established between the 21st century methodology of digital craftsmanship presented here, and the 19th century Arts & Crafts movement. Finally, the paper presents directions for further research which would allow the utilization of fractals in different design scales.

Keywords. Fractal geometry; computational design; digital manufacturing.

1. An introduction to Fractals

1.1. FRACTAL GEOMETRY

In modern mathematics, fractal geometry is a significant field of study both in pure mathematics, with links to complex analysis and chaos theory, as well as applied mathematics, with applications being found in various fields of physics, engineering and technology.

Given the importance and applicability of the topic one would expect that there would be a highly specific and precise definition of a fractal, as is typically the case with mathematical concepts. However, fractals remain notoriously difficult to define. Most however exhibit certain properties such as a fine structure on (arbitrary) small scales, irregularity, and often a recursive definition (Falconer, 2003).

In order to facilitate understanding, visualization and graphical representation has been used since the beginning of the study of the field. The advent of computers increased manifold the capabilities of representation and today there is a range of software that allow the design of fractals. The focus of such software is often mathematical, but various tools have provided a focus on the artistic by-products of the geometry, such as the popular Electric Sheep program or the landscape generator Terragen.

1.2. FRACTALS IN ARCHITECTURE

Given the visually complex and striking nature of the outputs of fractal geometry, one would expect that architecture would engage with it directly, following on the various precedents of mathematics-influenced architects. This however has not been the case. The literature describing the effect of fractals in design is limited, and it tends to concentrate more on the potential of fractals, than on built (or even designed) examples (Bovill, 1996). On the academic side there have been interesting efforts in establishing how the non-topological dimensions defined in fractal geometry affect architecture (Lorentz, 2012).

The fact remains however that fractals have had very limited application in architecture and design. This appears like a missed opportunity; fractals provide infinite form-generating possibilities while simultaneously raising thought-provoking questions about scale and dimensionality. Their mathematical nature means they can be generated via scripting and implemented via digital manufacturing techniques. In an era where the interest in morphogenesis (Roudavski, 2009; Menges and Ahlquist, 2011) and detailing (Loveridge and Strehlke, 2006; Picon, 2013) has remerged; when digital manufacturing has become a standard tool of the trade (Sheil, 2012); fractal geometry can be a powerful tool in introducing a new, mathematically- inspired and oriented design.

This paper intends to show that the potential is significant and the computing and manufacturing tools that exist today open a great range of possibilities.

2. A form-generating fractal for furniture design

2.1. MATHS & CRAFTS

The design presented here was a response to a challenging brief that demanded a modular table that would be easy to reconfigure, combine, and manufacture. Interestingly, the design ethos was meant to be "contemporary DIY"; 5-axis CNC machines were to be used for the manufacturing, but at the same time the aim was to maintain a non-industrial aesthetic.

The response presented here ascertained that this effectively called for a new aesthetic, intended for a 21st century "digital craftsman". The thesis is that, as the Machine has achieved technical perfection, the rules change. The craftsman is the designer; the attention to detail manifests in highly detailed design; expertise in one's tools means expertise in the Digital, allowing the creation of geometries that others strive to achieve. Simultaneously, the sources of inspiration differ. The Organic, the Minimal, and the Random are not discarded, but the 21st century craftsman has a wider range of tools and often these are mathematical, relying on complex equations and elaborate algorithms, as hinted by Kotnik (2010).

While at first glance the two approaches appear contrasting, there is some precedent. The notion of the craftsman's aesthetic infused with the modi operandi of a formally trained designer has echoes of the 19th century British Arts and Crafts movement (Cumming and Kaplan, 1991). The design presented here reinterprets the movement through a 21st century lens and, as it relies on mathematics, it is provocatively called "Maths & Crafts".

The implementation in a design however would require a mathematical concept to act as a form generator. Given the self-similarity, recursion, and fine detailing of fractals described above, fractal geometry presented an ideal starting point.

2.2. THE PYTHAGORAS TREE

The Pythagoras Tree is a simple 2D fractal generated by applying a set of iterated geometric rules. It relies on consecutive rotations utilizing Pythagoras's constant, and after a certain number of iterations the form resembles a tree (Figure 1).

As an introduction to developing a methodology, this fractal appears ideal. As a two-dimensional shape it allows for easier manipulation; its iterative properties are concentrated on a single plane, reducing the compositional complexity and programming difficulty. Thus, it is ideal for designs based on clearly defined planes. The Cartesian nature of the fractal, with exact $\pi/2$ rotations, indicates that the visual result will be more coherent if the composition is based on planes arranged on similarly strict Cartesian manner, ideally parallel and perpendicular, as is the case for a table.

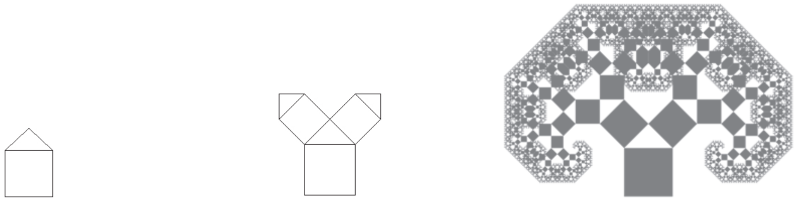

Figure 1: Development of the Pythagoras Tree

3. Crafting a wooden table from a Pythagoras tree

3.1. A METHODOLOGY

The constantly changing nature of the Pythagoras Tree with each iteration, gives different design outputs at different scales. To demonstrate this, a suitably sized Tree is drawn, and split into parts creating different tables. The fractal patterns serve as both form guides and decorative elements. The decoration is highlighted further by creating negative 3D extrusions. The legs of the table are then created from similar parts of the fractal.

Starting with a side dimension of 1524 mm (60 inches), a Pythagoras Tree fractal is drawn with 9 iterations (a simple script can be written to draw this). From these, six different tables can be manufactured; three designs and their mirror images, symmetrical on the y-y axis (Figure 2).

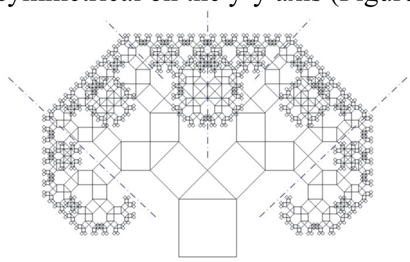

Figure 2: Pythagoras Tree with sections per table

These can be then sized into individual components, with suitable dimensions as to fit the working area of a typical milling machine (Figures 3 and 4).

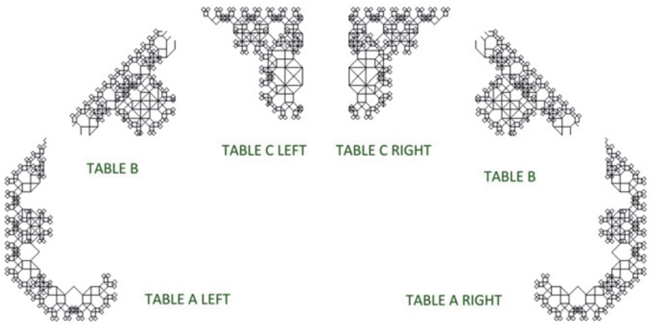

Figure 3: Fractal decorative pattern for each table

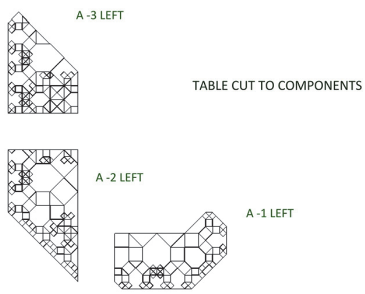

Figure 4: An example of a table cut to individual components

Similar scripts can be written to create programmatically offsets of square and triangle faces, offsetting to the interior of a rectangle and replacing the original points in triangular faces. Applied to all the faces of a component, the visual result is that of Figure 5.

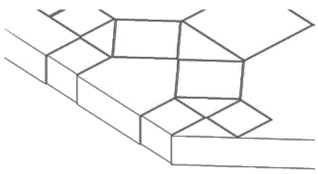

Figure 5: Detail

Thickness can also be applied algorithmically. Depending on the complexity and the efficiency of the algorithm in identifying the correct faces, some manual corrections via a 3D modelling package might be needed. In the application of the project presented here the aim was to achieve a balance between programming and modelling complexity. The 3D modelling package used, its capabilities and constraints, play a critical part in selecting the optimum combination of tools for each design.

Similar procedures are applied to design the legs of the table, which are built from the components of the fractal not used in the board. This aims to introduce a notion of a "digital sustainability". While this is meant as an allegorical gesture, without immediate practical effects, extensions of this in the physical realm are applicable in the manufacturing of the component (Figure 6).

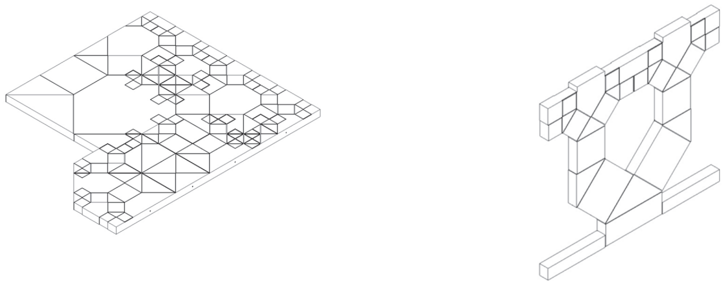

Figure 6: Isometrics of typical component and typical leg

Once the designs are completed, they can be exported to a suitable file format as to be readable by a milling machine. Any wood type of suitable quality and strength can be used; the texture and colour can be decided based on the selected end use. Other, non-organic, materials are not recommended.

3.2. MODULARITY AND FLEXIBILITY

The design concept behind the finished object is for it to work on three levels. On the macro scale the right angle contrast between the horizontal and vertical planes of the board and the legs creates a Cartesian contrast. This is supported by the 45 degree angles of the legs while their lineation suggests the development of the Pythagoras Tree, which continues in the horizontal plane. On the medium scale the aim is for the fractal pattern to pique the viewer's curiosity, inviting him/her to explore the object further, hinting at aesthetic not only functional considerations at the design. Finally, on the micro scale, the fine detailing of the design is meant to work together with the natural grain-based texture of the wood visually; a merge of the Organic and the Mathematical, the visual complexity of one both building on and contrasting with that of the other. Figure 7 shows some sample tables and a detail.

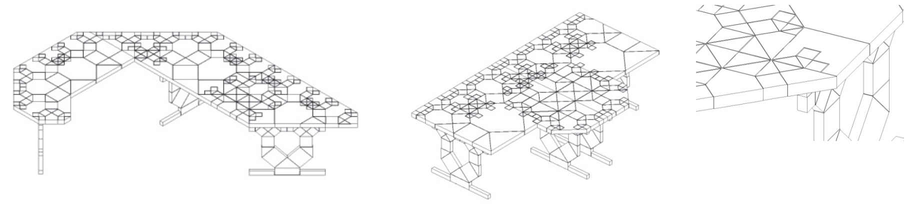

Figure 7: Sample tables and detailing

The size of the individual components is not simply a restriction imposed by the working surface of the milling machine; it allows for a modularity that enhances significantly the flexibility of the design. The mathematical underpinning of the design enables numerous combinations, seamlessly so. Often the board detailing acts as guidance for possible combinations at different scales. Figures 8 and 9 provide illustrations of sample combinations for different usages in the small, medium, large, and extra-large scales.

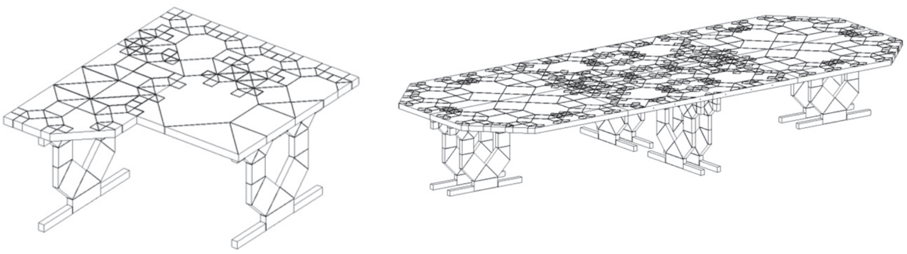

Figure 8: Small reading table and medium conference table

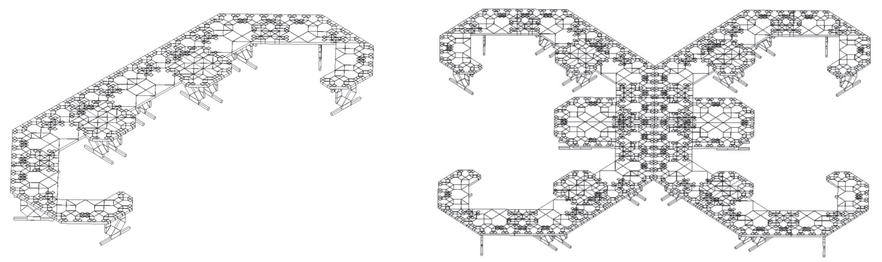

Figure 9: Large office table and extra-large exhibition table

This is achieved with one design, at one scale only; the same designs could be extended to different scales by changing the starting dimension, thus multiplying the usages. This also links with the key issue of ergonomics; the design presented here follows the guidance of the UK standards for desks (BSI, 2000). Specifically, a default height of 720 mm, with a default clear height of 690 mm has been adopted. Furniture design standards tend to err on the side of greater height, arguing that the key aspect is the clear height and user-specific height adjustments can be made with an adjustable chair. Experts suggest that the ideal is a height-adjustable desk (Lawson, 2013). The benefit of a parametric approach is evident here; the desktop supports allow for modularization of the height based on how deep they enter in the desktop, and how high the based is adjusted. This requires a small amount of further processing to tailor to specific users, especially in the detailing and the geometry of the legs, so both functionality and a visually uniform result is achieved. However, as the design approach is parametric, it is safe to assume that the demands placed by different scales can be satisfied without an effort that is disproportionate to the benefits, while the production process could be easily automated for mass production via a simple script that would link the leg parameters to user height data and/or personal preference.

3.3. DISCUSSING MATHS & CRAFTS

One would not expect a hundred years of modernist teachings to go quietly into the good night simply because of the emergence of new technologies. Indeed, despite the gradual re-emergence of the Ornament many seem to conform to the 1929 Loos maxim that "the evolution of culture is synonymous with the removal of ornamentation from everyday use". After all, in its original re-appearance in post-modernism, the Ornament served an ironic purpose, fully embracing the pastiche (Jencks, 2011). Today a concern is often voiced that the new utilization of the Ornament is just a return to Enlightenment architectural ideas of the "decorated shed"; Picon (2010) has voiced concern the new digital culture favours surface over the tectonic, envelope over structure. Kolarevic and Klinger (2008) identified three lines of enquiry: seamless materiality; outcome of digital craftsmanship; unity of skin, structure, and pattern.

This last observation sets some clear benchmarks to evaluate the Maths & Crafts paradigms against. It is self-evident that as a design paradigm it is directly connected with the potential and outcome of digital craftsmanship. Simultaneously it pursues materiality by utilizing techniques that showcase the potential of the material; the design example resented here focuses on an intricate surface pattern to highlight the granularity and uneven colouring and texture of solid wood. Designs in the Maths & Crafts vein with different materials will call upon the employment of different mathematical tools to celebrate this materiality. This is in line with the challenges identified by other digital craftsmen, calling for an engagement with the small scale and a rethinking of the material use, detailing, and manufacturing (Thomsen and Tamke, 2013).

It is in the last line of enquiry though, the elusive unity, that the mathematical underpinnings of Maths & Crafts show their greatest strength. The pattern is not simple decoration applied on a flat surface; the form of that surface itself derives from the same mathematical function. In the third dimension, the structure is derived from bending the function at a right angle; a design decision on what from the outside appears a semi-automatic pattern.

The theoretical background of this approach can be traced back to Christopher Alexander's ideas about systems (1968). Alexander separates a system as an output ("a whole…a way of looking at an object") and as a generator ("a kit of parts, with rules about the way these parts may be combined"). The example presented in this paper suggests both aspects. The design output itself can be understood as a multi-layered, multi-scale system of (in range of ascending size) decorative patterns (the detailing), structural parts (the individual parts manufactured in the CNC machine), and components that can be recombined to form different types of furniture. At the same time there is a clear *generating system* that provides the rules (a design vocabulary, grammar, and syntax) which the designer can follow to produce an object. It is a testament to the power of fractals that the designs in Figure 9, appearing externally intricate, are based on a generating system with a basic rule as simple as the one in Figure 1; a simple rotation on an isosceles right-angle triangle.

4. Conclusion

Fractal geometry provides an extremely powerful toolkit for computational and parametric design. It can act as a basis not only for simple form generation but also for a different approach to design. It appears that, though fractals are well understood and extensively analysed from a mathematical perspective, their use in design has been very limited. The continually increasing adoption of CAM and CNC techniques in architecture means that there is significant potential in making a considerably greater use of fractal geometry in design.

The project presented here concentrated in one use type, one material, and one fractal. By changing these three parameters, a wide range of combinations can be generated in a variety of scales.

Beyond showcasing the potential of geometry, the project presented here aspires to make a more general point on the reintroduction of detail in design. Though it describes a piece of furniture, the principles apply to different scales and are easily extendable to architectural design. The underlying notion is that Detail works; it is simply a matter of having the technological capacity to reintegrate it in the design. As the capabilities of the tools available to architects increase, the Detail can be reintroduced in the design vocabulary, challenging the core tenants of Modernism which for one century have been largely taken for granted.

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