UNIVERSAL DOVETAIL JOINT

A geometric investigation

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Abstract. The paper presents the geometrical investigation of a three-dimensional dovetail joint that can lead (timber) frame construction to more than two-dimensional frames; the creation of timber construction with timber members meeting at irregular angles can be shown to be feasible, simplifying overall construction. Traditional joints in timber construction usually work only in two dimensions, in other words in planar surfaces, resulting thus in complicated assemblies in three-dimensions. Stemming from traditional timber dovetail joints, the universal joint under investigation is produced under revolution of the geometry of a dovetail fastener through its middle axis. The resulting concave disk can connect timber elements under irregular angles, without the need for the structural members to lie in the same plane. The joint works due to friction between members rather than using any other element of bonding, allowing for the assembly of joints and structural members with no specialized tools. The paper explores the geometric constraints and degrees of freedom that such a disk creates in timber construction, and consequently in similar linear construction systems.

Keywords. Universal Joint; timber construction; geometric investigation.

1. Introduction

Timber joints such as dovetail joints or tenon and mortise, have long been a feature in timber construction in manifold traditional and vernacular architectures not only in Europe but also more elaborated in Asia especially in China and Japan. The creation of timber only joints which act as fasteners themselves without using additional materials like metal nails has been ac-
complished by artistic and fascinating craftsmanship many centuries ago. The results of this accomplishment can be marveled at timber frame constructions in medieval city centers of Europe or at the amazing monastery Koyashan in Japan built in 1593. Since industrialization manual labor has become expensive and therefore these elaborate and complex timber joints have had the tendency to disappear from construction. With the development of computer driven manufacturing methods and the ability of low cost mass customization it seems to be worthwhile to reinvestigate the performance of traditional timber joints in modern fabrication.

Traditionally dovetail joints only allow for the creation of two-dimensional frames that are then assembled in three dimensions, almost always in a parallel piped or triangulated form in the case of roofs. The paper describes the geometric investigation of a universal case for a three-dimensional dovetail joint that expands the capabilities of timber construction to free-form shell surfaces by using bar-shape elements. Initially the background is mentioned, then the geometric generation of the universal tie joint and its accompanying beam-ends is developed, leading to an investigation of parameters for the system and exploration of geometric constraints and boundary conditions. The paper concludes with an initial geometric case study and sets avenues of investigation to be explored in terms of materiality and loading, physical testing and construction.

2. Free form shells and dovetail joints

Timber structures are also used in 3-dimensional structures such as grid shell structures. In the Multihalle in Mannheim, Germany there is an example of a regular grid structure with slender elements designed by Frei Otto. The underlying structural idea is behind Shigeru Ban’s pavilion for Japan at the Expo 2000 in Hanover where the slender elements are not subjected to bending but mainly under longitudinal forces. The regularity of the structures support the smooth transfer of forces whereas with irregular surfaces the stresses in the grid members are not usually equally distributed. Through optimization processes it is possible to reduce specific stresses in the members. (Dimcic, Knippers, 2011). Thus although the stress in the members itself can be optimized in or through irregular grids immediately the need for customized nodes or members appears when talking about irregular geometry.

In traditional timber constructions the dovetail joints are used to join elements in various ways. Either the joint is used to connect construction members in longitudinal direction to extend the length of a natural limited timber tension element. The joint is also used to join two elements perpendicularly or almost perpendicularly. The third major category is the use of
the dovetail joint as a corner connection. It is used both in carpentry and joinery equally. Thus not only corner joints of i.e. wooden drawers but also beam joints in timber constructions are carried out with this connection type. The specific geometry of the dovetail joint enables to bear not only compression forces but also tension forces in the direction of the longitudinal axis of the element. The specific force fit geometry of the dovetail joint keeps the elements also in their defined position without the use of screws or nails or other fasteners as long as forces not affect perpendicular to the dovetail lock. (Figure 1)

In traditional constructions and furniture the dovetails are carried out through manual craftsman work. In modern timber constructions it almost vanished due to the labour intensive manual production and through the replacement by metal fastener elements. However the joint has enjoyed a renaissance through the development of computer driven machinery that were able to produce the joints mechanically. By employing special milling heads dovetails can be produced easily by machines. This capacity of machine-made dovetail joint was adopted to create a robot fabricated shell where the plate joints were produced by a milling robot where the joints are used to connect non-coplanar plates (Figure 2).
The force fit character, the capacity to take both tension and compression forces and the simplicity of the joint itself make the joint a perfect connection method to resolve grid shells restricted to only tension and compression elements. The above-mentioned example is using plate shape elements to create a compression only shell. This paper focuses on bar-shape elements creating a compression and tension containing shell structure.

3. Geometric analysis of the joint

The force bearing capacities are dependent on the quality of the used timber but more important on the geometry of the dovetail since through the geometry the size of the force transferring surfaces are defined. In traditional handcrafted joints the geometry of the dovetail is carried out according to thumb rules rather than on a real calculation basis. These rules have developed through experience and practice and might differ in different regions in the world. The geometry shown in (figure 3) is based on the second author’s experience and training as a carpenter in Germany.

Timber is not a homogeneous material having differentiated load bearing capacities in a member but which has distinct structural properties according to its fibre directions. Timber compression strength perpendicular to the fibre amounts 2.0 N/mm² and in parallel direction to the axis the compression strength amounts to 8.5 N/mm² significantly higher. Tension forces parallel to the fabric direction are possible up to 8.5 N/mm². Shear forces can only be
added up to a maximum of 0.9 N/mm². If forces are affecting from different angle than 0° or 90° to the fibre direction the maximum strength could be approximated by the following formula:
\[
\sigma_{\alpha} = \sigma_1 - (\sigma_1 - \sigma_\perp) \cdot \sin \alpha \quad [N/mm^2]
\]
Since the thresholds for \(\sigma_1\) and \(\sigma_\perp\) are constant the allowable stress is proportional to \(\sin (\alpha)\). This dependency is affective to the geometry of the joint. To create a most effective joint system this force geometry dependency can be implemented into the grammar of the joint where not the traditional thumb rules are applied but location specific geometry according to loads and forces.

4. The Universal Dovetail Joint

An example of a shell created with the universal dovetail joint presented below is shown on (figure 4) to help the reader understand the simple yet elegant construction the grammar and the joint attempts to create.

The universal dovetail joint (Figure 5) is developed through geometric transformations of a classic dovetail starting from a single trapezoid, mirroring, extending the ends and then revolving 180 degrees around the central axis. The result is a disk with two concave cones missing from top and bottom centre. Purposes of construction force the subtraction of a wedge from the joint so that appropriate beams can be inserted and locked into it. (Figure 6)
By fabricating the joint with milling machines, we can accomplish a gradually wider curved area that will be used to lock the beams into place, much like the traditional methods of timber construction where the joint and beam are hammered into position and then stay there under friction through loading without glue or nails.

5. Geometric Boundaries

The investigation includes the fitting of a beam to the joint (Figure 7):

Here the end of the beam is treated with a subtraction of the negative of the tie joint, with variations on the rotation that the subtraction can take. The different rotations allow for the beam-joint system to accommodate various angles in a free surface construction, embedding however constraints in terms of the angles of position and dimensions of the members. From figure 7 it is evident that the critical dimensions of the beam will be i, j where ma-
terial failure under loading will lead to the beam failing. As such the grammar needs to take into account the dimensions of the members as parameters of geometric constraints, enabling the geometrical configuration of joint and beam to avoid any structural failures. Positioning of the members on the joint can happen symmetrically and asymmetrically, starting from two members per joint (Figure 8).

![Figure 8: Symmetric and asymmetric members and joints configurations in plan drawing](image)

The classic dovetail joint is equivalent to the case where \( n=2 \), where all the loading of the joint is only tension. In essence the universal dovetail joint is a global case of the stereo-ids that would be produced for \( n=i \), where \( i \) is the number of the members joining each time (Figure 9).

![Figure 9: Alternative types of the joint, with the dovetail joint transformed into 3d to accommodate 3-4-7 or 25 members joining](image)

In essence the geometric transformation that creates the universal dovetail joint is a global grammar covering the cases where the dovetail joint is transformed for polygonal arrangements of members. The equivalent in Euclidean geometry happens when the polygon is transformed to a circle with the \( n \)-, where \( n \) is the number of sides, tends to infinity.
Free form shell structures can be produced using beams either with a rectangular configuration or with a triangulation of all rectangles in the shell. A grammar or set of rules (not examined here) can determine the strategy of triangulation. In both a triangle and rectangle configuration the number of beams converging into the joint is specific, either in symmetric or asymmetric form as in (Figure 8). In triangle configurations a joint will be at the point of convergence of 8 members, when the joint is located inside the grid of the shell structure, or 5 members when located on the edge. In rectangle configurations the joint will be at the point of converge of 4 members inside the grid, and 3 members on the edge of the grid. The convergence of 8 members is revealing in the limitations of the system (Figure 10)

![Figure 10: 8 Members converging asymmetrically on one universal tie joint.](image)

Increasing the number of members connected on one joint will not be possible above a certain number, as the width of each individual member will accumulate, filling the periphery of the joint. The dependency of the maximum number of members from their width can be given by the following function, since the maximum number of beams will form a polygon with their widths.

\[
\text{radius} = \frac{s}{2 \sin \left(\frac{180}{n}\right)}
\]

Where \( S \) is the width of the beam, \( n \) is the number of sides, and \( \sin \) is the sine function calculated by degrees. To be able to explore these constraints, the joint and beam were analyzed parametrically taking into account the material constraints that timber members impose, the constraints created by the number of members and the positioning of the members on the joint.
From Figure 11, the extraction of relationships between dimensions to define the grammar parametrically is possible where:

\[
\text{Width } b = 2c_1 + 2d_1 + e_1 \quad (2)
\]

\[
\text{Radius of joint } r = \frac{b}{2} = c_1 + d_1 + \frac{e_1}{2} \quad (3)
\]

\[\text{Angle } \theta \text{ trigonometric functions:} \]
\[
\sin \theta = \frac{f_1}{h}, \quad (4)
\]
\[
\cos \theta = \frac{d_1}{h}, \quad (5)
\]
\[
\tan \theta = \frac{f_1}{d_1} \quad (6)
\]

\[
\text{Angle } \phi = 90 - \theta \quad (7)
\]

\[\text{Structural height of beam } S = \frac{a}{\sin \omega} \quad (8)
\]

The dimension in the middle, e, is a critical dimension, since it could expand the size of the joint considerably allowing for more beams to join on the joint, avoiding at the same time one beam converging unto each other.

5.1. CONSTRAINTS

The perfect theoretical shape for a joint should be a point. Stemming from this the dimensions for the universal joint should be as small as feasibly possible. The geometric constraints applying on the joint are the number of beams that converge to the joint, the structural height of the beam and the expected surface that the beam will engage on the joint, as this will have repercussions on the behaviour of the stability of the system. Accordingly, the optimized beam would be one with minimal width but as high as possible.
The critical geometric constraints on the beam are the dimension ‘h’, that determines the surfaces that the beam engages with the joint transferring load. Although we have described the joint as a timber construction element, modern fabrication techniques can allow its production with ease, for example by using milling 5-axis machines or robots, or produce the joint from other alternative materials. An obvious case the authors would like to explore is a 3d printed version out of plastic or a 3d milled one out of metal. The initial geometric investigation has been based on connections that either work via tension or compression. However free-form shell structures can develop more than axial forces leading to moment loading. The i and j dimensions of the member in (Figure 11) are critical in making the joint-beam system as efficient as in axial loading situations.

6. Conclusions

The paper has described the geometric investigation to generate a novel universal dovetail joint for use in free-form shell construction, including the constraints and parameters that connect the joint and beam system. The clear advantage is the potential to create a free-form shell without the need to customize all joints according to the directions the beams converging to the joint. We suppose that this would allow an easier construction without nails or glue, employing the joint using relatively untrained personnel, and industrialization of production in a large scale, without the need to customise the joint for each connection. We believe that by exploring the possibilities of the parametric grammars and optimising according to the constraints type of joints can be constructed and tested for almost every conceivable free-form shell.

The authors look forward in exploring the issues and constraints the paper frames depending on the joint -beam system, in terms of materiality, loading and fabrication in depth.

References