PLAYING BY THE RULES

Design reasoning in Escher’s creativity

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Abstract. Despite reservations for the rule-based perspective of design with regards to creativity, the role and the potential of rules is instrumental in reflective design thinking. Rules enable sensible and functional conversations about the process. Working with (and mastering) a small rule set is common amongst artists as well. Escher developed rules based on the shape combinations in Majolica tiles to investigate regular plane divisions in his own work. Dwelling on a rule set allows for explorations in a rich design space that result in new series of works. Escher’s published sketches show that throughout his artworks, keenness in seeing shape relations and an ability in manipulating simple shape rules leads to numerous artworks from the same tessellations. Through a visual and rule-based analysis of artistic tiling in Escher’s work, the paper provides a computational perspective on analytical methods for design.

Keywords. Shape; Shape Grammars; Art; Rule-based Design; Rules as templates; Escher.

1. Introduction

Designing is a multifaceted activity and cannot be defined just as a straightforward mechanism. Mitchell (1993) writes that "any successful attempt to describe the mechanism of some 'creative' design activity will have the immediate effect of redefining that activity as 'noncreative'. The more success we have, the more we can be accused of dealing only with the noncreative aspects of design…" The generation of a work of art and design is a creative process. Rule-based and grammar-based perspectives into design acknowledge the repetitive acts that yield to creativity within design. Rules represent acts (visually or spatially manifested decisions) whereas
grammars stand for rule sets. Individually, rules are more versatile than their
generalized instances in grammars. Nevertheless rules are able to epitomize
repetitions but do not explicitly capture how they contribute to creativity.
The key is in how rules are applied and utilized. On the limits and potentials
of rules and grammars, Chase (1997) observes that "creative design using a
grammar does not have to be limited to a grammatical derivation; it also
occurs in the development of a grammar. Moreover, restrictions are in the
eye of the beholder." The emergence and comparative use of rules are part of
the creative process. Stiny’s writings (2006, 2011) are indicative of the
comparative and creative use of rules and categories of rules, namely
schemas, in visual design. Following this understanding of visual rules, we
review Escher’s artworks to reflect on the pedagogical function of rules as
tools of comparison and understanding in a creative process.

The Dutch graphic artist M. C. Escher was best known for his
mathematically inspired artworks. He usually worked with geometric
patterns to generate his art. When viewed closely, the mathematical rules and
tessellations behind his works are clearly seen. Escher used these
tessellations as templates for his graphic art.

Escher continually searched for new rules, variations and systems in his
art works. The concept of infinity was one of the key ideas he wanted to
visualise. His motivational question was "which possibilities does a flat
surface offer if we want to fill it entirely with congruent figures?" (Ernst,
1998). Continuous fields, only possible with systems of mathematical
relations and tessellations, fit well with the aim of achieving visual infinity.
Nevertheless, Escher’s works present creative aspects just as much as
mathematical relations. In his efforts to seek and achieve variations, Escher
is keen in adapting shape relations into new ones in his mind’s eye. Within
the scope of this paper, creative endeavours in Escher’s works are generally
understood to be in reference to seeking variation, a common act in basic
design.

Our study suggests that working on the same rules or repeating the same
shape manipulations is not what limits creative thinking in a rule-based
approach. Fixation on a certain application of a rule set may indeed trap one
in what already exists. Ironically, most artists first master available rules
prior to achieving further discoveries, new techniques, and adding new rules
to their work. There is value in repeating a rule for the purpose of
understanding how it may change. Escher’s works, a small group of which is
explored below as a series, provide a showcase for such a purposeful
approach to rules.

The selection of Escher’s works presented and investigated in this paper
is based on a logical lineage. The works are studied mainly with
consideration to the repeating shapes and the underlying tessellation rules. Starting with the artist’s earlier works, analysed images are selected to consecutively include more shape types or transformative rules in the tessellation than in the previous one. As we follow through the examples, the number of repeating shapes increases, the repeating shapes gradually morph or entirely change into more complex forms, and finally, the Euclidean transformations (specifically mirror reflections) alter how the tessellation works in each case.

2. Certain Shape Rules as Drawing Templates

Escher is among the best-known artists to explore mathematical rules. His earlier prints as a graphic art student shows that he is interested in strong symmetrical rules from the beginning (Piller et al., 2013) and that he develops more complicated relations of symmetry over time.

In an early work from 1922, Escher tries to create a continuous field with eight head figures (Figure 1a). Continuity is achieved through form repetition and form similarity as well as through forms flowing into one another. For example, heads 1 and 2 complement one another. The exception is the disruption between head 3 and the neighbouring ones (Figure 1b). Compared to his later works, this is a noticeable disruption in the continuity.

The following works displays Escher’s fascinations with the shape rules of Majolica tiles in Alhambra. That Escher copied the Alhambra patterns on his visits (Piller et al., 2013) helped him in developing his own pattern sketches to study shape combinations. He was captivated by the traditional artists’ ability to create such complex geometric relations. As he read on crystallography and conversed with mathematicians, he started creating his own ideas for similar shape relations (Schattschneider, 2010). He based his graphic art on geometrical patterns and created new regular plane tilings. Some of these matched the tiling of the crystallographic approach (Souvignier, 2009). Figure 2 shows a Moorish design from Alhambra together with an artwork by Escher. As in many other early works from the
same time, Escher copied the geometrical pattern of the Alhambra tiling but adapted the shapes to beetle figures (Stevens, 1980).

![Figure 2. a. Moorish tiles from Alhambra, b. Escher's drawings with beetles, 1953 (both taken from Stevens, 1980).]

The tessellations behind Escher’s drawings worked as templates that not only organized the visual field but also sustained the visual continuity. The relation between the shapes in the template provided for the outlines of Escher’s figures. Escher’s own definition (1947) for his technique gives clues in "comply[ing] with the rule" and repeating forms in the plane that "can be endlessly varied." The idea was to fill up the plane with repetition and without an empty space. The rules for tessellation and variation of figures in each artwork worked towards this goal. The creative potential existed "both because it offers endless possibilities and the huge restrictions imposed by the rules of play." (Piller et al., 2013) Both the tessellation rules and the rules behind how the figures vary are of interest for this paper.

Figure 3 is a copy of a page from Escher’s sketchbook with five different symmetry applications for rhombus in tessellation. On the right side are sketches for new repeating units based on one of the systems (Schattschneider, 2010). Escher marks where the connections to neighbouring shapes are with hollow points. Top and bottom sketches show his exploration with boundaries while keeping the connections between units intact.

Although Escher used certain geometric guidelines, these templates did not stop him from being creative and producing variance. On the contrary, focusing on the same rules allowed him to incrementally and systematically alternate shape relations. The sketches in Figure 3 illustrate that he defined a new rule by adapting the existing template.
In these sketches, the underlying tessellation of the rhombus is the same. All the examples below use this pattern but vary in shape rules as Escher plays with the shapes and the orientations of the shapes. The hollow points marked on the straight lines are guides for orienting the repeating shapes. The two sketches on the right are two alternatives. The sketch in the upper right corner is based on the rhombus system labelled as $IVB$ from the second top left sketch. Here, Escher creates a new shape from the rhombus and breaks its symmetry.

As shown in Figure 4, visual rules vary the shape while preserving the tessellation rule in $IVB$. Rule 1 alters the corner and creates two curved lines $A$ and $B$. Rule 2 brings together two of $B$ around another corner and Rule 3 brings together two of $A$ around the opposite corner. $A'$ and $B'$ join just like
in Rule 1 to generate a closed shape. In an alternate representation, Rules 1-3 come together in Rule 4 (Figure 5). Rules 5 and 6 are additive rules that achieve the same tessellation in Figure 4. The curved shape is copied and connected from its vertices in the horizontal axis (rule 5) and vertical axis (rule 6). The area that emerges between these four shapes is a vertically mirrored copy of each.

Figure 5. The rules to draw the template from the shape in Figure 4.

Additional to the two illustrated in his sketches in Figure 3, Escher produced more complex structures through shape modifications on the same tessellation. Figure 6 shows an artwork with a "winged lion" figure set in the same tessellation order in Figures 4 and 5. As explicitly shown in Figure 7, the rules forming the figure around a rhombus are similar to those given in Figures 4 and 5. These rules basically are of the same schema but vary in the shapes they employ. The obvious difference is that the figure boundaries of the winged lion are more embellished.

Figure 6. Winged Lion, 1945 by M.C. Escher (taken from Souvignier, 2009).
Escher was able to create numerous works using the same schemas with shapes varying across tessellations. He also created tessellations where shapes vary within.

Figure 8 shows two different artworks. In Figure 8a, rules are similar to those in the drawing in Figures 6 and 7. Differently in Figure 8b, Escher plays with variations of the repeating shape, the fish. There is a transition between three different fish figures (Stevens, 1980). Only small details change such as the fins and the tail. This example is a showcase for Escher’s variations on the shape rule within one tessellation.
In other works, Escher increased the number of unit types by introducing entirely different shapes to the tessellation. Figure 9 shows such an example where Escher used two completely different shapes in one design. The duck and the fish figures share four unique boundaries. The same tessellation is thus sustained, but each boundary segment requires a different rule as opposed to the earlier singular Rules 1 and 7 (in Figures 4 and 7).

![Diagram of Escher's Bird Fish artwork, 1938 (Redrawn based on the images in Stevens, 1980)](image1)

**Figure 9.** a. Escher’s Bird Fish artwork, 1938 (Redrawn based on the images in Stevens, 1980)  b. Illustration of the four different spatial relations between two shapes in a tiling c. Escher’s rules as interpreted by authors.

![Diagram of Escher's Periodic drawing 63, 1944](image2)

**Figure 10.** Two primary figures in Periodic drawing 63, Escher, 1944 (taken from Schattschneider, 1977),

In yet another artwork, shown in Figure 10, two different figures come together but in alternating orientations. This arrangement is similar to the rhombus systems in Escher’s sketch VI B, the second from left in the middle row in Figure 3. However it is an advanced version since there are two different shapes meeting on the boundaries of the rhombus.

Finally, in one of his famous prints, *Metamorphosis*, Escher takes a further step and continually alters the repeating shapes (Figure 11). Shapes
facing right on the canvas, *i.e.* bees, butterflies and birds, go through a gradual transformation. The bees to the left, drawn with minute detail, alter into birds represented only as black toned shapes. The fish figures facing left on the canvas, differently, only slightly change in dimensions and detail. Their outlines gradually disappear into the white background on the left side of the canvas. In *Metamorphosis*, the number of shapes, and correspondingly shape rules, multiplies beyond count. Nevertheless, the tessellation is similar to that of Figure 9.

![Figure 11. Detail from Escher's Metamorphosis, 1939-1940 print (taken from Piller et al., 2013)](image)

3. Discussion and Conclusions

Restrictions attributed to rules are just in the eye of the beholder. Rules can be created from scratch, can be applied in different ways visually, and moreover, can be comparative tools to understand what repeats and what changes in design. This study intended to show that working with shape rules and templates does not necessarily limit a pursuit of variance but on the contrary supports it. Escher’s graphic art with both repetition and variety showcases this argument.

Escher, famous for his systematically designed prints, rigorously employed rules but offered infinite possibilities. Throughout his artworks, he sought for new tessellations and used them as templates for his drawings. He was aware that he had to form his figures according to shape relations and rules. His keen eye led him to creating numerous artworks from the same tessellations. By comparatively looking at Escher’s various works and deciphering some of the rules behind the visual organization in each, the study illustrates how Escher operated systematically on certain shape rules and augmented them with alternatives and variations.

Although the word *rule* comes into conflict with creative processes to some extent, rule-based systems can operate as inventive thinking tools in design. Design, by its nature, is an act where decisions get manifested, and constraints are defined. In reflective analysis, rules benefit the designer as live documentation of possibly infinite yet interconnected alternatives of manifestations. Escher’s shape rules always link back to the tessellation rule
and vary systematically. This helps him explore other artistic aspects such as visual perceptions of continuous fields of figures. Just as the large corpus of his work, his analytic sketches and notes provide a guide to playing creatively by the rules, rule-based approach to design holds potential in talking about design creatively and, with awareness.

References
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