Abstract. Typical horizontal building types, open spaces, and human activities have yet to competently overcome or adapt to the constraints of vertically-oriented dense urban environments. Designing the built environment to the oblique, or more than two axes at once, is a required strategy for the future of city planning, the advancement of body-space interaction in architecture, and to reinforce the interconnectedness of the natural environment with human activities. With the development and increasing use of 3D modeling software, parametric and generative design processes, and the progressive investigation of complex geometry, it is likely that the oblique will be envisioned more and more as a functional architectural device. This research investigates the tessellation of minimal surface geometry to achieve a folded, multi-use surface capable of connecting disparate urban program and which can enable a city to offer amenities that are typically available in horizontally-oriented suburbs. The geometric family of the helicoid is found as the optimal formal generator because of its ability to create a continuous surface while allowing for both horizontal and vertical circulation.

Keywords. Oblique; helicoid; tessellation; surface; urban.

1. Introduction
Vertical and oblique conditions – inclines, gradients, curved or twisted surfaces – are frequently found in nature and provide a multitude of geometric scenarios for human movement. Through experimentation, humans have invented equipment and technology, formed social communities, and created commercial enterprises centered upon the manipulation of non-horizontal surfaces in nature.
As a result of growing urban populations, advances in building technology, and the increasing interconnectedness of urban communities, there is an opportunity to provide these types of spaces in the built environment as well to lessen several of the most common burdens cities face today: the shortage of parks and open space, the need for alternative means of exercise, the separation of land uses, dependence on vehicular transportation, lack of pedestrian thoroughfares, and social segregation.

An essential element in the next stage of urban growth, oblique surfaces or conditions can provide urban facilities for vibrant niche sports communities, increase the volume and variety of open space in cities, and provide program that can generate income by offering unusual experiences in an urban environment (Figure 1). Perhaps even more vital to the longevity of a city, oblique transportation methods and public space opportunities can serve to increase density, without building ‘out’, and while uniting traditionally disconnected spaces and activities.

![Figure 1. Concept rendering: the Matterhorn superimposed onto downtown Los Angeles.](image)

2. Theoretical Motivations

The following explorations are used to illustrate means of overcoming various limitations inherent to oblique environments. The oblique as an architectural strategy must overcome certain restraints concerning city planning
methods, architectural design conventions, and even the human biology of perception.

2.1. THREE-DIMENSIONAL ADAPTATIONS AND TENDENCIES

Almost all creatures have evolved to take advantage of three dimensional environments given incentives like food or protection from predators. The modern model of niche sports requires oblique surfaces (Figure 2) and demonstrates that humans are not only capable of navigating in 3 dimensions, but that we have an inherent desire to travel in the $z$ axis whether for ritual, defense, or fun.

![Figure 2. Bouldering: an activity that involves navigating oblique surfaces.](image)

It is important to consider the tendency to assume that humans are able to experience a three dimensional environment in its entirety. While the spatial world essentially has no boundaries, visual perception in this environment is limited. The world surrounds us in $360^\circ$, whereas the human visual field is restricted to less than $180^\circ$ in each dimension (Thiel, 1997). We are capable of designing environments which we can only experience one half of at any given time. There is a large disconnect between how modern architectural environments are digitally designed versus how they are physically experi-
enced. This circumstance can result in architectural solutions that do not consider the full extents or possibilities of the human perception of a 3D environment.

2.2. TRADITIONAL METHODS

Most aspects of modern technology, architecture, and city planning have a horizontal bias. The industrialization of cities has resulted in the mass-implementation of highway, subway, and train networks – semi-permanent horizontal systems – all over the world. Since the implementation of the elevator, mobility in an urban setting has become sleeker and faster. However, the majority of movement in cities still occurs in one direction at a time. When travelling to an adjacent high-rise, one must travel in a series of perpendicular vectors. Would it not be more efficient to travel diagonally in all three axes at once, possibly even using gravity as a propellant? This example demonstrates the shortcomings of designing architecture to only one dimension at a time, whichever dimension that may be.

Modern urban architecture also demonstrates the effect of traditional 2D trends in city planning. Buildings are simple extrusions, mostly squares and rectangles, which make up the urban Cartesian ground plane. There are rarely extrusions that don’t originate from this plane. Buildings which deviate from this method can provide pedestrian transportation and usable open space, as well as create opportunities to deal with the mass separation of urban programs popular in modernist planning policy. As urban architecture transitions into more complex forms, utilizing extrusions in all 3 dimensions, or alternative form-making methods altogether, the community within becomes richer and the intimacy of human interaction with architecture deepens (Figure 3).

![Figure 3. Diagram: extrusion type vs. bodily orientation.](image-url)
3. Precedents

A successfully interconnected urban environment exhibits the key tectonic methods shown in Figure 4 and also displays a vague distinction between surface and volume. Relevant built examples typically involve transit centres like Osanbashi Pier (Yokohama Port Terminal), 2002, by Foreign Office Architects and the Arnhem Central Transportation Hub, 2014, by UNStudio. These examples exhibit the formal strategies below.

![Figure 4. Diagram: key tectonic methods.](image)

3.1. SURFACE & VOLUME: DEFINING NECESSARY EXPERIENCES

The solution to connecting disparate urban spheres and adapting horizontal program to a vertical urban environment requires the strong architectural presence of a multi-use oblique surface which defines and connects several volumes. Cues can be taken from natural forms and adapted to the restraints of a dense urban environment. A hill corresponds to a surface, while a cave corresponds to a volume. When square footage or construction material cannot be wasted, both can be pared down to minimize mass and increase usable surface area, as shown in Figure 5.
The appropriate formal strategy requires a continuous surface which organizes several volumes, as shown as "surface + volume" in Figure 6, rather than a volume in which several discrete surfaces are defined, which is the current architectural status quo. The surface must provide opportunities for users to navigate and negotiate its terrain, while functioning as an envelope that bounds activity volumes on each of its sides. The concept of a folded, multi-use surface, as prescribed in Figure 6, can be created via any tessellation of minimal surfaces.

3.2. CASE STUDIES

A major consequence to designing with repetitive 3D structures using minimal surfaces is that there are often "upside down" and "disconnectedness" conditions. This is shown in geometry created from a common Schwarz P surface (Figure 7). Schoen, Neovius, batwing, and disphenoid surface tessellations produce similar problems.

These tessellations result in edges or holes where the surface is not continuously navigable under normal conditions. In an architectural scenario, this approach mimics the traditional horizontal slab approach, especially since a secondary independent system is required to travel vertically. This
consequence is apparent in Toyo Ito’s Taichung Opera House. Though the form is generated by a minimal surface tessellation, it does not produce a continuously navigable surface and lacks perceptual interconnectedness of discrete volumes (Figure 8).

While The Broad art museum by Diller, Scofidio + Renfro (Figure 9) also employs a traditional ‘slab’ approach, the project displays how minimal surface tessellations can be used to facilitate horizontal and vertical movement at once, in this case via stairs or escalator. The ‘holes’ in the tessellation follow the natural incline of the staircase, and thus, facilitate movement in 3 axes. This can be achieved by warping an unfavourable tessellation, or more genuinely by tessellating certain geometric families of minimal surfaces.

Figure 8. Taichung Opera House, Toyo Ito, exhibition model (Kim, 2010).

Figure 9. The Broad museum, Diller, Scofidio + Renfro, construction photos (Daniels, 2013).
4. The Helicoid as Solution

Certain tessellations of the helicoids do not result in holes or disconnectedness. Based on research by Daniel Piker, former employee of the Arup Advanced Geometry Unit, one can technically join helicoids smoothly into one continuous surface (Piker, 2009). This is possible due to the nature of the shape itself. A helicoid section encompassing one quarter, or 90 degrees, of a full 360 degree spiral rotation can be tessellated to provide simultaneous vertical and horizontal circulation (Figure 10).

![Figure 10. The helicoid module and tessellation.](image)

4.1. SHAPE GRAMMARS

Helicoid sections can still be joined in ways that create dangerous edges as a result of a discontinuous ground plane. Shape grammars are a useful form generation strategy given that a specific combination of mirroring, rotating, and copying a helicoid section will result in either desirable or undesirable spatial conditions. A successful operation is shown in Figure 11.

![Figure 11. A successful helicoid tessellation displaying no holes or edges.](image)
4.2. SITE CONDITIONS

Once a desirable pattern is found, the tessellation can be warped to fit the established boundary of a site, creating both a dynamic multi-use surface and several volumes (Figure 12), servicing a wide range of activities involving intimate physical interaction with an architectural surface.

![Image](image1.png)  
*Figure 12. Architectural application of tessellations as urban environments.*

5. Conclusions

Given the growing population in developed cities, there is an overwhelming need for a more efficient and thoughtful way to design architecture that reacts to restraints in the $x$ and $y$ axes. Forms that follow the formal strategies established in this study do not need to be generated out of minimal surface geometry to be effective. Oblique environments have the ability to provide needed square footage for urban communities of interwoven systems and ideologies.

Oblique surfaces created by helicoid tessellations offer a formal basis for this new urban typology, identifying a means of simultaneous vertical and horizontal transportation to be a necessity. Given that segments, or tessellations, of one geometric family can often be joined to segments of another family, continuous navigation of a form should be possible as long as manual attention is given to key navigation points when the helicoid pattern language is not the sole generator.

Future benefit of the research of minimal surface geometry in urban development scenarios lies in the advancement of environmental building technologies, human transportation systems, and new models of urban open space.
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