SEARCHING ON RESIDENTIAL ARCHITECTURE DESIGN BASED ON INTEGER PROGRAMMING

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Abstract. This paper describes an approach to using integer programming algorithms for computer-aided architecture design, taking residential buildings as an example. The research realized the dense arrangement of multiple shape templates in a certain domain of orthogonal grids. In addition, combined with the topological relationship of building functions, a single-story residential building layout is generated. The architectural design problems at different levels are solved by changing the objective function and restrictions in the integer programming algorithm. The algorithm can be expanded and employed to other fields of architecture, and may provide new architectural methodologies.

Keywords. Generative Design; Architectural layout planning; Integer Programming; Topology.

1. Introduction

In the research of functional layout design in architecture, with the diversification of modeling methods, a large number of generating algorithms have emerged. The commonly used algorithms can be broadly classified into the following types: rule system, multi-agent system, evolutionary algorithm and mathematical programming. Multi-agent systems and evolutionary algorithms focus on searching for better solutions. Rule systems and mathematical programming methods tend to find the optimal solution of a problem. Given the input conditions, one solution is usually output. Many feasible solutions can be obtained by changing the less relevant conditions.

Integer programming belongs to the category of mathematical programming. In linear programming model, the variables are limited to integers, which is called integer linear programming. Integer programming is a research on the model, algorithm and application of the optimization problem in which all or part of the decision variables are integers. It is one of the most widely used optimization models in operational research and management science. In the integer programming problem, its objective function and restrictions are linear, and the integer programming problem is NP-complete. Integer programming is mainly applicable to two kinds of mathematical models. One is that variables can only be expressed in integers, such as the number of machines, vehicles, workers and so on. The other is 01 programming, where variables are expressed as “yes”
or “no” with 0 or 1. The following is the expression of integer programming with three parameters (values 0 or 1): Maximize, \( x + y + 2z \); Subject to, \( x + 2y + 3z \leq 4; x + y \geq 1 \) \((x, y, z\) binary).

Based on the characteristics of integer programming, this paper describes the generation and arrangement of building space in given volume. The research is divided into three parts: (1) A variety of templates to achieve a maximum number of fills in the domain, including experiments in the plane and three-dimensional space. (2) An integer programming model based on functional topological relationships is implemented in the residential floor plan. (3) Changes in the shape and size of the entire home and functional rooms create different layout results that can be used to build a library.

2. Previous Research

There have been some research using integer programming to solve some problems related to urban layout modeling and architecture space planning.

Urban layout modeling. Peng (2016) studied the generation of urban road network. The block is first gridded and represented by a half-edge data structure. The problem of road network generation has been transformed into the problem of selection and combination of “edges” in grid. The location of the first level road network is determined manually. Secondly, the position of secondary level is generated by integer programming algorithm. The restrictions include accessibility of land, avoidance of “isolated islands” surrounded by roads, avoidance of forks and end roads, etc. The objective function is set to the shortest overall length of the road network. Under the combined action of restrictions and objective function, the road network generated by the selected edges in the grid is calculated.

Architectural space planning. Keatruangkamala (2005) proposed a topological relationship layout based on integer programming. The room unit is represented by the coordinate points in the upper left corner of the room and the length and width of the room. The size and location of a room are variables in the equation. The objective function is the minimum distance between rooms and the maximum total area. Michalek (2002) proposed a quadratic programming based method for building layout. Each room contains a central coordinate point, the length and width of the room, and the size of four windows. The objective function consists of weighted summation of several subfunctions, each of which represents a design factor. The complexity of architectural design makes it difficult to formulate the plane layout problem. In this model, the room is recorded by position and length and width parameters, so the planar layout is composed of rectangular rooms.

3. Template Filling Experiment

The main goal of template filling is to arrange the fixed shape templates as densely as possible in the given domain, while ensuring that the templates do not overlap (Figure 1). The domain to be filled and templates are divided into orthogonal grids. In the template, the grid cells where the core points are located are recorded as 1, and the rest are recorded as 0. There is a core point in a template to calculate
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whether the templates overlap. The total number of filling templates in the domain is the sum of the values of all elements of the template. The objective function of dense arrangement is the maximum of the sum of parameters.

![Figure 1. Experiments of filling planar domains with different templates. (a) 7×7 domain range. (b) Three different templates. (c) The maximum number of templates is filled in the domain and each template is guaranteed to exist. (d) 20×20 domain range.](image)

The step of defining the restrictions of template filling is critical. Take template T as an example. As shown in Figure 2, core point P is recorded as 1 and the remaining cells are recorded as 0. When a cell in the grid has a current value of 0, it means that it is not filled by the template; 1 means that it is covered by a unique template; when it is greater than 1, the unit is covered by multiple templates. The position of P1 in the Figure 2 is an eligible occupancy. The following two conditions do not satisfy the restrictions: (1) Template overlap. The template P2 of the red dotted line overlaps with P1. When the core point is in the lattice of the fork, it is not satisfied. (2) The template is out of the range of the area, as shown by the positions of P3 and P4 in the Figure 2. That is, the grid at the edge of the domain needs to be restricted.

![Figure 2. Restrictions diagram.](image)

Filling experiments are also carried out on the three-dimensional template. The volume is abstracted into three-dimensional lattices in an orthogonal system according to the modulus of the building. The room is abstracted into a block occupying a certain space in three-dimensional space. The set conditions increase the restriction in Z-direction compared with the two-dimensional layout, and the templates are in the domain and do not overlap. The objective function is the
compact arrangement of the maximum number of templates. Figure 3 is the result of filling two kinds of templates in the space range of $7 \times 7 \times 8$.

![Figure 3. Three-dimensional template filling experiment.](image)

4. Layout Design based on Topological Relationships

4.1. POSITIONAL RELATION

Residential building design contains many practical problems, so it is not easy to abstract a room into a rectangle. The functional topology relationship defines the building functional units and the connections between them. Transforming the topological relationship into a qualification in the integer programming makes the result more reasonable and closer to reality. The topological relationship of the room is expressed in the model as the positional relationship between the room and other rooms. One is the adjacent relationship, such as the living room and bedroom, dining room and kitchen. The other is the separation, such as the hall and bedroom.

**Adjacent.** For rooms A and B, the relationship between them is shown graphically as having at least one cell adjacent. Taking Figure 4 as an example, room A is a $3 \times 2$ gray rectangle, and room B is $2 \times 2$. Using $S$ to indicate a situation in which room B is arranged adjacent to A, then all occurrences of adjacent rooms around A are formulated as: $Adjacent = \sum(S(i - B.x, j - B.y) + ... + S(i - B.x, j + A.y) + ... + S(i + A.x, j - B.y) + ... + S(i + A.x, j + A.y))$.

![Figure 4. The arrangement of adjacent relation.](image)

Restrictions include: (1) All rooms do not exceed the domain. (2) Multiple rooms B do not overlap. (3) Room B is within the range that can exist. (4) Limit the minimum and maximum number of different templates. When the objective function is the largest number of rooms B, the experimental results are shown in
Figure 4 (left). If multiple templates appear, the additional condition is to limit the number of templates, such as at least 2 occurrences per template. Figure 4 (right) shows the results of the adjacent layout of the room of 3 × 1 and 2 × 2 and room A. **Separated.** The relationship between the two rooms is shown graphically as the distance between the closest points of the two rooms is greater than the minimum of the target spacing. The relationship between room A and room B is expressed as: \( \min \text{Distance}(R_a, R_b) > \text{tarDistance} \).

The \( \min \text{Distance}(R_a, R_b) \) in the formula is a function of calculating the closest distance between room A and room B. It can traverse all the corner points on A and B, and the distance between the nearest points is the minimum distance. Restrictions include: (1) All rooms do not exceed the scope of the domain. (2) The rooms do not overlap. (3) The minimum distance is greater than the target distance. Figure 5 (left) shows the target distance between the 2 × 2 room and the room A as a result of the separation relationship of one cell. Figure 5 (right) shows the result of the layout of the separation of the target spacing between two cells.

### 4.2. TOPOLOGICAL RELATIONSHIPS AND RESULTS

According to the topological relationship between rooms, the connection relationship of the building space is expressed in the form of a matrix. Assuming that the building contains \( N \) rooms, the connection relationship of the room can be expressed as an \( N \times N \) matrix. By querying the value of \((i, j)\), it can be obtained whether the \( i \)-th room is connected to the \( j \)-th room. The connection relationship is divided into three ways: disconnected, connected through the door hole and completely open, and the values are 0, 1, and 2 (Figure 6).

![Connection matrices represented by integers](Rodrigues E, Gaspar A R, Gomes A 2013).
Five different functional rooms, including hall, living room, bedroom, bathroom, and kitchen, were set up in the research. Set the living room as the core function of the layout, and the rest of the rooms are adjacent to it. As shown in Figure 7 (left), the living room is adjacent to each room. However, with the expansion of the total area, there will be adjacent subsidiary cases as shown in Figure 7 (right). The toilet of $2 \times 2$ is attached to the bedroom of $3 \times 3$, which is separated from the living room. The restrictions in Figure 7 (left) include the following: (1) All rooms are in a certain domain. (2) The living room is adjacent to the rest of the rooms. Considering the orientation problem, one side of the room is not placed. (3) There is at least one room for each function. The difference in Figure 7 (right) is that the total area is enlarged, the number of bedrooms is set to at least two and one of them is adjacent to bathroom.

![Figure 7. The layout of single-storey residential buildings with different total area based on the topological relationship.](image)

Each time the objective function and restrictions are set, a result can be obtained. When the less relevant conditions are changed, several feasible solutions can be obtained (Figure 8), such as the length and width of the domain or the position of the template in the domain. Designers can change the conditions according to their needs or reality, and choose the layout that suits their needs.

![Figure 8. Different layout planning results of a topological relation.](image)
5. Research Extension

The research also attempted to solve the problem of other residential buildings using integer programming algorithms. Mainly in three directions to expand: the establishment of the residential units library, residential planning, vertical spatial layout.

The establishment of the residential units library. Extending the study in 4.2, changing the size and length of the room, you can get a variety of results. Encode different results and store them in the library to provide searchable services for future designs.

Residential planning. Plane elements in residential areas are decomposed into buildings, green spaces, roads and so on. Each element is represented by a kind of template, which can be multiple shapes. The block where the residential area is located is transformed into a domain in the geometric plane, and the objective function is set to be densely arranged. Building and green space are matched to form a collage unit according to a certain number of proportional relations. The distance between the units is the width of the road.

Vertical spatial layout. In addition to plane layout, integer programming can also be used to solve some design problems in vertical space. As shown in Figure 9, the location of the two traffic cores is emptied, and each layer is arranged according to the maximum area of use, so a rich vertical spatial layout can be obtained. Designers only need to change the objective function and restrictions to obtain the required layout results.

![Figure 9. Attempts at vertical spatial layout.](image)

6. Conclusion

The development of computer-aided architectural design has made computers no longer just drawings tools. More and more models and algorithms have been developed to solve various problems in architectural design. The core of the research of generative design method is “the process of producing excellent architectural schemes”, which is not limited to producing a single architectural scheme. Therefore, reasonable rules and algorithms can produce a large number of high-quality results according to different input conditions, and improve the quality and efficiency of architectural design.

Architectural functional layout design is one of the core tasks of architectural design. This research explores the feasibility of applying integer programming to residential building design. The process of abstracting architectural problems into computational models can be understood as the process of gradually reducing
the solution space to the optimal solution. Changing the objective function and restrictions in integer programming can solve different architectural problems. The layout based on the topological relationship makes the computational model more reasonable and the results more valuable. On the one hand, the current research is limited to residential buildings with relatively simple topological relationship. For buildings with complex functional relationship, the corresponding generation methods still have a lot of research space. On the other hand, this research is limited to orthogonal grids, and the plane layout and three-dimensional spatial layout of non-orthogonal grids still need further research.

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