4D PRINTING: COMPUTATIONAL MECHANICAL DESIGN OF BI-DIMENSIONAL 3D PRINTED PATTERNS OVER TENSIONED TEXTILES FOR LOW-ENERGY THREE-DIMENSIONAL VOLUMES.

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Abstract. From the distribution of the embedded energy in materials, can be operated in order to design and produce optimized material systems with minimum use of external energy to achieve its maximum three-dimensional capacity within their mechanical constraints. This research studies the process of 3D printing bidimensional layers over a tensioned fabric to generate three-dimensional shapes. After the tension of the fabric is released, the printed pattern generates tension and compression over the textile, which conduce and distribute the internal forces generating a controlled deformation with a final form. Digital simulation of finite anticlastic shapes and parametric design under mechanical constraints of the material used to predict and compare both physical and digital forms. These allow us to evaluate and optimize the printed pattern in order to decrease the amount of used energy and material to produce a performative shape.

Keywords. 4d printing; material computation; digital fabrication.

1. Introduction

Material Computation doesn’t require the use of computers (Vivanco, Yuan, 2019) to compute physical forms, nevertheless, computational tools might be needed to explore, evaluate and simulate formal, behavioural and mechanical properties (Terzidis, 2006), of the potential outcomes.

Understanding and controlling the distributed forces of different materials allows computing its mechanical and formal behaviours pursuing performativity. Formal reactions, transitions and variations led by the information embedded in materials are conditioned to environmental, structural and/or mechanical constraints that affect matter in different interactions. Giving it a gradient continuity, promoting heterogeneity and fluidity instead of rigid and static development (Oxman, 2010). A material entity embeds its material characteristics, geometric behaviour, and manufacturing processes by integrating a wide understanding of form, structure and behaviour (Mengues, 2011).

Computational tools enable the incorporation of mechanical constraints into the formal exploratory process, which can be categorized into a morphogenetic space where all the possible and probable geometries that the internal and external systems allow (Mengues, 2018) are located. The selection of forms within the morphogenetic space must be accompanied by an evaluation process of the formal objectives and mechanical behaviours that might be required (Hensel, Mengues, 2008), associated to the physical representation capacity of those forms, conditioned by the available tools that and material properties. Overall, it is determined as a deterministic selection process (Leach, 2019) in which constructive viability is critical.

By activating the information of material through mechanical stimuli (Tibits, 2017), such as stretching, new formal compositions can be explored under the conditions of physical and mechanical constraints. These activated forms can be stabilized by the use of a second material or element, achieving an equilibrium state of the tension. Allowing to decrease the amount of energy invested into the configuration of three-dimensional forms by operating the material in two dimensions, in this case, the use of horizontal forces to create vertical deformations.

Previous studies in the three-dimensional behaviour of bi-dimensional patterns to create 3D forms by applying a resin pattern over pre-stretched latex sheets (Oxman, Rossemberg, 2007) have led to the development of digital simulators to study the relationship between geometry and material behaviour in both digital and physical prototypes. Defining the transitional process form a bidimensional to three-dimensional form as four-dimensional. Due to the self, but a controlled material changes, in this case, the latex sheet.

A similar research, but based on external forces to keep the three-dimensional material activation in position, an exploration into 3D printing over the pre-stretched fabric, based on the analysis of 2D patterns that provide relief with 3D printing, generating volumes of figures (Nervous Systems, 2018). Where an algorithm (Rohan Sawhney and Keenan Crane, 2018) is used to flatten a 3D model to generate a bi-dimensional figure. Then, the figure was processed to generate patterns with areas of contraction and compression of different points, printed over a sheet of pre-stretched fabric to create volumes.

2. Aims and objectives.

The concept of operating materials through their information raises the question of which could be the minimum energy (information and/or material) invested to maximize three-dimensional performativity. At the same time, how much control of the physical final form could be achieved and how the mechanical constraints can be incorporated into the design and digital production process to predict the outcome.

The aim of this research is to study both physical and digital prototypes through the develop a simulator tool to predict three-dimensional forms working only in two dimensions in order to decrease the amount of energy (and material) invested. The main function of the simulator is in first instance, to predict the resultant
three-dimensional shapes designed in two dimensions. In a second instance, to study the deflection digital and physical prototypes, and last to compare both physical and digital prototypes.


The design methodology developed in this research is primarily divided into five steps (Figure 1). The first step is to define the input pattern by form-finding studies using physical prototypes with both analogue and digital fabrication processes. Second, the application of the mechanical theory of buckling columns. Third, the development of a digital simulator that makes visible an approximation to the form generated from the input pattern and the output form. Fourth, the generation of the physical outcome form. As last, the mechanical analysis of both digital and physical outcomes, a critical step for the detection of patterns and discoveries will serve for further developments.

![Figure 1. Summary table of the 5 steps of the design methodology developed in this research.](image)

3.1. INPUT PATTERN

As the elastic membrane is horizontally pre-stretched to distribute the force which creates a volume once is released, the input pattern is a stiff flat geometry that will work in compression when the tension is released. Through this, the membrane will contract to generate a three-dimensional deformation (Figure 2).

Firsts prototypes were done with latex sheets and epoxy resin patterns. By the end of these tests, two issues stood out:

- A large amount of time invested in the preparation of the materials to tests the prototypes.
- The latex sheets weren’t homogeneous in their height and size, making the prototypes not comparable with others.

To have standardized measurements of deformed latex sheets were replaced by elastic textiles. Also, a form-finding process lead to define basic patterns geometries (crosses, squares, circles and parallel lines) with different widths and orientations over the tension of the fabric. Where the density and rigidity of the resin played a key role in setting the final shape due to the compression stress which is subjected, considering a prestretch length of 50% of the textile sheet size was considered for the development of initial prototypes.
3.2. DIGITAL INPUT PATTERN

Based on the previous step, 3D printed patterns with precise control over the printing variables of thickness, linearity and height of the input-form needed to refine. Initial forms are generated to contrast or validate the behaviour of the previous prototypes. After several tests, the configuration of the 3D printing machine presets for ABS plastic with Infill: 80%; Number of shells: 3; Shell Height: 0.1 mm; Speed while Extruding: 70 mm/s.

![Figure 2. Firsts Latex prototypes.](image)

3.3. MECHANICAL PRINCIPLES

3D printed patterns in shrunken flexible textiles have similar mechanical behaviour as deflected buckled columns. The analysis of the column’s deflection based on how a finite linear element with known mechanical properties, like density, elastic or Young’s modulus and geometric dimensions, is influenced by restrictive forces bent buckling principle. Conditioned by the freedom of the ends of the column, defined as pivoted, fixed or free (Gere, 2009). The bending columns theory is based on the solutions of the differential bending moment equation:

\[ EIvPv = 0 \]  \hspace{1cm} (1)

Where EI represents the flexural rigidity of the column, in which E is the material’s Young’s Modulus and I its inertia, \(v\) is the lateral deformation of the column and \(P\) the bucking pressure associated with its deflection. Also:

\[ k^2 = \frac{P}{E} \]  \hspace{1cm} (2)

Which is a factor that relates the bending pressure to the flexural rigidity of the column. The main equation is a homogeneous, linear, second order differential equation with constant coefficients, which allows to determine the magnitude of the critical load and the deflected shape of the buckled column. With this in mind, the solution to the bending moment equation can be written as:

\[ v = C_1 \sin (kx) + C_2 \cos (kx) \]  \hspace{1cm} (3)

With \(C_1\) and \(C_2\) the constant parameters of the solution. Fixing the border conditions (deformation in the ends of the columns are equal to 0) it’s obtained that \(C_2 = 0\). This leads to the main buckling equation:

\[ C_1 \sin (kL) = 0 \]  \hspace{1cm} (4)

That has 2 cases for solving. The first one, being the mathematical trivial scenario, is \(C_1\) be equal to zero, physically relevant in order to explain the equilibrium state.
of the column, thus it has not been deformed. The second scenario is \( \sin(kL) = 0 \), which will occur when

\[
kL = n\pi, n = 1, 2, 3, \ldots
\]

Defining \( n \) as buckling mode, \( P_{cr} = n^2 \pi^2 EI / L^2 \).

\( P_{cr} \) is called critical bending pressure and becomes relevant for design, since determines the material to be used and the wanted deformation, related to flexural rigidity and the cross-sectional area of inertia, respectively.

Compared with the central buckling equation, it can be noticed that this theoretical value will only describe the column’s behaviour when the bending pressure is equal to \( P_{cr} \). Thus, for higher or lower values, the bending equilibrium will be present only if the column remains with no external perturbations, except the bending pressure.

Given this explanation, it is possible to contrast the deformation of the 3D printings in lycra through the \( n \) parameter (bucking mode), that will define the number of curves that the printed figures will have when the textile is released. Through the static study of columns, specific combinations of extremes are analyzed, for their structural relevance and their mechanical feasibility. Considering two extremes, there are:

- Column with one end fixed and the other free
- Column with one end fixed and other hinged
- Column with both end hinged
- Column with both end fixed

Lycra Dupont as a textile material, its mechanical properties are measured in a non-standardized way. The simile to the young module in textile engineering is the comparison between the stretching of the fabric with the force applied to stretch it, is understood as the comparison between the stress and the strain of the material. (Goldade and Vinidiktova, 2017). Specifically, Lycra Dupont can be stretched up to 100% of its size in one direction, but in the other direction, it can be only stretched 75% of its size due to its fibre direction. This defines a stretched length criteria of considering its most unfavourable elasticity of the size of each sheet.

Understanding the key properties of the materials that give structure and form of the prototypes, deformations obtained were compared (Table 1) with the deflection curves named above. An adapted column buckling theory was used to predict the forms that were printed and then compared.
Table 1. Comparative table of prototypes and different modalities \((n=1, n=2)\) of the pivot-pivot configuration of column buckling theory.

<table>
<thead>
<tr>
<th>(P&lt;P_{cr})</th>
<th>(P&gt;P_{cr}) (n=1)</th>
<th>(P&gt;P_{cr}) (n=2)</th>
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<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
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3.4. OUTPUT SIMULATED FORM

Initially, to mechanically simulate the static conditions of the prototypes, the base geometry of the membrane and input pattern were parameterized and the fixed vertices from the flat parametric surface were extracted. From this initial setup, a five-step computational definition was made (Figure 3):

- Mesh and base points input for the printed figure (Figure 3, color magenta).
- Mechanical and Young modules of each element. (Figure 3, color blue)
- Anchor points from base points (Figure 3, color orange)
- Stresses, tensions and deflections following column buckling theory with Karamba 3D solvers (Figure 3, color yellow)
- Calculation of the deflection curves of the fabric (Figure 3, color cyan)

Figure 3. Grasshopper diagram of the simulator of the output simulated form. In colors divisions of elements in the code.

The definition starts with the selection of the finite elements that will
participate in the deformed body-defined from the basic parameterized geometry and the input pattern. Next, the single developable curves are generated from the base geometry; after that, mechanical and geometrical properties are defined in MatProps and Cross Section functions of Karamba3D plugin. In order to transform the developable curves to non-developable double curves, which in this case will be ABS beams pattern, using the Line to Beam component. The mechanical and physical properties of Lycra Dupont are given to the mesh, considering its contraction according to the predefined stretch distance.

The guiding key for the form-finding process within the simulator is the assignation of mechanical reactions to each anchor points of the input pattern, similar to the reactions they would have in a physical prototype. Based on initial prototypes, it was observed that there are mainly three types of lycra-supports (fixed, hinged and free) since their mechanical behaviour was relatable to the column buckling theory. These combinations will generate deformation in the simulator (Figure 4). Considering that the pivot type support generates parallel and perpendicular reactions, the fixed support generates the same reaction as the fixed one, added to moment or torsion reactions towards the centre of the bar. The free support generates reactions only on the axis towards which it is an external force applied (Gere, 2009).

Figure 4. Simulated outcomes generated using Grasshopper and Karamba 3D definition.

### 3.5. OUTCOME PHYSICAL FORMS

For the development of the outcome prototypes (Figure 5), tension, input shape, thickness and height of the print and the number of ‘hinges’ or segments of variations in height of the figure were varied to influence the performance of the outcome form relatable to critical pressure and inertia in theory of columns buckling. With small variations in the length of each beam, new equilibrium solutions are achieved (Kilian and Ochoendorf, 2005), for example, for three-dimensional networks, a spherical dome can become a conical shell by adjusting the length of each element.
Figure 5. Comparison between outcome’s physical prototypes varying in the number of hinges with the same thickness with the achieved height (mm) after tension was released. Prototypes name code: T or C (width-large (in cms)) F(number of segments) A(segment thickness pattern (in mm)) T prototypes: two parallel sides with the same thickness; the remaining sides with hinges with different thickness according to the pattern. C prototypes: same size and same thickness on four sides but with variable thickness and different hinge patterns. The last row shows the maximum height in mm achieved by the input pattern after the tension was released.

By varying the height of two of the sides of a squared printed pattern, the final outcome was deformed in only one direction. (T-prototypes) Even more, when segmenting the length of the pattern and giving each section a different height, thinner parts had a lower resistance to compression, a greater deflection and deformation of the output physical form. The geometrical interaction between the textile tensile stress and the input form generated variable controlled outcomes that allows to pursue a functional final form. The development of a functional prototype (Figure 6) was possible due to the incorporation of the different behaviours of C and T prototypes mentioned above. Making possible to predict the outcome form and its geometrical assimilation of the input figure.

Figure 6. Evolution of a functional prototype. From a planar form (right) to the three-dimensional prototype (right) generated an increase in height after tension was released.

4. Mechanical comparison between the digital and physical prototypes.
Printed patterns work as a continuous and closed geometry under the mechanical principle of deflected buckled columns, transmitting forces between them until they achieve a static equilibrium. Depending on the height and/or segment of a trace the energy will distribute until it finds its own stiffness balance. As shown in the image above (Figure 7) the deflection principle finds its own
equilibrium by distributing forces all across the fabric, generating a deformation and resultant geometry. With and accurate printing over the textile, the outcome physical prototype will be structurally adapted keeping a force balance, similar to the simulated form.

The control of the outcome’s curvature through the pattern section depends on the size of the segments of each pattern line, where smaller deformations are when fewer segments in the structure length. The behaviours of the physical prototypes are reciprocal with the simulated digital prototypes; therefore, the input behaviour can be projected without need to make physical prototype or by defining a pre-descriptive geometry.

5. Conclusions

Computation can contribute to understand and inform materials. Which only can be operated in the physical world, where material belongs. The idea of predefined forms questions the basis of creativity, which can only navigate within specific material constrains and production processes.

Developing an integrated research between physical and digital prototypes based on mechanical constraints, material strength can be considerably be optimized through geometry, decreasing the amount of embedded required energy of three-dimensional form. The input pattern through the force of the textile controls and adapts the embedded energy, generating a actuated transformation from a bi dimensional to tridimensional form, defined as a 4D printed.

As shown in prototype C10-10 F15 A21, this research reveals that by segmenting the printed bidimensional input pattern length its three dimensional achievement can increase considerable. This change of thickness and hinges gives a better resolution and control over the outcome physical form, opening new
exploration for design considering angle changes and straight elements, as shown in initial further developments (Figure 8).

![Figure 8. Experimental prototypes with rectilinear patterns and deformations.](image)

This open new research questions based on the potential change of scale of the prototypes and outcome control to generate functional forms combining both curve and linear based forms in a continuous pattern with different resolution.

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