EMERGENT SYMMETRIES: VISUAL COMPUTATIONS
A group theoretical analysis of the Smith House by Richard Meier

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ABSTRACT: This work proposes the use of partial order lattices along with representational schemes to account for patterns of ambiguity and emergence in the description of designs. The complexity of such designs is viewed as an aggregation of spatial layers that can all be decomposed by the subgroup relations of the symmetry of the configuration. At the end, this methodology points to a combinatorial approach that generates visual prototypes for future use in design synthesis. Here, Meier’s work is just a case study that validates the group theoretical approach.

KEYWORDS: Abstraction, partial order lattice, recombinant, visual prototype

RÉSUMÉ: Cette recherche propose l’utilisation de semi-treillis combinés à des systèmes de notation propres à dénoter les phénomènes d’ambiguïté et d’émergence dans la description des compositions architecturales. Des designs à forte complexité sont à lire comme un agrégat de strates spatiales qui sont décomposables en termes de sous-groupes des symétries de la configuration. À cette fin, la méthodologie vise à la production d’un répertoire de prototypes visuels à adopter pour composition. L’ouvrage de Meier sert de cas d’étude.

MOTS-CLÉS: Abstraction, semi-treillis, recombinant, prototype visuel
1. INTRODUCTION

A fascinating aspect of certain classes of architectural works is their ability to escape easy interpretations based upon existing formal tools. This is especially true for several architecture works of the modern movement that feature asymmetrical arrangements and diverse kinds of complexity. A new look at existing formal tools can perform the task, if applied, though differently. Currently formal analyses using group theoretical tools focus on repetitive designs that show immediately their recursive structure. It is suggested here that complex designs can still be described and analyzed in a group theoretical manner. Some first steps towards the extension of the tools of group theory to explain such designs have been taken by March (1976), Economou (1999) and Park (2000). This work builds upon these methodological approaches and proposes a model that investigates whether the combination of existing group theoretical formalisms with appropriate systems of representation can cast light in the analysis of such designs.

More specifically the work proposes the use of partial order lattices along with representational schemes to account for patterns of ambiguity and emergence in the description of such designs. The object of analysis has been polemically selected here to be the Richard Meier’s Smith House, a design that clearly exemplifies formal qualities of late modernism architecture such as abstraction, layering, complexity, depth, collage and so on, all aesthetic categories appearing impenetrable to a systematic and rigorous analysis using existing group theoretical formal methods.

Design computation discourse relies upon a long tradition of architecture discourse that foregrounds specific vocabularies of form, kits-of-parts, assembly thinking, structural components and so on, including for example, Durand’s Précis (1819), Chernikhov’s Architectural Fantasy (1931), etc. Recently, Mitchell (1974) and MacCormac (1974) refer to Neo-Vitruvian tendencies in architectural design using modern computing techniques—a revival of the classical sense of order. If Clark and Pause’s Precedents in Architecture (1985) can relate to this tradition, then it is equally more plausible at the age of automated design to envision more effectively the use of cyclic and dihedral symmetries to assist in the analysis and synthesis architectural design. Currently CAD operations are limited to non-automated basic isometries. These representational modes can describe forms in an automated manner. One can imagine automata that detect axes of symmetry, and construct dihedrals in complex configurations. These can be used to manipulate formal qualities of assemblies during design process to construct hierarchies from ordering relations.

Finally, this paper is divided into five sections, including the introduction. In section 2, a subsymmetry analysis shows how to parse a design in layered identical parts. In section 3, an application of this methodology is instantiated on the
Smith House. The visual computations and sub-symmetry relations are captured in partial order lattices. In section 4, the recombinant vision exhausts all possible ways of combining the parts of the house; it also captures the theoretical statements on the symmetry of the house. In the last section, a discussion is opened on the degree to which the symmetry decompositions can support visually the established discourse on the house and the possibility of using such formal tools in CAD applications for analysis.

2. FORMAL MODEL

The formal analysis model proposed in this work requires: a) a system of representational conventions to codify the aspects of design that are of interest to the analysis; and b) a group theoretic formalism to parse the design to identical parts. The system of representational conventions is built upon three features of architectural representation including abstraction, projection and weight. Other aspects of representation conventions are omitted. The group theoretical formalism relies on subsymmetry analysis and uses partial order lattices as the primary mode of pictorial representation. A brief discussion of all these characteristics follows below.

2.1. Architectural Notations

A three-dimensional model or a two-dimensional drawing typically provides an abstracted representation of an imaginary or a real architectural object. This visual representation does not depend solely on formal similarities. This representation is not a mapping of an object's complete form but a mapping of certain privileged or relevant aspects. Depending on the number of features being deleted from the original or alternatively being added or transformed in the mapping a level of abstraction is then arbitrarily defined. Here abstraction is used entirely in terms of essential and accidental properties (Mitchell 1990). An abstracted version in any mapping is the one that keeps certain characteristics of the original object while dropping others. All such different levels of representation allow the analysis to describe shapes at different generative stages and establish links which are not immediately available to the viewer. In this sense plans, sketches, and working drawings are all scaled analogs and highly abstract notational scores that condense visual and non-visual amounts of information into a codified language of symbols.

A second aspect of architectural representation is its ability to describe architectural space through the deployment of depthless drawings. Among the various techniques of projection including plans, sections and elevations and systems of lines linking them all one to another, a specific composite projection technique is quite unique in architecture and not in other engineering fields. This technique combines essentially orthographic and affine transformations
and has the unique feature to represent abstract space both visually and metrically. This type of composite projection, also known as an axonometric paraline and oblique paraline projection has attracted the interest of twentieth century architects and theorists because seeing in oblique views is essential for depth perception (Evans 1995). It is significant that such composite axonometric projections foregrounded two functions initially thought as one: the axonometric as a representational device and/or as a conceptual device. The former is progressively enriched by the methods of multimedia presentation; the latter is able to support the communication of spatial qualities of an architectural object during the process of conceptualization. In all cases the exploitation of the fundamental ambiguity of the axonometric projection has been the discourse of much architectural theory in the twentieth century and it is suggested here that this should be the primary mode of representation to adequately describe and analyze the architecture works of that period.

The third aspect of architectural representation discussed here is its ability to represent materiality. A pen stroke has a width. Lines are of different thickness. Typically these widths and weights of the lines refer to what is represented and work upon a set of conventions widely shared and understood. Different types of conventions and assumptions often give the clues about what is represented and weights distinguish formal attributes from physical properties. Shapes made up of basic elements and weights answer to the Vitruvian categories: physical properties being included in firmness, functional properties in commodity, and spatial properties in delight. This particular understanding of weights as a characteristic of representation itself is taken here as a powerful construct for formal analysis. In this view, weights suggest a different kind of projection directly alluding to transparency, a key feature of the twentieth century architecture, whereas the three-dimensional qualities of the design are illustrated through the weighting of lines (Klee 1953). Varying degrees of transparency and opacity can be strategically deployed to hierarchically differentiate preselected conditions important to the development of the work.

These three aspects of representation can be combined in various ways to create a large number of possible notational systems for architecture design. In principle, there can be a large number of such notational models that privilege diverse aspects of the design and capture some, but of course not all, conventions characterizing a design. Here, three levels of notational languages are suggested as a minimum for formal analysis.

The first level of notation, called here the architectonic level, is the level that approximates in some way the original notation used for the language of the actual design model into its closest geometric representation. The notation privileges functional elements such as walls, slabs, columns, and beams, walled furniture, handrails, and openings of various kinds such as windows, doors, stairwells, chimneys, and so forth.
The second level of notation, called here the *spatial* level, privileges space divisions and corresponding openings in these boundaries, and discards all other information. This level essentially picks up planes that function as walls and slabs and so on and the connections between them and their interface, such as ventilation, light and so forth. At this level, a spatial model that emerges is a spatial decomposition of the building with geometric shapes that bound the space.

The third level of notation, called here the *diagrammatic* level, foregrounds underlying, emergent boundaries of space and discards all connections between them. This level of notation, closely related to the parti of a design, the geometrical diagram or pattern that emerges when all details have been dropped out, is the most abstract version of the model and functions as a scaffolding of the design.

### 2.2. Order

*‘The semi-lattice is the structure of the complex fabric; it is the structure of living things—of great paintings and symphonies.’* Alexander (1965)

In formal analysis, the similarity relation permits overlap between elements of the set, whereas the equivalence relation separates the set into disjoint classes. Equivalence relations are represented by trees and similarity relations, by lattices (March 1982). Here the key operator that parses the representations is the partial order relation defined by the symmetry group that describes the maximum subsymmetries of the configuration. The fundamental significance of symmetry arises here from its capacity to reveal two opposing aspects of form: transformation (change) and conservation (invariance). That which is conserved during a change is an invariant; the set of transformations which keeps something invariant is its symmetry group. The set of elements and their structural relationships forming the complete system are conserved as a single whole and this order identifies all the nested parts in any configuration that have a group theoretical relationship to the overall group of the configuration.

The key idea is that spatial representations of complex objects can be understood as layered compositions of simpler parts and these parts can all be related through symmetry values from group theory. These values can be structured as a partial order lattice that pictorially presents the symmetry structure of any spatial configuration; the number and qualities of the symmetry subgroups found in any given configuration provide the maximum number of layers that can be found in a spatial configuration; for example, in any spatial arrangement that is based on the structure of the rectangle, the maximum number of layers and spatial constructs that can be built upon those is five because this is the number of symmetry subgroups of the rectangle. Typically, Hasse diagrams are used to represent such order and show the nested relations of the subgroups in graph (Ho 1982).
An example of a graph of the symmetry group of the rectangle is given in Figure 1. The complete group of the rectangle with four symmetries is on top, three subgroups with two symmetries are in the middle row and the single group consisting of only the identity symmetry completes the graph in the lower row.

**FIGURE 1. A GRAPH SHOWING THE PARTIAL ORDER OF THE SYMMETRY GROUP OF THE RECTANGLE.**

3. APPLICATION: THE SMITH HOUSE

An application of this methodology in formal analysis is presented here using Richard Meier’s Smith House as a case study (Meier 1984). For a succinct account of the discourse developed about the house and its critical role in the formation of contemporary architecture discourse, see Din (2009). An axonometric view of a 3D digital model is shown in Figure 2.

**FIGURE 2. AXONOMETRIC VIEW OF THE SMITH HOUSE.**

The initial shape that starts the computation is the three-dimensional model of the Smith house that is built upon existing plans, elevations and sections of the house. This model consists of three parts each corresponding to one of the
three floors of the building and all of them together comprise the architectonic model of the Smith House. A series of shape rules are constructed to replace parts of the model with simpler parts in order to construct the model in the other two notational schemes. The symmetry partition of the house occurs at the diagrammatic level, the simplest possible notation for the design, and this partitioning specifies how the higher level notations correspond to that level. The spatial correspondences in the diagrammatic level are easy to identify and compute; they are all mapped through the same shape rules that gave the original partition of the model to the complex spatial relations that are found in the architectonic level.

The key idea here behind these applications of shape rules and their reversals is that the decomposition and recomposition of the design helps to evaluate the basic assumptions about the system itself. The visual computations and the subsymmetry relations are nicely captured in partial order lattices that pictorially present the subsymmetry structure of the underlying spatial configuration of the rectangle. Two different ordered relations for two different floors of the house are shown in Figure 3 and 4 respectively. The spatial notation is omitted for clarity of representation.

4. THE SMITH HOUSE RECOMBINANT

The subsymmetry analysis parses the design in layered identical parts that foreground qualities that are hidden within the overall structure of the design. Here a somewhat different approach is taken and the focus switches on the juxtaposition of all these correspondences, one with another, at the architectonic level. More specifically the goal here is to examine all partial group theoretic descriptions of the Smith house one by one and in doing so foreground specific relationships that a straightforward application of group theory wouldn’t do.

The lattice of the symmetry group of the rectangle consists of five symmetry subgroups. These subgroups can be combined one with another to comprise a set of $2^5$ or thirty-two possible design worlds that are differentiated one another with respect to the number of elements that belong in each subset. The number of combinations of symmetry elements $r$ among a set of $s$ elements is given in (1).

$$\frac{s!}{r!(s-r)}$$  \hspace{1cm} (1)

For example for a set $s$ comprised of five elements the number of subsets $r$ comprised of three elements is given in (2).

$$\frac{5!}{3!(5-3)} = 10$$  \hspace{1cm} (2)
FIGURE 3. PARTIAL ORDER LATTICE OF THE FIRST FLOOR: A) DIAGRAMMATIC; B) ARCHITECTONIC.
FIGURE 4. PARTIAL ORDER LATTICE OF THE SECOND FLOOR: A) DIAGRAMMATIC; B) ARCHITECTONIC.
The complete list for all subsets comprised of five elements including dihedral symmetries (D), vertical reflections (V), horizontal reflections (H), half-turn rotations (S) and identity transformations (C) is given in Table 1.

**Table 1. Complete Listing of Recombination of Subsets of Symmetry Group for the Rectangle.**

<table>
<thead>
<tr>
<th>n-ary set</th>
<th>Recombination list</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null</td>
<td>Ø</td>
<td>1</td>
</tr>
<tr>
<td>Unary</td>
<td>D, V, S, H, C</td>
<td>5</td>
</tr>
<tr>
<td>Binary</td>
<td>DV, DS, DH, DC, VS, VH, VC, SH, SC, HC</td>
<td>10</td>
</tr>
<tr>
<td>Ternary</td>
<td>DVS, DVH, DVC, DSH, DSC, DHC, VSH, VSC, VHC, SHC</td>
<td>10</td>
</tr>
<tr>
<td>Quaternary</td>
<td>DVSH, DVSC, DVHC, DSHC, VSHC</td>
<td>5</td>
</tr>
<tr>
<td>Quinary</td>
<td>DVSHC</td>
<td>1</td>
</tr>
</tbody>
</table>

Furthermore all these design worlds, a total of thirty-two sets, can be individually augmented with three representations each corresponding to the three floors of the house, making then a total of ninety-six drawings including of course the empty set. And still, all these drawings can be further given in three different versions corresponding to the architectonic, spatial and diagrammatic notation bringing the total number of representations to two hundred and eighty-eight. This recombinant vision exhausts all possible ways that the parts of the house can be combined and should therefore be able to capture all theoretical statements on symmetry that have been said or could be said about the house. The thirty two sets of subsymmetries of the dihedral group are depicted in Figure 5.

**Figure 5. A Complete List of the Diagrams of All Combinations of Symmetry Parts.**
A nice outcome of this constructive combinatorial approach is its ability to capture and reflect on existing debates on formal analysis of the house. For example Frampton (1975) suggested that the theme of simultaneous frontal and rotational development of composition pervades Meier work—and as a matter of fact the whole work of the NY5 architects that Meier was a part of (Eisenman et al. 1975)—but it is not resolved in Meier’s work. Rosemarie Bletter (1979) critiques Frampton that his categories of frontality and rotation are in the end too broadly defined and too general to help precise analysis and that the applications of the two categories in the analysis of buildings are somewhat non-systematic. She further claims that Frampton’s notion of frontality is applied in different ways for each of the NY5 architects: in Hejduk’s projects it is used to refer to overall massing, in Graves’ work it is used to refer to the entrance while in Eisenman’s and Meier’s work it is used to describe the interior gridding of the space. Such arguments and similar ones can be nicely captured and discussed using the group theoretic combinatorial approach suggested here. The ten possible ternary models of the house foregrounding characteristics such as rotary movement (S), collage elements (C) and so on, are all given in Figure 6.

All these models are interpreted here using two distinct descriptive systems: facts and values, or forms and functions (Ho 1982). Functions are suggested by values such as reflection, rotation, identity and so on, and a relation $\lambda$ represents the mapping of one system to another with logical variables 0 and 1. The models juxtapose the qualities one against the other and examine how the presence of the one clarifies or obscures the significance and role of the other. For example the recombinant model DSC in Figure 6(e) foregrounds the individual values of parallel layering (D), rotary movement (S) and collage elements (C). Clearly, the house is partitioned in terms of frontal symmetries—in the original model the (H) symmetries. The corresponding partition to the side symmetries, in the model the (V) symmetries, shows clearly the part of the design that is subject to these transformations. The embedded relationship of the rotations to the two systems of reflection is a byproduct. Furthermore the relatively dense partition of the design given the dihedral, rotational and identity transformations suggests that the house is conditioned in a great extent by both orthogonal axes and not just one.

The DSC model, and in fact several other subsets of the building can help investigate questions and criticisms of the kind. These properties are clearly foregrounded in the representation suggested here. Three issues are important here: a) the isolation and foregrounding of specific subsets of symmetries happens in the diagrammatic level and all lessons learned transfer then to the architectonic level; b) all partial and incomplete correspondences that may be observed in the Smith House are captured by the group theoretic decomposition of the parti of the house, that is the rectangle and therefore the five sym-
FIGURE 6. THE GROUP THEORETIC TERNARY MODELS OF THE SMITH HOUSE
A) DVS; B) DVH; C) DVC; D) DSH; E) DSC.

metrical subgroups and the ninety-six subsets of the complete set of transformations of the rectangle; and c) the interpretation of a set of chosen recombinant parts suggests a constructive understanding of the design and provides a view on how these recombinants can be used toward synthesis as a visual prototype to start with.
5. DISCUSSION

A major motivation of this work is that there is a correspondence between the evolution of architectural languages and the formalisms that can be used to describe, interpret and evaluate them. Classical modern buildings can be and have already successfully been described by group theoretical techniques. In the same way, Richard Meier’s work constitutes a hyper-refinement of the
modernist imagery that has been inspired not by machines but by other architecture that was in turn inspired by machines. The group formalism that can describe Meier’s architecture relies on precise representations and mappings that foreground internal complex relationships of the structure itself, i.e. the symmetry subgroups and supergroups of any configuration.

It is clear that the complexity suggested in the reading of the Smith house could be contextualized within a wider set of designs with similar properties and especially the corpus of the NY5 (Deamer 2001). Other houses could have been selected to test the formal method suggested here. Still, it is argued here that among all such candidates, the Smith House stands out as the best candidate. The house has a long legacy: Frampton (1975) has nominated the Smith House as a classic and selected the young Meier as the one architect out of five who knows history the most and learns from it. Rykwert (1999) has asserted that the house is a classic case of an architectural typology that uses a formal vocabulary whose elements are all abstracted from the repertory of early modernism and juxtaposed back as a collage. Jencks (1990) has asserted that Meier uses a mixture of traditional forms of modernist architecture. And still many other key discourses have been suggested to include the themes of compositional grid and patterned frames (Goldberger 1999), the discipline of the Dom-ino and Citrohan structures (Kupper 1977), and Mies’ aesthetic of rhythmic linear elements (Hildner 1997). Clearly, Meier’s language, iconography, and elemental categories force comparison and differentiation with the work of other architects and at the same time call attention for a precise analysis in itself.

At the end, this research points to a series of other extensions and domains. These extensions generally fall into two categories; a) on the improvement of the system itself; and b) on the interpretative capabilities it affords for the construction and evaluation of critical languages of design.

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REFERENCES

Din, E., 2009, Emergent Symmetries - A Group Theoretical Analysis of an Exemplar of Late Modernism: The Smith House by Richard Meier, Saarbrucken, Germany, VDM Verlag Dr. Muller.


Evans, R., 1995, The Projective Cast, Cambridge, MIT.


