Parametric Design with Standard Elements for Non-Standard Architecture

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Abstract. The development of non-standard architecture is often combined with the use of non-standard elements. But for economical or sustainable reasons, the use of standard elements may be particularly useful. The introduction of standard elements adapted to geometries far from parallelepipeds and freely designed raises a specific problem. The aim of this paper is to explore some ways offered by computing tools in order to help architects in the design process of non-standard shapes using standard elements. An approach is proposed for a specific typology of systems composed of constant length elements. The method used herein is based on parametric modeling associated with constraint resolution algorithms.

1. Introduction

The growing use of computers in architectural design, associated with the development of the computer numerical controlled (CNC) machines, make possible the industrial production of non-standard elements. This evolution leads some authors to name as "non-standard" the architectures using forms far from parallelepipeds that are traditionally associated with standardization [1]. The freedom of forms allowed by the computing design (NURBS, BLOBS, mathematic modeling, parametric modeling), however, can be associated with a certain amount of standardization, especially in order to limit costs and constraints due to the use of CNC process. For economical or sustainable reasons, some architectural projects can deliberately be oriented to standard constructive solutions using predefined or prefabricated elements. This approach can also be useful for adaptation of construction to different configurations like temporary
structures that can be dismantled. In this case, the freedom in forms is restricted but remains important thanks to the possible combinatorial assemblies. During the design process computing approaches make possible forms exploration composed of standard elements. These approaches can be different according to the way of taking into account the discrete nature of models composed of standard elements. Discretizing a form in standard elements imposes a particular cutting scale that influences the resulting shape. To be convinced, let’s draw an arabesque curve and try to approach it by a polygonal curve composed of constant length segments. For long segments the polygonal curve is distant from the initial curve (arabesque). For shorter segments the polygonal curve is closer to the initial curve. Moreover, for a fixed length of segments, the polygon vertices position can be chosen according several criterions. A first way may consists of beginning design from a predefined geometry designed by the architect (surface model for instance) independently from any standardization constraint. Then the model is discretized with an algorithm embedding all the specific standardization constraints or an algorithm that aims to standardize the maximum of elements (for example, this process has been used by Morphosis to design the Tour Phare, [2]). Such algorithms need the definition of tolerance thresholds with respect to the difference with the initial surface and the proportion of standard elements used in the final model. A less restricting process with respect to the initial shape definition consists of beginning with only a few geometrical elements (limit curves, inside or outside gauge volumes) and to organize more precisely the standard elements fitting. We chose this last approach to explore the ways to adapt a predefined system of standard elements to multiple configurations.

In the first part of this paper a modeling process with standard elements using solving constraints algorithms is presented. For a survey of Constraint modeling for curves and surfaces see [3]. In the second part of this paper an experimental tool based on parametrical modeling (developed with Grasshopper and Rhinoceros with Visual Basic scripts) and some applications to form finding are presented. These works are carried out in ARIAM-LAREA laboratory in the research field of the computer aided design using parametric approaches [4, 5].

2. Definition of a modeling process with standard elements

In order to illustrate the proposed modeling process, we consider structural systems composed of identical linear standard elements. Their number and length may be considered as constant or variable parameters. Among all the possible polygons only some of them satisfy the constraints defined by the architect (designer). The computing problem consists of determining these solutions. The constraints may be vertices position, angles between segments, mechanical constraints (funicular polygon). A class of solution models is defined by these
constraints and we aim to explore it by instances. The proposed process to do this is to build a 3D model composed of planar polygonal curves according to geometrical constraints given by the designer. So, the first step consists of defining the generative process to construct planar polygonal curves composed of constant length segments. The second step consists of defining a process of fitting planar curves in order to build a standardized 3D model according to geometrical constraints.

2.1. Planar polygonal curves construction composed of standard elements

The problem is to define and construct polygonal curves in a plane P composed of n segments of Ls length. These are three input parameters. Other input parameters define other constraints relatively to the curve position and shape:

1. Curve extremities parameters: we note Pdeb the beginning extremity of the curve (this point is supposed to be fixed) and Pfin the ending extremity of the curve. The Pfin point may be supposed to be fixed (input parameter) but it also may be considered as a partially defined point and computed according other constraints. A way to partially define the Pfin position is to give a line D (input parameter) passing through Pdeb where Pfin must be located. Pfin is in this last case an output parameter.

2. Polygonal curve type parameters: the polygonal curve may be characterized by the angles between two successive segments of the curve. The polygonal curve also may be characterized by the curve type in which the polygonal curve vertices are located. These parameters are or not connected with the Pfin parameter.

We study specific moments in architectural design when the architect’s choices lead to fix some constraints. These constraints are expressed by input parameters. The problem consists of proposing a process taking into account the defined constraints in order to construct a usable model to the architect. In order to analyse the constraints’ impact on models, we study different basic cases defined by parameters lists.

2.1.1. Partially or fully constraint extremities:

Case 1: « one free extremity ».

The length and the number of elements are fixed and one extremity is fixed too (input parameters : Ls, n, Pdeb). The curve is fixed on only one extremity and can unfold in every direction. A total unfolding corresponds to a line. Conversely a complete folding corresponds to a unique segment. Between theses two limit configurations, the curves may take various shapes. Theses remarks show that the curves are included in a circle centered on Pdeb with a radius of $L=n.Ls$. 
Case 2 : « constraint extremities ». 

The length and the number of elements are fixed and the two extremities are fixed too (input parameters : \( L_s, n, P_{deb}, P_{fin} \)). The extra parameter \( P_{fin} \) involves a diminution of the possible shapes of the curves. In Figure 1 is presented the case of polygons constructed with 4 constant length segments, beginning from the fixed point \( P_0=P_{deb} \) and ending on the fixed point \( P_4=P_{fin} \). In Figure 1, on the left, are presented the possible positions of the polygonal curves for a constant direction of \( P_1P_2 \) and, on the right, all the possible positions without this last constraint. It can be noticed that all the curves stay inside the ellipse which foci are \( P_{deb}, P_{fin} \) and constructed with the gardener’s method with a string of \( L=n.L_s \) length. The possible shapes of the curves are more restraint than in case 1, but the field of possibilities remains very important. The designer must give more constraints in order to completely define the curve. A way to do this is to choose a type of curve.

![Fig. 1. Possible positions of the polygonal curves as a function of the angle \( P_4P_0P_1 \) variation for a constant direction of \( P_1P_2 \) (left) and all possible positions (right).](image)

2.1.2. Definition of polygonal curve type:

Among all the possible geometries of a polygonal curve, the designer can choose to prioritize a certain shape or a curve type. The influence of some curve types on the constraints satisfaction is developed below. \( P_{deb}, n \) and \( L_s \) are supposed to be fixed. In order to describe the polygonal curve type, we characterize the curve which goes through the polygon vertices. Some non exhaustive cases are studied in this section :

- **Line** : all polygon segments are aligned. This case does not allow imposing the ending point \( P_{fin} \) because it depends on the curve length \( L \). Only the line inclination can be given. This case corresponds to the classical type of ruled surface.
• **Arc of a circle**: two consecutive segments make a constant angle \( \alpha \) all along the curve. This angle \( \alpha \) characterizes the arc of a circle curvature. Two cases can be considered:
  a) The angle \( \alpha \) is fixed (the constant curvature is hence also fixed) and is an input parameter. Like the case of a line, this case does not allow imposing the ending point \( P_{fin} \) but only the inclination of the line \( (P_{deb}, P_{fin}) \). An algorithm involving rotation process allows the polygon construction.
  b) The angle \( \alpha \) must be determined (and consequently the curvature too), and is an output parameter. The ending point \( P_{fin} \) is fixed to be a new constraint (and input parameter). The problem is now to determine \( \alpha \) according to the constraints. The computing process is quite more difficult because the geometry depends on the unknown parameter \( \alpha \). In order to solve the problem a dichotomy algorithm is used.

• **Arc of a spiral**: the angle between the \( i^{th} \) and \( (i+1)^{th} \) segment is \( \alpha_i \). These angles decrease according to a regular law (for instance for \( i>0 \), \( \alpha_i = \alpha_0/(i+1) \)). Like the case of an arc of a circle, two cases can be considered:
  a) The angle \( \alpha_0 \) is fixed and is an input parameter. An algorithm involving rotation process allows the polygon construction.
  b) The angle \( \alpha_0 \) must be determined and is an output parameter. The same type of algorithm as for an arc of a circle is used.

• **Model curve**: for any curve chosen as a model it is possible to define a method to generate a polygonal curve close to it. The process consists of discretizing the initial curve into \( n \) segments and calculating the angles \( \alpha_i \) between consecutive segments. The polygonal curve is constructed using these angles. Hence the ending point \( P_{fin} \) can’t be imposed. Only the inclination of the line \( (P_{deb}, P_{fin}) \) can be forced. A transformation of \( \alpha_i \) can force the ending point to pass through a precise point.

For all these cases the polygonal curve shapes are constructed according to the angles between two consecutive segments. For an arc of a circle or an arc of a spiral it is possible to force the ending points to pass through precise positions (if the total length of the curve is long enough). These kinds of curves can be useful for technical reasons. For any curve chosen as a model it is impossible the ending point without transform significantly the shape. The designer has to manage with his/her priorities in order to precise the constraints. The computed model can help to make a choice.

### 2.1.3. **Polygonal curve transformation: Folding**

Transformations of the angles between consecutive segments can help polygonal curve to pass through defined ending points. Folding is one among the possible
transformations. This transformation consists of inverting alternatively the direction of angles (the new angles $\alpha' = (-1)^n \alpha$, for instance).

In Figure 2, some examples of polygonal curves are presented. For the defined ending points Pdeb and Pfin (corners of a rectangle of 10 units width and 20 units length in this example), polygonal curves of $n=7$ segments of $L_s=4$ units length are constructed.

The different curve types are:

a: Polygonal curve as a line on the line $D=(P_{deb}, P_{fin})$. The ending point can’t be $P_{fin}$ because of the curve length.

b: Polygonal curve as an arc of a circle.

c: Polygonal curve as an arc of a spiral. For b and c an iterative process allows determining the curves passing through Pdeb, Pfin.

d: Polygonal curve based on a model curve. Ending points pass trough the line $D$. The model curve (Figure 2 right) is freely drawn by the designer and discretized in $n$ segments.

e: Folded polygonal curve between Pdeb and Pfin. This folding transformation is always possible even for very long curves.

These various processes can be useful to study different configurations of a parametric model adapted to different geometrical cases.

Fig. 2. Examples of polygonal curves defined by $n=7$ segments of $L_s=4$ units length. Polygonal curves a) as a line, b) as an arc of a circle, c) as an arc of a spiral, d) from a curve model, and e) folded.
2.2. Construction of 3D structures from standard elements

The resolution of constraints systems for 3D structures constructed with standard elements is even more difficult to carry out than for 2D structures. With too many constraints the system may have no solution. For instance geodesic domes constructed with equal length elements can only be platonic polyhedrons and don’t really look like domes. So, even if the fine discretization of the Buckminster Fullers domes gives the feeling of equal length elements, it is not the actual case. In Figure 3 (left) a geodesic dome constructed from a discretized icosahedron projected on a sphere from its center is presented. It can be noticed that the discretized icosahedron is constituted of equilateral triangles but the projected ones are not equilateral any more. Thanks to symmetries, some elements have the same length: the structure is partially standardized.

If the designer (architect) chooses the spherical shape as a priority, he/she must lose the choice of the same length for all the elements. The new goal becomes to determine the number, the length and the arrangement of the elements. Conversely, if the priority is to have the same length of elements, the determination of the non spherical shape becomes the goal. So the designer has to give the priority in the constraints in order to solve the problem. In this apparently simple example of a spherical dome, the solutions of structures constructed with standard elements can be only partial. Another way to construct a partially standardized dome consists of choosing particular polygonal curve on it as defined in 2.1. In fig. 3 (right) such a discretized dome is presented with standard elements on longitudes. This is a classical alternative way to discretize a sphere.

![Fig. 3. Geodesic dome constructed from a discretized icosahedron (left) and geodesic dome discretized with standard elements on longitudes.](image)

More generally, the designer is faced with the choice between the shape and the standardization. In both cases a realistic solution consists of a partially standardized structure. If a shape is primary chosen the arrangement of standard
elements is deduced. Conversely, if the arrangement of standard elements is primary chosen, the shape is deduced. It is precisely this second approach that is proposed in this paper to construct partially standard models. The standardization constraints are limited to particular sections of the structure (according to their structural impact for instance). This involves less constraints and more freedom in shape definition. The shape is defined from a set of planar polygonal curves constructed with standard elements (as described in 2.1). Each curve plane can be different. If each curve has the same number of vertices, a mesh can be constructed. The transversal curves of this mesh are not standardized. This is illustrated with the non standard latitudes in Figure 3 right in the case of a geodesic dome. An experimental design tool using this approach is presented below.

3. Experimental design tool

The experimental design tool presented here is based on the process described in section 2. The aim of this tool is 1) to help the designer to define the standardization constraints (number, dimensions and geometry of elements) and other constraints chosen according to the architectural project (geometrical limits, morphology type, mechanics), 2) to define models satisfying these constraints and 3) to give the possibility to dynamically evolve the model. To develop the tool we used Rhinoceros software in association with Grasshopper plug-in. The resolution of constraints involves the writing of scripts in Visual Basic in Grasshopper.

![Fig. 4. Two different configurations of a same parametric standard model supported by defined curves (in blue).](image)
The process described in section 2 gives to the designer the possibility to construct polygons according to the chosen input parameters. This section shows how these parameters can be deduced from the input data and linked to form a complex model. For example, Figures 4 and 5 show parametric models satisfying constraints like supporting curves (C1, C2, etc.) or constraints in association with morphologic constraints (elements length, belonging to planes, curve kind, etc.).

3.1. Definition of 3D model constraints

Modeling software like Rhinoceros allows to freely model curves in space. The designer can choose these curves as supporting curves for the standardized polygons. In the following examples the ending points on supporting curves are defined by the same method but other methods could be used. This method consists of regularly dividing the supporting curves in p segments. Hence, in the case of two supporting curves (C1 and C2), p+1 points are created on each supporting curve and p+1 standardized polygons have to be defined to constitute the model. For instance each polygonal curve can be in vertical planes or in other directions to be defined by parameters.

An example of such a model is given in Figure 4 on the left, in a perspective view. Two supporting curves (C1 and C2) are freely built in Rhinoceros in 3D. Eleven ending points are created on each curve and eleven standard vertical polygonal curves in arc of a circle, composed of n=5 same length segments are constructed. The same parametric model is applied to two other supporting curves (C’1 and C’2) in fig. 4 on the right. The curve C’1 is identical to C1 but C’2 is obtained by a rotation of C2 in order to be almost horizontal. The two instances of
the same parametric model only differ considering the elements positions but not considering their number or their length. It illustrates how a standard parametric model can be adapted to the designer’s requirements.

The same procedure can be extended to any number of supporting curves. In Figure 5 an example is given with 3 supporting curves and standard polygonal curves composed of 5 elements.

3.2. Applications to form finding

The design tool developed in Grasshopper (Figure 6 on the right) allows defining the standardization parameters with cursors and buttons (number of elements, length, curve type, etc.) and computes elements position. The designer can freely build the supporting curves in Rhinoceros (Figure 6 on the left) and eventually change their shape and position at any moment. The 3D standard model is constructed in real time in Rhinoceros and the designer can visually evaluate the computed shape.

Once the supporting curves and standardization parameters are defined, a lot of possible models can be proposed according to the possible polygonal curve types. In Figure 7 different instances of the same parametric standardized model with the same supporting curves C1 and C2, the same number of polygonal curves (11 curves coloured in black) and the same number of elements per polygonal curve (10 segments) and element length are presented. Only the polygonal curve types differ. Each model is presented in wire frame and underneath a surface model generated from its vertices is also presented. The models A to E are based on the construction methods presented in section 2.1 and in Figure 2. The models F, G and H are based on the same constructions as respectively E, B and C,
excepting the inverted concavity. It can be noticed that even if every model is composed with the same number and kind of elements, the shapes are all different.

Even if all the models are composed with the same standard elements, their shapes are all different and controlled by the designer. The models are geometrically limited by the curves $C_1$ and $C_2$ excepting for models A and D for which $C_2$ gives the orientation of the polygonal curves. In these two last cases the ending curve is computed and constructed. This may be a help to the designer if the limits are not precisely supposed.

Fig. 7. Different instances of the same parametric standardized model with the same supporting curves, number and element length. Only the polygonal curve types differ.
3.2.1. Use of this approach to surface generation:

All the modeling software (Rhinoceros for instance) give a lot of possibilities to generate 3D surfaces. Operations like "surfaces by sections", "sweep", etc. give intuitive ways to model surfaces from curves. Geometrical constraints are taken into account with these operations, but metric constraints (length, curvature) are almost totally missing. The modeling approach with standard elements (hence with metric constraints) gives the possibility to construct surfaces with the metric control of chosen elements. As an example, the surfaces models presented in fig. 7 show how a surface can be deformed while the polygonal curve lengths remain constant. Particularly, in Figure 7, surfaces E and F illustrate a kind of folding of surface A. Actually it is not a real folding because surface areas are not constant. Only some curve lengths are constant. The parametric model allows the control of the shape and the length of plane sections (planes of polygonal curves) of generated surfaces. A thin discretization involves a more accurate control of the section curves length.

4. Conclusion

This paper proposed a parametric approach to construct models composed with standard elements of non-standard surfaces and architectures. All the constructed models have to satisfy a lot of constraints and restrictive hypotheses (one type of standard elements, planar polygonal curves). The examples show that a lot of shapes can satisfy all these constraints and allow a very large field of solutions to the designer. The two main reasons for this variety are that a same set of standard elements can be adapted to different geometrical limits (ending curves), and that the models are composed of polygonal curves (of different types) that can be combined in a lot of different ways. Hence the designer’s place is predominant because he/she controls geometry limits, standardization constraints, curve types, and curves combinations. The parametric model gives instances of compatible structures with all these constraints. It also can give solutions when geometrical limits are not totally restrictive, in the case of one ending curve for instance. The proposed tool gives an assistance to solve a set of constraints defined by the designer.

An application induced by this approach is the possibility to generate surfaces with the control of some metric constraints as input data. It is possible to impose the length (and curvature) of planar section curves of a surface. This is a way of surface construction that is not available in most modeling software.

Among the possible future developments of this approach, the case of different lengths for standard elements can be implemented. Some specific constraints can be added according to technical necessity (specific angles between two consecutive segments for linking components for instance). Last, technical
evaluations in mechanics, thermic, can be carried out to optimize the standardized model.

References