Parametric analysis in Islamic geometric designs

Carlos Barrios and Mostafa Alani

Clemson University
{crbh, mostafh} @clemson.edu

Abstract: A method for the analysis of metamorphosis in traditional Islamic geometrical patterns using a parametric model is presented. The method uses traditional Islamic geometry as a starting point and performs an analysis of pattern's fundamental units and cells and presents metamorphosis operation based on parametric variations of geometry and color.

Keywords: Islamic Geometric Patterns, Pattern Generation, Parametric Modeling, Color, Parametric Variations, Metamorphosis.

1 Introduction

A considerable amount of research has been conducted to answer the question of how to create both traditional and new Islamic Geometrical Patterns (IGP). One of the biggest challenges is the lack of comprehensive manuals from the original designers of the geometrical patterns since some of the traditions date back many centuries and was transmitted by word of mouth. Some scholars have attempted to answer this question turning their attention to the process of replicating traditional patterns and making inferences by observation and analysis. Others have aimed to create new patterns from scratch that look similar to the traditional counterparts using the rules of symmetry groups in the plane. Recent studies show how computation may enrich the understanding of the IGPs by simulating the methods presumably used by the original artists and craftsman while creating new designs that might have not been done before.

In the last few year parametric modeling has served the purpose to create digital models with variable geometry for design. So far there are limited applications of parametric modeling outside the realm of generative design. This is perhaps due to the unpredictable manner in which design exploration is conducted in the context of computational design, thus making parametric models ideal for the initial phases of the design process when many decisions are yet to be made. It helps the designer to know that changes to the model can be made with ease while certain decisions are still temporary.

In this paper we aim to use parametric models as aids in analyzing traditional design patterns. The focus on the study herein presented discusses a methodology that uses parametric models to analyze Islamic geometry and the traditional patterns of

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Islamic decoration and reconstructs the traditional patterns and new original IGP designs in the language. Our exploration focuses on two aspects of the original designs: the geometry of the patterns and the use of color as a parametric attribute in the pattern. Furthermore, we explore the implications of the parametric variations in the topology of the IGPs. This paper presents and discusses an approach towards the use of parametric models to study traditional design patterns and create new designs in the language by means of metamorphosis.

2 Analysis of Islamic Geometric Patterns (IGP)

In general terms, Islamic Geometric Patterns (IGP) are decorative elements that make extensive use of geometric shapes. According to Syed Abas and Amer Salman (Abas et al. 1995) Islamic geometric patterns contain any of the following:

1. Arabic calligraphy
2. Created between 900 AD and 1500 AD by Muslims or non-Muslims in a society where the common practiced religion was Islam
3. Patterns derived from Arabic calligraphy or traditional patterns

For the purpose of our research we build upon the last definition to explore traditional IGPs by observation and use the results of this analysis to derive new IGPs from existing geometries.

Our process starts with screening a set of selected IGPs and decomposing the existing traditional Islamic geometry by examining at the two most important qualities of traditional IGPs: the seed geometry or basic unit for the pattern which we will call *the cell*; and arrangement or *tessellation* which is the actual pattern generated by the repetition of the cell in one of the 17 plane symmetry groups. **Figure 1** shows a pattern and the corresponding cell. When these elements are parameterized it is possible to make variations to generate *new patterns*. Variations can be done by making changes in the geometry of the cell and the tessellation. For the purpose of this research we will focus on making parametric variations on the cell only. Both Issam El-Said (El-Said et al. 1993) and Rima Al Ajlouni (Al Ajlouni 2012) have clearly distinguished between the repeat unit or cell, which is the basic geometrical composition, and the pattern structure, which is the product of systematically repeating the cell.

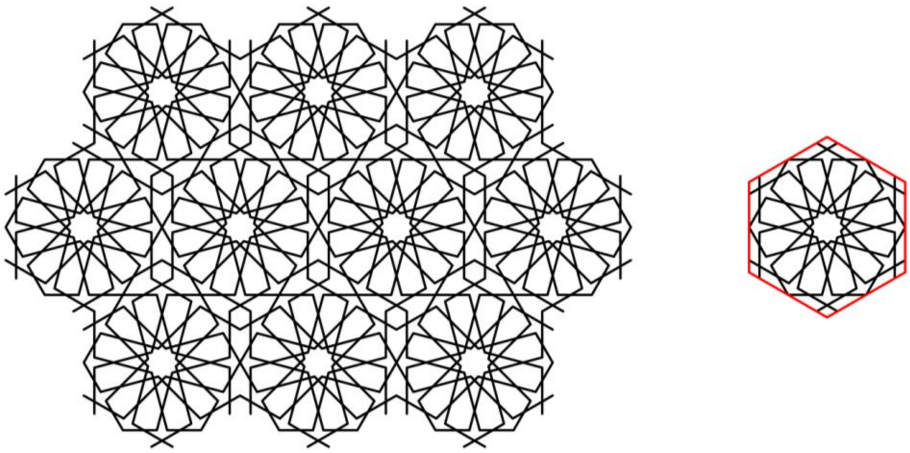

Fig. 1. Islamic geometric pattern and cell

The Fundamental Unit

In most cases, it is possible to examine the IGP cells and find symmetry within the geometry of the cell. It is also possible to dissect the cell into smaller units until the non-symmetrical part is found. We call this part *the fundamental unit.*The fundamental unit in the cell is the group of geometrical elements with non-repeating components(Alani 2015). **Figure 2** shows a cell and the extraction of the non-symmetrical geometrical component, *the fundamental unit*. In principle every cell for every pattern will contain a single fundamental unit which defines the *minimum motif* that is not attainable with symmetry. For the present study, finding the fundamental unit is crucial since this is the place where the parametric variations will start.

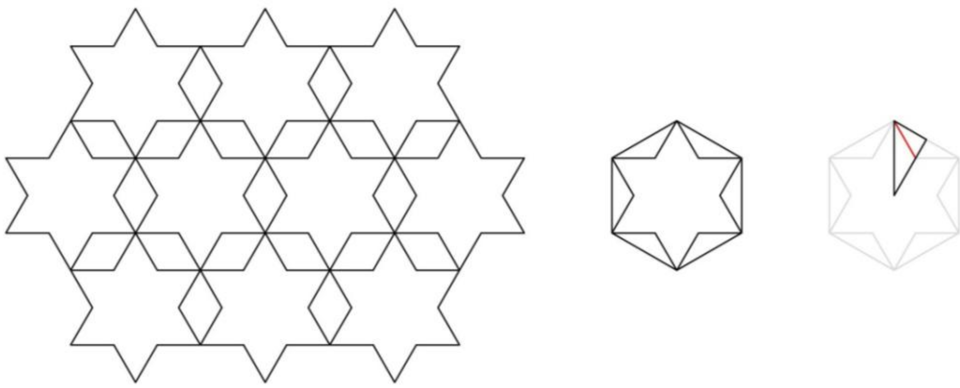

Fig. 2. Islamic Geometric Pattern (IGP), Cell and Fundamental Unit

According to Abas and Salman (Abas et al. 1995), *the cell* is the region with the motif that may be repeated to create the whole geometry, and*the fundamental unit*, that is also a repeated geometrical composition, are essentiallydissimilar. The difference is that the cell creates an *unbounded* design while the fundamental unit does not.Therefore, we differentiate between *the cell* and the *fundamental unit* by the results they create when repeated. Because the method we are defining does need a completed geometry to begin, it is always a good idea to break down the steps of finding the fundamental unit by analyzing the parts of the geometry. The geometrical components of the Cells can be subdivided intotriangular polygons that enclose the fundamental unit, with one of the vertices located at the center of the cell, and the other two vertices located at the edges of the cell **Figure 3** shows a cell subdivision with a corresponding polygon enclosing the fundamental unit [1].

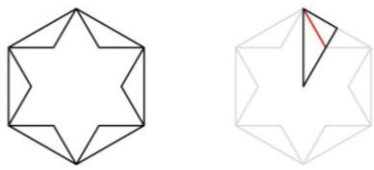

Fig. 3. Subdivision of Cell with polygon enclosing Fundamental Unit

There are no specific rules that govern the relationship between the fundamental unit and the cell; in fact, it actually depends on who originally designed the geometry. Aljamali (Aljamali and Banissi 2004) proved that point by breaking up the steps of creating Islamic geometric patterns into four stages: the planer surface stage, the divisional stage, the artistic stage, and the extension stage. The artistic stage is an important factor to consider in determining the fundamental unit. The combined cell units that contains the fundamental unit are defined as the fundamental region.Aljamali(Aljamali and Banissi 2004) takes the approach of defining the radius of the cell and the angle of rotation of the fundamental unit inside the cell.

Parameterization

Many scholars have done work analyzing the geometry of the IGP by isolating cells and populating them to reconstruct the corresponding pattern. Recently designers have used this knowledge to create modern versions of IGP designs from scratch. Such is the case of *Al Bahar* in Abu Dhabi, a 29 stories tower complex designed by Aedas. This award winning project features a double façade with a triangular pattern simulating a *mashrabiya*, a traditional Islamic lattice typically used to enclose second story large openings in houses and buildings. The method here proposedis able to do both simultaneously. By performing a post-design analysis of an existing geometry

[1] As a principle, cells can hold only one fundamental unit, although we have found some exceptions to this.

and parameterizing the geometrical elements of the fundamental unit, we are able to recreate the existing patterns and at the same time, produce new unique designs from the same parametric model.The proposed methodologydoes not create new geometries from scratch, but relies on the geometry of the existing IGPs. We create a parametric model and performparametricvariations on the existing Islamic geometry based on specific rules to control the results.

The first step is to isolate the cell to delineate the fundamental unit. The fundamental unit is found by decomposing the cell to its constructional non repeating components. This operation will generate a fundamental unit for the pattern, which is defined as the minimum motif that cannot be reached with symmetry. Once the fundamental unit is attained we proceed to reconstruct its geometry with a parametric model; a geometrical construct with variable attributes (properties) that allows the exploration of design variations with ease (Barrios Hernandez 2006). By defining certain rules that govern the parameters, the designer can to explore the patterns in a manual manner.

The variations performed on the fundamental unit will populate to the cell and consequently to the pattern. Thus, the parameterization of the fundamental unit will allow designers to manipulate the whole pattern. We have built a computer representation model to construct the entities of the geometrical patterns with modifiable attributes. The parametric model we constructed consist on parameterizing the geometrical points located inside the boundaries of the fundamental unit's bounding triangle. **Figure 4** shows the parameterized geometry in the fundamental unit.

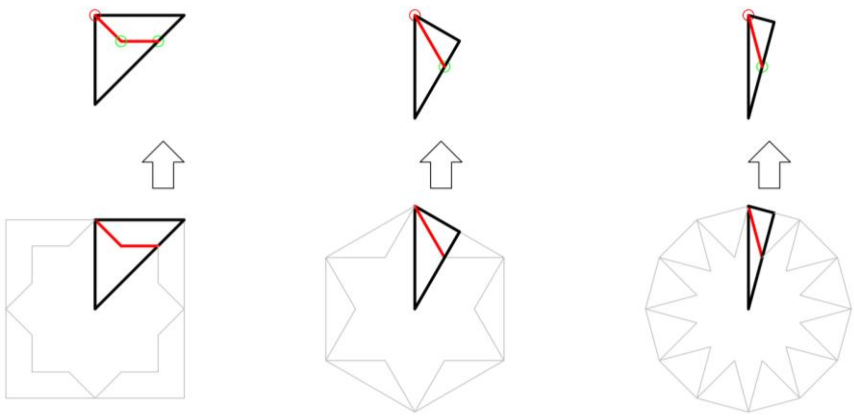

Fig. 4. Parameterization of the Fundamental Unit for IGP cells for 4, 6 and 12 pointed stars

Parametric Variations

Parametric variations are carried on the parametric model of the fundamental unit. These are done by moving the position of the free points to change the geometry of the fundamental unit. As a result, the geometry of the cell and the design of the pattern is transformed. Our research revealed two possible outcomes from thisparticular parameterization: The first enables the preservation of the topology of the fundamental unit and consequently of the cell and, by extension, the pattern. The second outcome transforms creates a topological variation in the cells and corresponding patterns. Both outcomes have advantages and limitations when using them to study the design language of the IGPs, or when creating new designs, which are subject of current research. Nonethelessit is quite possible to derive new patterns from both of them.

Since the parametric model creates a very large number of geometrical variations it is reasonablyimpossible to show all of them. To see all of them would beimpractical, hence designers typically select a representative group of instances that show enough variations and arrange them in a table or matrix form. As a consequence the parametric model becomes the *container* of all possible designs[2]. We found that a convenient way to visualize the variations is through geometric*metamorphosis*[3].A Geometric Metamorphosis, or change in the geometry,provides a convenient way to explore design variations in the form of a timeline showing progressive variations in the parametric model. There are several advantages of applying this process. First, the changes are done in an incremental manner and one parameter at a time, thus providing continuity to the transformations. Second, it becomes possible for the designer to selectively extract the instances that are of interest. This process is akin to computer animation were the instances are extracted as *key-frames* while the rest of the variations are the *in-between* frames. These key instances or *key-shapes*[4]can be *frozen* in time for further study. **Figure 5** shows parametric variations of a fundamental unit and corresponding cells. Any of these instances can be selected to become a key-shape for further studies. In theory there are infinite *key-shapes* in any parametric model, as many as design instances. But it would be impractical to treat them as equal.[5].

[2]While a very large, or even infinite, number of possible designs can be contained with a single parametric model, a typical problem is that designers are only able to see one at a time.

[3]Metamorphosis refers to a biological process that occurs in some living creatures in which changes to appearance occur. In our case we refer to changes in geometry.

[4]The idea of a *key-shape* refers to the state of the geometry at a particular point in time (Kolarevic 2004).

[5]The importance of a *key-shape* is given by the designer and there are many ways in which this can happen. This would constitute a whole research in itself beyond the scope of the present study.

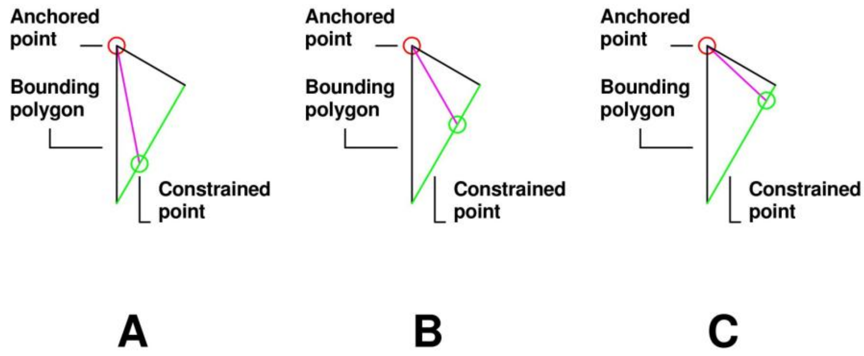

Fig. 5. Parametric variations of the Fundamental Unit for the six point star cell. Anchored points are fix and constrained points move along the green line in the bounding polygon

3 Metamorphosis

In this stage of the process we try to move beyond the re-construction of the patternsin order to explore the geometry and the outcomes of the variationsof the parametric model, both in the cell and the pattern.We had discussed before that the IGPs can be produced by imitating the existing traditional patterns, or that new patterns can be created from scratch. Since our parametric model is able to do both,the metamorphosis will be used to conduct an exploratory study of the geometrical transformations of the cells and the implications on patterns to reconstruct historical IGPs as well as new patterns. We believe that this will help to enhance our understanding of formal qualities of IGPs in relation to the cells and fundamental units

The experiment will be divided into two categories based on the two aforementioned outcomes: 1) *fixed topology*; and 2) *variable topology*. On each of these categories two different kinds of parametric variations will be conducted: a) a parametric variation on geometry; and b) a parametric variation on color.The combination yields four (4) possible kinds of parametric variations as follows:

1. Fixed Topology: parametric variations will change the geometry but not the topology of the cell
2. Variable Topology: parametric variations will change the geometry and the topology of the cell
3. Fixed Topology and Color: Color is introduced as a variable attribute in the parametric model with the fixed topology.
4. Variable Topology and Color: color is introduced as a variable attribute in the parametric model with variable topology.

In the next few sections we will look in detail at each of these possible variations and the effect produced by them when applied to generate the IGPs.

Rules of Spatial Transformation for the Parametric Model

Of the three kinds of points used in the parametric model, anchored (A), constrained (C) and free (F), there are some rules to follow on how they will be varied. 1) Anchored points (A) are fixed and cannot move. 2) Constrained points (C) can travel toward and against the center of the cell along a line. 3) Free points (F) can move anywhere inside the boundary of the fundamental unit All the points located on the outer edge of the repeated polygon are considered anchored points, and the rest are either constrained or linked constrained.One final requirement is that all parametric transformations will start on the same stage of the parametric model. This will help keep track of how parametric variations alter the cell and the IGP. We call this the *initial state.*

Fixed Topology

In the first stage we perform parametric variations where the new geometry should be always be topologically identical to the starting point of the variations. In other words, the total number of points and edges remain identical at every stage in the parametric variation. While the geometry changes, any parametric variation should not result in a topological transformation.

The geometry of the new designs can be generated by adhering to the following rules, for all the points within the fundamental region: 1)Point overlap is not allowed; 2) line overlap is not allowed; 3) Intersections are allowed; and 4) Points should not leave the fundamental region.**Figure 6A** shows parametric variations in the fundamental unit for two examples, the six pointed star and the eight pointed star. We see a change in the geometry of the cell and the corresponding IGP. While the geometry changes, the topology remains identical in all designs.**Figure 6B** shows the corresponding IGP generated by the parametric variations.

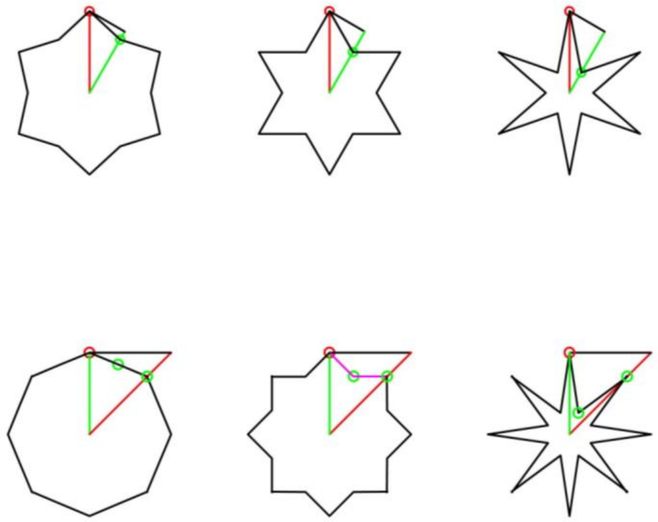

Fig. 6A. Parametric variations of cells and fundamental units of the six (6) and eight (8) pointed stars.

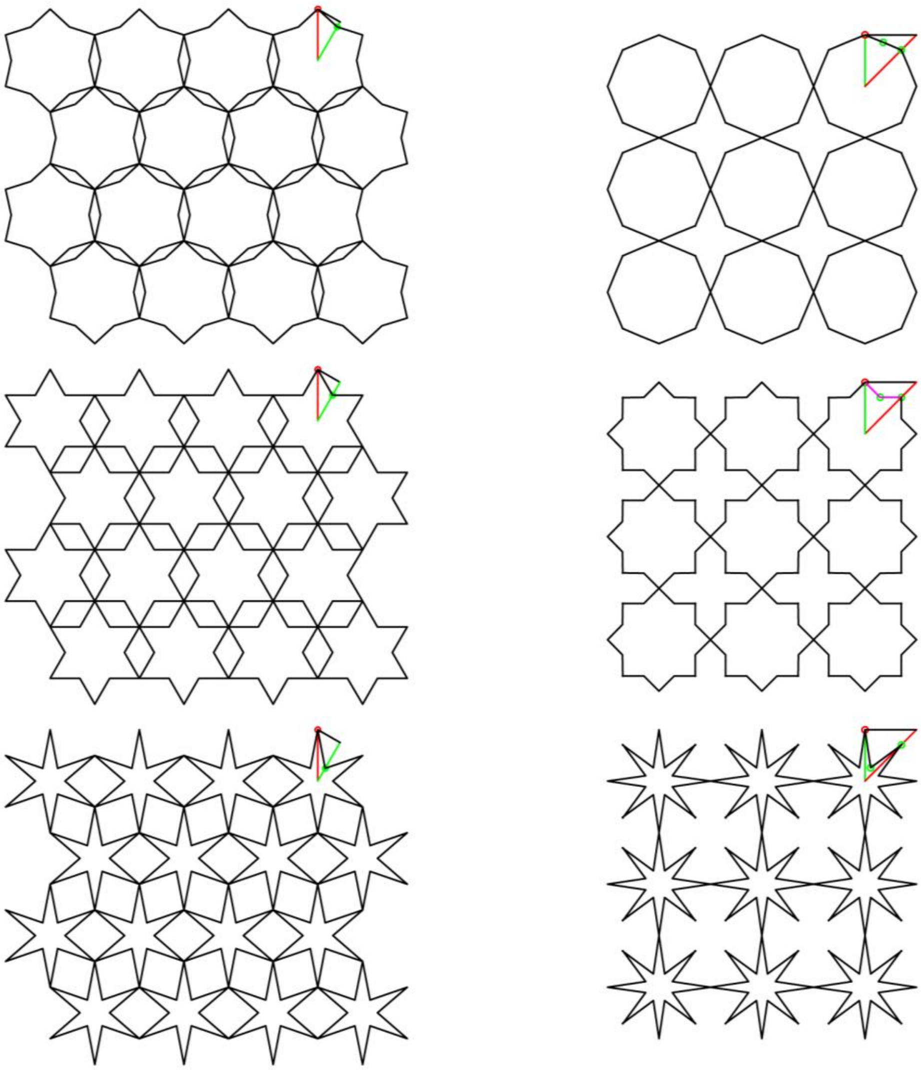

Fig. 6B. Parametric variations of patterns for the six (6) and eight (8) pointed stars IGPs. Note how the topology of the pattern does not change.

Variable Topology

In this section we discuss the second kind of parametric variation. In this case the variations will result in a new geometry and a new topology. We start to perform parametric variations to the initial state of the parametric model. The initial state should be always be topologically identical to the starting point of the variations.

New geometries can be generated by following rules: 1). At least one point should overlap; 2) Lines are not allowed to overlap; 3) Intersections are allowed; and 4) Points should not leave the fundamental region.**Figure 7A** shows parametric variations in the fundamental unit for the previously discussed examples, the six pointed star and the eight pointed star. In this case we see a change in the geometry of the cell and in some circumstances a change in the topology as well. This propagates to the corresponding IGP as shown in **Figure7B**

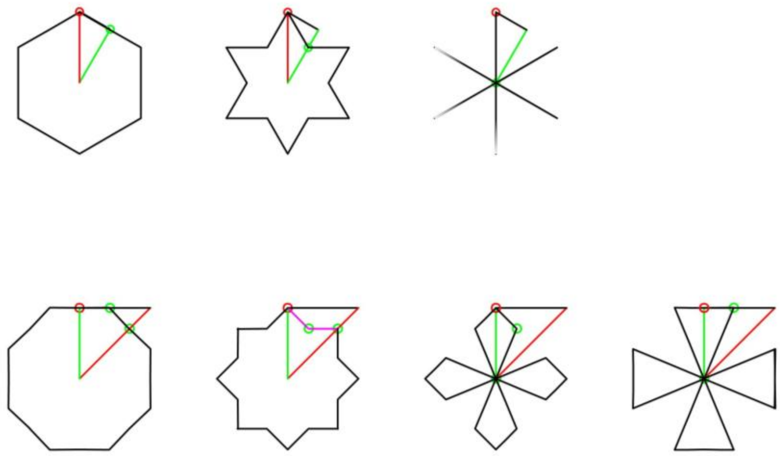

Fig. 7A. Parametric variations in the fundamental units of the six (6) and eight (8) pointed stars. Notice variations in topology.

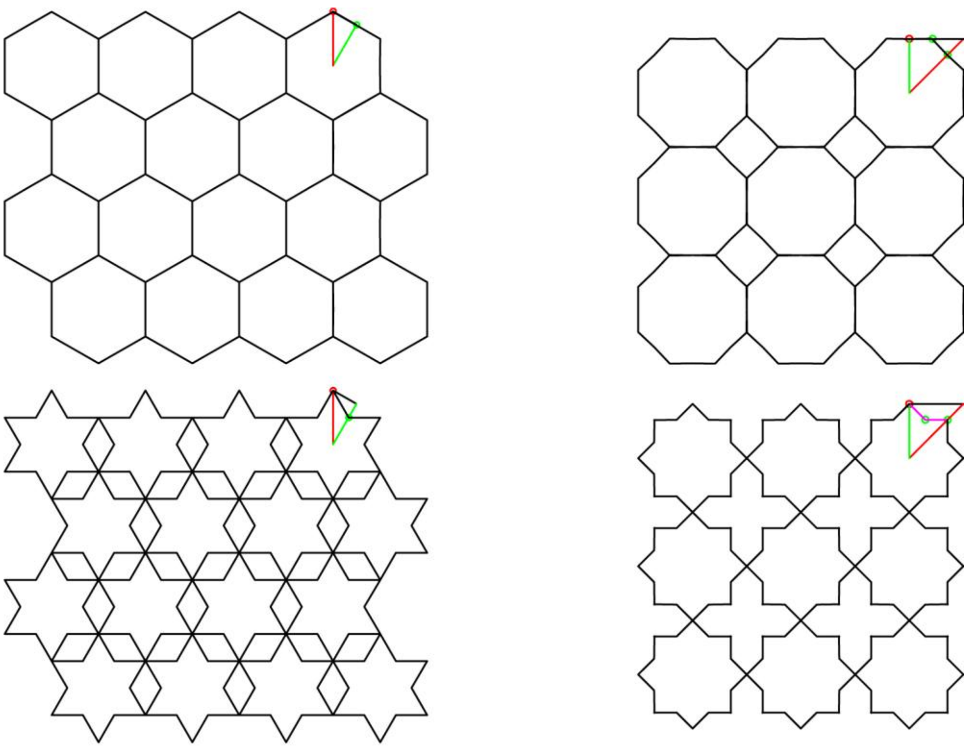

Fig. 7B. Parametric variations in the six (6) and eight (8) pointed stars IGPs. Notice variations in the topology

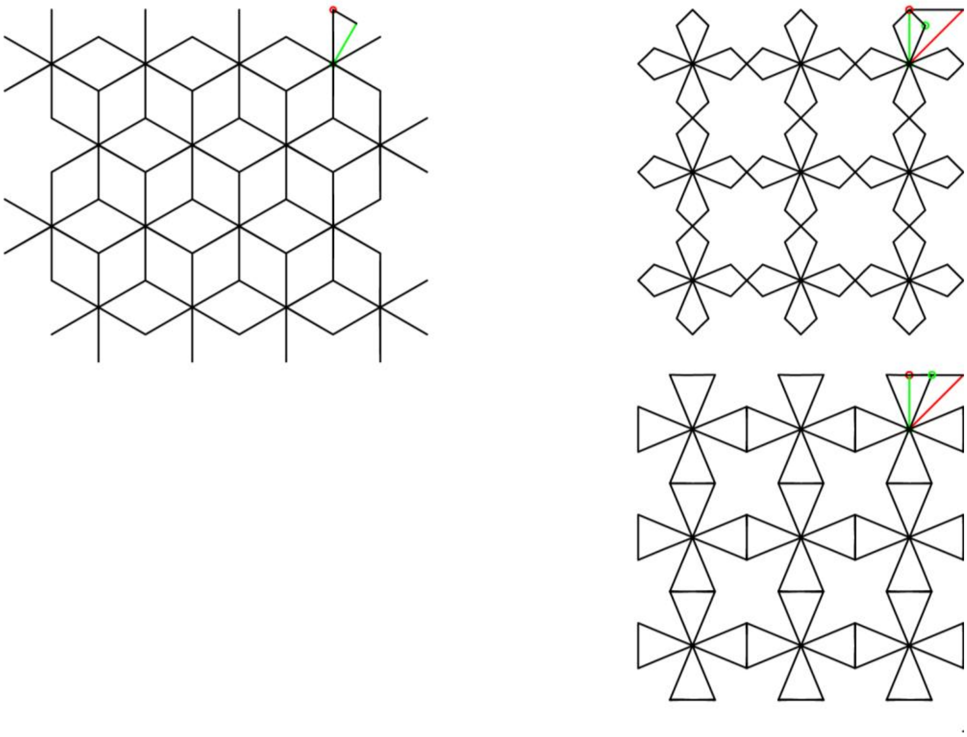

Fig. 7B. Parametric variations in the six (6) and eight (8) pointed stars IGPs. Notice variations in the topology

For the six point star, there is only one parameter to control because it contains only one constrained (C) point. However, the eight-point star has two points to control, one is a constrained point (C) and the other is a free point (F). This extra degree of freedom results in more geometrical variations as we will discuss later. An important aspect is to note the changes in topology of the IGPs as noted in **Figure 7B**. The number of points and line segments can change when points collapse and lines overlap.

Fixed Topology and Color

Another type of variations occurs when we introduce color as a variable attribute. In this case color is added as a parameter in the model. The use of color in the cell will results in a change of symmetry and will have an effect on the overall pattern. For the purpose of our exercise we will only use one of the patterns of the six pointed star

Take for instance the cell in **Figure8**and see how the variations in color changes the symmetry group of the cell, and consequently the pattern. While the geometry remains unchanged, the color schemes in the cell changes the patterns, therefore generating a new one.In our example color becomes an attribute that is subject to parametric changes. Color can be used as a way to distinguish patterns with similar geometry but subtle differences. Color can also be used to generate variations in the pattern and break the regularity of the symmetrical IGPs to create semi-periodic patterns out of regular geometries, or even irregular patterns. Color can also be used as a design attribute to indicate changes in materials, textures or other properties in a building,, to indicate layers in a composition, or to design elements in a Figure/ground manner

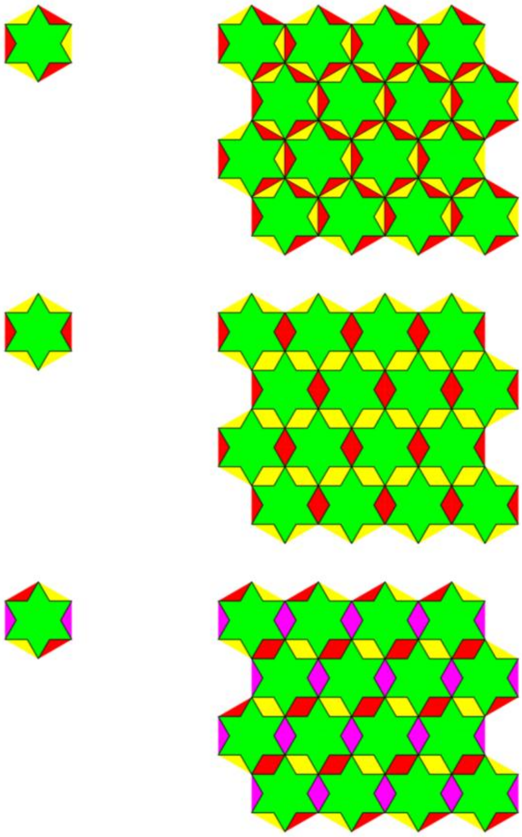

Fig. 8. Parametric variations of the six (6) point star cell due to changes in color and corresponding patterns.

Variable Topology and Color

Introducing color as a variable attribute to the variable topology case can be a little more complicated. This is due to the fact that for this case there are two independent parameters that can change the symmetry group of the IGP. When color is introduced to the fixed topology IGP, all colors in the cell will be preserved in the pattern. This is not the case with the variable topology.

In the variable topology, some of the parametric variations will produce new shapes that might not have an associated color. Or some of the actual colored shapes will disappear when the parametric transformations evolve to change the topology. If colors represent specific properties they may not be present in the pattern. This does not mean that it might not be useful, but it probably requires an extensive study of the implications of the parametric variations in the cell and the corresponding patterns when the topology of the cell and the pattern changes. We found that this might have an application in analyzing the evolution of the IGPs in time, or when making comparisons between different IGPs.

Continuous Metamorphosis

In section 2.3 when parametric variations where introduced, we spoke about extracting some of the designs as *key-shapes* for further analysis. This process will allow the isolation of a specific design or group of design instances of interest. However, in research we are also drawn to look at the process of continuous and discrete transformations. When parametric variations are done in very small increments the result is akin to morphing. Morphing is an effect used in animation to actively change one form into another in a seamless transition. Morphing has also been used in computational design to explore intermediate stages between two different shapes. Morphing is studied in mathematics as part of set theory and topology.

Morphing caught our interest in this research as a tool for finding relations between similar and different IGPsas it would allow us to stop the transformation process at any convenient time. To morph the IGPs, we wrote a code that started with two extreme design conditions and created all in-between transformations in relation to a timeline. The program allows the user to assign different values to the parameters at different points in time and let the computer find all intermediate stages. Furthermore, the program allows the user to freeze (pause) any instance for further analysis.

In **Figure 9**we present a few snapshots of one of the animations of the six point star cell morphing procedure. In this example we start by selecting a cell unit and determine the point types. Anchored points are drawn in red, and constrained points are drawn in green (Fig. 8A). The code proceeds to *collapse* all the constrained points to the center of the polygon (Fig. 8B) where all green points are at the center of the cell.

The next step is to release one of the green points at a time (Fig. 8C) and have the point travel away from the center of the cell until it reaches the outer boundary of the cell (Fig. 8D). Once this happens, a second constrained point is released and the first point travels towards the center of the cell (Figs. 8E-8F), until it reaches the center of the cell (Fig. 8G) to start the process again. This back and forth motion of the constrained points produces all the variations according to the preset number of in-between units in the code.

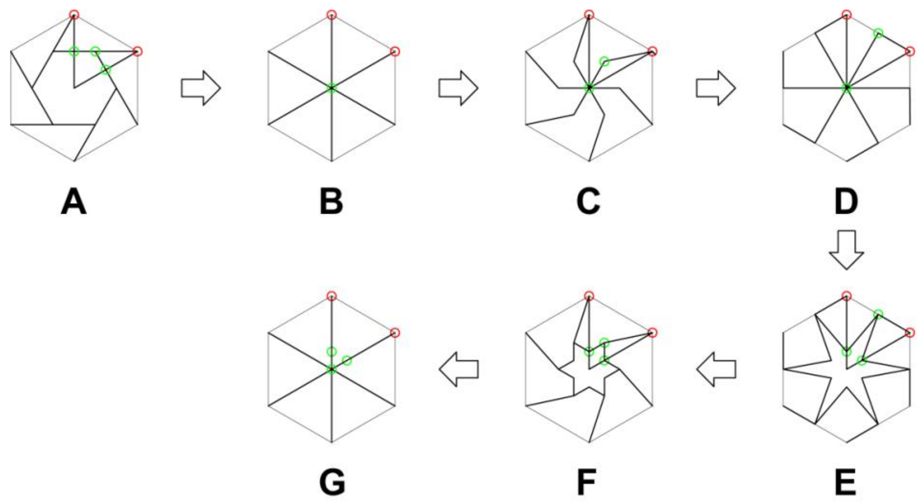

Fig. 9. Six point star cell morphing. Anchored points are indicated in red. Constrained points indicated in green.

A point can travel a specific distance within a specific preset amount of time, thus making the morphing process go faster or slower. The time and distance are variables making the morphing process smoother. The point stops if it intersects a line, overlaps another point, or the point leaves the fundamental region. This procedure will allow us to explore the design domain all possible designs within the selected boundaries. As a result we can obtain not only the original geometries in the IGPs, but many others that are not historical designs. **Figure 10** shows a few of the *key-shapes* of this particular cell where the IGPs enclosed in a green box are existing IGPs and the others are *new designs.*

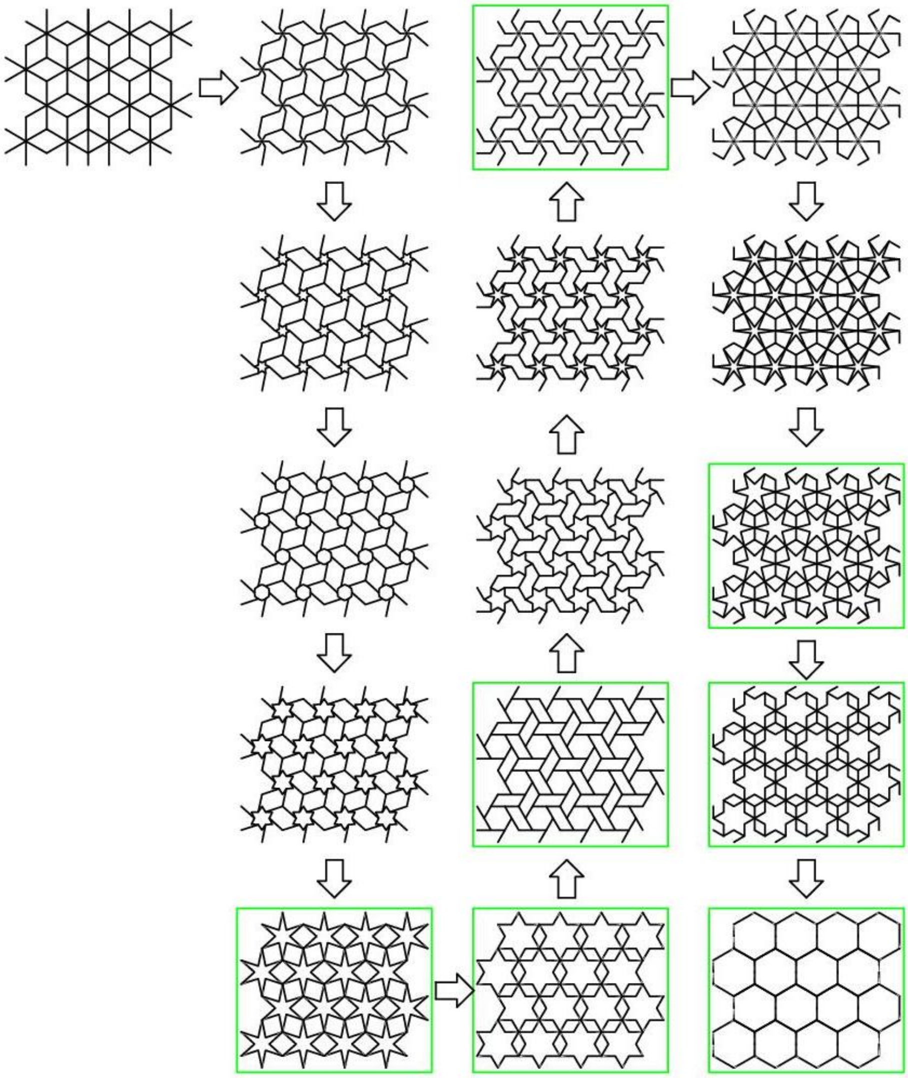

Fig. 10.A selection of key-shapes from a morphing process of the IGPs based on a six pointed star.Shown here are some of the original IGPs indicated with the green square, and the others are new patterns.

4 Discussion

The question that needs to be asked at this point is,*is it possible for a key-shape of one geometry to be equal to an existing Islamic geometry*? The answer is yes. In the case presented in **Figure 10** we found that of the selected 16 key-shapes, 7 of them exist. The rest are considered new patterns[6].

Moving an anchored point (A) can break the continuity of the pattern as well as create additional intersections. In addition to changing the topology, this yields different results, some interesting and some chaotic. This is the subject of further studies.

By exploring all possible combinations of the geometric components within a pattern, it will be possible to identify the desirable systems of proportion for a specific case. It is possible for two distinct geometries that exist in traditional Islamic patterns to have same geometry and topologically, but each one represents a different point in time. Consider this, El-said, in his book Islamic Art and Architecture, demonstrates how to generate an eight-point star. Later in the same chapter, he explains how to generate the octagonal pattern. In other words, he shows two different geometries with two different set of rules to generate them, but both have the same topology and belong to the same symmetry group. El-said expresses the eight-point star as A:B:A, which represents the proportions of the constructional grid, while he expresses the octagon as A+B:B. However, using the method presented in this paper, it will be possible for a designer to manipulate these proportions to reach the octagon from the eight-point star and vice versa with the morphing method(El-Said et al. 1993).

If the original IGPs can be derived from the morphing process, what else can we derive? The answer is that a seemingly unlimited number of geometries can be derived by considering fractions of distance in relation to time. Now, to differentiate geometries, we need a new system that can classify based on when they occur. This predicament implies the second question as to considering the new generated IGPs as genuinely Islamic. If we consider the third proviso that prescribes Islamic patterns as ones derived from other Islamic patterns, then we can say with confidence that the new IGPs are in fact Islamic. Furthermore, some of the new patterns seem to fit the visual imagery of the traditional Islamic counterparts, but this perhaps will require further studies to determine the degree of likeness to the traditional patterns. “Just like nature, there is a universal code, there must be one like this for architecture,” Lalvani (Lalvani 2010) said in a TEDx Brooklyn talk. This method is a step toward in finding the code of the original Islamic geometries, and in generating new geometries through a guided exploration of Islamic geometry.

[6]At the time of writing this paper, we have not found a match for an existing IGP for the other 9 key-shapes. However it is quite possible that they actually exist. On the other hand, we only selected 16 of many more key-shapes, thus it is possible that many more new designs can be created.

Acknowledgments

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