Spherical Perspective
Notational Drawing System for non-Euclidean Geometry

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Abstract. As a traditional design media, drawing usually has limitations in dealing with non-Euclidean geometrical problem, and therefore is highly challenged by the digital tools in contemporary architecture. This paper offers an explanation of the working mechanism of spherical perspective, an alternative projection instrument, to explore the potential of drawing in digital design scenario. Firstly, the paper reviews how architects notated non-orthogonal geometry by introducing perspective projection into the drawing system of Stereotomy in history. Then based on the conclusion from historical research, the paper develops a design tool, which would be able to translate geometry from orthogonal projection system to spherical one to generate non-Euclidean form. In the end, the paper brings further discussions about the formal and spatial effects brought by this new tool, and its potential and difficulty to be developed into professional design and representation media for architectural practice.

Keywords: Form Study, Spherical Perspective, Projective Geometry, Non-Euclidean Geometry, Notational Drawing

1 Introduction

Architect as a profession, in both historical and contemporary context, is relying on certain medias to conceive and represent space. In the historical context, the medias were usually architectural drawings and their associated projection systems. Depending on their different capabilities, these drawings could be defined as either notational or representational tools [1]. The former were usually employed as abstract machines to conceive and notate building form in virtual space, i.e. plan, section, axonometric; the latter were basically used as visual medias to simulate spatial experience, i.e. perspective. In contemporary, drawings have been highly challenged by the development of 3D modeling and rendering technique in architectural digital turn. And they are usually treated as optional instruments for architectural design,
representation and construction. Confronting this situation, the paper aims to explore the potential of drawing in digital environment and to utilize its working mechanism to generate new formal effects.

One fundamental reason of the contemporary situation of drawing would be its limitation as a design media in producing and notating non-Euclidean geometry. And in this paper, we would argue that this limitation is caused by the usually adopted parallel projection system in drawing.

When architecture became an allographic art in Alberti’s time, drawing set was invented for the first time to separate design and construction. Architects was able to design in virtual space by using notational drawings firstly, and then the drawing set would be delivered to building site to instruct the real construction without the presence of the architects themselves [2]. The separation between design and construction required high fidelity in the translation of geometric information from 2D drawing to 3D building. And it is the employment of parallel projection that ensured the correlation between drawing and object in a highly precise way. In this situation, because what could be constructed was highly depending on whether it could be notated by a set of drawings, the mechanism of parallel projection started to have strong limitations on building form.

In his book Projective Cast, Robin Evans elaborated the limitation of notational drawing on the classical building form through the lens of the working efficiency of design [3]. In the tradition of architectural profession, the design process was usually interpreted as being operated in an imaginary cube of “architecture working space” (Fig. 1). In this space, plan, section and elevation was imagined as being distributed onto each face of the cube and correlated by invisible parallel projection lines. Based on the relation between drawing set and object, Robin Evans developed a particular theory explaining the orthogonal feature of classical building form. As he stated in his text,

‘It is easiest to deal with three types of drawing if they are perpendicular to each other, and it is easiest to align the principal surfaces of a project with the surfaces on which it is drawn; in consequence, a building will be a box in a box of pictures. So planar, rectangular form is economical too, within the confines of the technique.’ [3]
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If we expand our scope to contemporary architectural practice, we could also find demonstrations for Evans' argument. The architects, who still use drawings as design generator, usually prefer to use certain types of drawing as the dominant media to conceive form. And as the side effect of the choices, the drawing types would affect the form of their projects. In the case that section is the dominant design media, the vertical distribution of space would usually be more complicated than the horizontal; in contrast, project designed mainly through plan drawing usually shows more spatial complexity in the horizontal arrangement than in the vertical. For example, in 21st Century Museum of Contemporary Art designed by SANAA, the distinct character in the horizontal configuration of the project would indicate that the form was conceived mainly through a series of plan drawings.

Generally speaking, contemporary architectural design could be considered as operating in two distinct categories with different approaches of conceiving form. Based on drawing system, one is a problem of translation and transformation from 2D drawing to 3D object. And relying on the digital tool, the other could be described as a solely 3D form problem. Projects in the former category are the consequence of a series of drawings. Projects in the latter category are independent from any kind of 2D media. In the first category, drawings are still the primary generator of design, and therefore have to contain all the geometric information of 3D building. And because the mechanism of informational translation in architectural drawing system, either orthogonal drawing like plan or oblique drawing like axonometric, is based on parallel projection, the building form would be limited as Evans argued. As a result, when architects try to conceive non-Euclidean geometry, drawing would usually cost much more labors than other medias. So it would usually be abandoned in design phase.

However, despite this negative impact, drawing system still has some valuable features, which could be transformed and then utilized in contemporary architectural
design. The geometrical projection from 2D drawing to 3D object involves a process of adding geometric information in the third dimension. So when architect conceives form through a 2D drawing, it usually opens up countless unpredictable possibilities for the 3D result. At the same time, because the way of working on a 2D media to generate 3D object stretches the design process longer just like what generative coding does, and architect does not have to deal with the 3D object directly, the requirement of imagination would be diminished.

Confronting the drawing’s contemporary situation, the paper will try to transform the generative power of drawing into a form-finding instrument by combining the drawing’s mechanism with new digital modeling technologies. And in order to accomplish the aim, the research will be subdivided into three parts. In the first part, by interpreting the drawing mechanism of Stereotomy as an analogical precedent, we will make a comparison analysis between the working mechanism of perspective and parallel projection, and then explicate the key features required for developing a new projection tool. In the second part of the research, we will try to construct a generative design instrument by adopting a particular drawing/projection system—spherical perspective, and then try to use this new tool to produce a series of non-Euclidean forms. Eventually, in the third part, we will conduct an experiment using the developed instrument to represent architecture interior space, and then discuss the tool’s potential and limitation to become a professional design media for architectural practice.

2 Analogical Research

2.1 Notational System in Stereotomy

To some extent, architecture history could be interpreted as a history of mutual motivation between the development of design tool and the evolution of formal features. In the history, the drawing technique of Stereotomy was a highly advanced tool for craft man at that time to deal with the complicated geometrical problems, even non-Euclidean problems, of stone cutting. So by investigating the internal mechanism of Stereotomy, we might be able to locate some key requirements for setting up the new drawing system to generate non-Euclidean form.

During the period from Renaissance to Baroque, French architects invented a technique of correlating multiple drawing types and projection methods in a single 2D media in order to define the complicated geometry of stone vault of some projects in both design and construction process. This complex notational system for the cutting and assembly process of stone vault was named later as Stereotomy. At that time, Stereotomy was used to solve non-Euclidean geometric problems, which usually involved angular measurement and surface intersection. As José Calvo-López illustrated in his article “From Mediaeval Stonecutting to Projective Geometry”,

'Such powerful graphical instruments were indispensable in Renaissance and Baroque ashlar construction, since the geometrical challenges posed by the
architecture of the period were quite complex. Archs opened in oblique or sloping walls or at the junction of two walls generate elliptical openings; lunette vaults and arches in round walls bring about cylinder intersections; windows opened in domes involve cylinder and sphere intersections…”[4]

2.2 Reinterpreting Stereotomy

In Stereotomy, using different types of projection lines to correlate multiple drawings in a single 2D media was a great improvement of geometrical control for architects. What made Stereotomy being capable of notating non-Euclidean geometry was that its drawing system contained both perspective projection and parallel projection at the same time. In the history of Stereotomy, the most famous example would be the tromp at Anet by Philibert Delorme. In this case, both plan and section drawing were unfolded into 2D, and correlated by the “invisible lines” of the projection system to describe the complicated geometry of each component of the vault (Fig. 2). If we fold these drawings back in three-dimensional digital space, we would find out clearly that the capability of Stereotomy in notating the complicated form derived from the perspective projection in its system [5] (Fig. 3, 4). The Fig. 3 on the left shows the situation, in which only parallel projection lines correlate the plan and section of the vault. And coinciding with Robin Evans’ argument, the projective result in Fig. 3 tends to be more orthogonal. In contrast, the system presented by Fig. 4 on the right follows perspective projection lines to extrude the section drawing of the vault into 3D space. And as a result, the result in Fig. 4 shows more freedom in terms of the formal condition.

![Fig. 2. Philibert Delorme, the tromp at Anet, geometrical construction. Redrawn by the author](image-url)
Although parallel projection can provide precise information for geometrical construction in both virtual and physical process without any distortion, [6] it also confines the formal result into a more orthogonal condition. In another word, the projective geometry tends to be exactly the parallel extrusion of the 2D drawing. Perspective projection, on the contrary, offers architect more freedom in geometrical manipulation because of the changeable relative positions of viewpoint, object and projection plane. Meanwhile, even the geometrical distortion in perspective projection process could be utilized to produce non-orthogonal features in design result. In contemporary architecture, we can also find the examples of using perspective projection in geometrical construction. The works of Preston Scott Cohen could be interpreted as the transformation of Stereotomy. Cohen developed a generative design process in which the distortion of perspective image was utilized to conceive non-orthogonal 3D geometries from 2D media [7]. Therefore, perspective projection has more potential to become a design tool to notate non-Euclidean geometry.

3 Methodology

3.1 Spherical Perspective

According to the research on Stereotomy, the distortion caused by the projection lines in perspective could be utilized to provide more freedom and generative power for architect to produce non-Euclidean form.
So at the beginning of the exploration, a particular type of perspective, spherical perspective, was chosen and then analyzed with the focus on its working mechanism. Spherical perspective as a technique has been highly developed in global mapping or panorama drawing process. And it contains some important features, which could ensure both precision and freedom in the process of notating geometry.

First, spherical perspective could be considered as a particular type of perspective projection. There is a single viewpoint located at the center of the spherical system. And all the projection lines are emitted from the center toward the sphere. In contrast with parallel projection, perspective contains geometrical distortion, which is determined by the relative positions of viewpoint, object and projection plane. However, as object and sphere (projection plane) should be always concentric in a spherical perspective, and as the viewpoint should be always at the shared geometrical center, the distortion in spherical perspective would be actually fixed within the system, and could be utilized as a constant variable to manipulate form.

Second, spherical perspective involves the process of geometrical translation between drawing and object as other projection interfaces do. Therefore, it provides a possibility of using drawing to either notate an existing 3D geometry or generate a new one. And in either way, spherical perspective is more capable of dealing with non-Euclidean form than parallel projection, because of their different working mechanism. Parallel projection transmits the geometric information in certain amount of discrete directions, such as orthogonal projection system only containing three axes. In contrast, projection lines emitted from the central viewpoint of spherical perspective would be capable of covering the object’s geometric information from all the directions in a continuous way, so it would have more freedom and capability in notating the forms that contain continuous variation features such as double curvature condition (Fig. 5).

![Diagram: comparison of working mechanism between orthogonal and spherical projection system. Drawing by the author.](image-url)
Besides the difference between the two systems, there’s another observation from Fig.5. In order to define the position of a point in 3D space based on projection system, usually at least two drawings containing the point’s information have to be provided. For example, in the left drawing of Fig.5, parallel projection system would not be able to define the position of point C in space unless an extra drawing containing the point’s information is provided. However, in spherical perspective, there is only one drawing, which is the sphere. The way of using intersection result between projection lines from different drawings to generate 3D form would not be suitable with spherical perspective. So a new working parameter, besides the 2D configuration of drawing, has to be provided.

3.2 “Distance” as Geometrical Parameter

In the Stereotomy drawing by Philibert Delorme, the vertical distance from each vertex of vault stone to the ground surface was utilized to transmit geometric information from section to side view elevation. And as Tomás García-Salgado also discussed distance as a measurable relationship in his article “distance to the perspective plane”,

“When quattrocento artists sought a sensitive representation of the object seen—not its measurement as a visual or practical problem—they introduced the concept of distance as a measurable relationship between object, pictorial plane, and observer. This is the first geometric foundation upon which the theory of perspective is built.” [8]

So in both parallel and perspective projection, the “distance” between object and projection plane could be considered as a measurable parametric variable, which is able to store useful geometric information. As we know, in the imaginary cube of “architectural working space” (orthogonal projection system), the geometric information of 3D object requires at least two drawings, which are perpendicular to each other, to be determined. For example, a plan drawing on the ground surface describes the 2D configuration of the object in horizontal, and then an elevation or a section drawing provides the object’s geometric information in the vertical direction. However, if there is a working mechanism, in which the distance between object and drawing could be utilized as a parameter to provide the geometric information in the vertical direction as elevation or section drawing does, only one drawing would be adequate for notating the fully 3D geometric information of an object (Fig. 6).
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![Fig. 6. Distance as a variable cooperating with a single drawing to provide the geometric information for a three-dimensional form. Drawing by the author.](image)

In the parallel projection system of Fig. 6, we established a particular projection tool by using parametric modeling technique. When an 3D object is projected onto a flat surface and become a 2D drawing through the new projection tool, the distance between the drawing and the object would be calculated and stored. So the drawing contains both visible configuration and invisible “distance” parameters. For example, in Fig. 6, the plan drawing on the ground, which is generated by the parametric projection tool, contains full geometric information of the 3D object in space. In the system, each vertex X (X presenting all the capital letters in Fig. 6) on the object has a corresponding point X' in the drawing. Point X' in the plan drawing is capable of determining the position of point X on the ground, and then the distance XX' is utilized as a parameter to store the “height” of point X in space. When the plan drawing is projected back to space, the distance XX’ will control how long point X’ should be “moved away” from the ground along the projection line to become point X. And following this operation, after all the points are projected out from the plan drawing and precisely located in space, the 3D object will be generated automatically.

3.3 Developing a Generative Tool based on Spherical Perspective

Based on the parametric projection tool described above, the way of correlating one drawing and the parameter “distance” to store geometric information provides a possibility of notating 3D object in spherical perspective system. And with this drawing-distance mechanism, we developed a generative design tool, which is able to translate any geometrical object from orthogonal projection system to spherical perspective system, to produce non-Euclidean effects. The tool contains two projection systems, orthogonal projection and spherical perspective (Fig. 7). As Fig. 7 shows, the orthogonal projection system is presented as a plane on the left, and the
spherical perspective system is presented as a sphere on the right. At the beginning of the generative process, a primitive geometry is placed on top of the plane in the orthogonal projection system. Meanwhile, the tool will automatically create the same geometry and place it at the center of the spherical perspective system. (The second geometry is invisible in Fig.7, for the clarity.) Then the tool will project these two geometries onto the plane and sphere surfaces based on the different projection mechanisms of the two systems respectively. So in Fig.7, the plan drawing in the left system and the spherical drawing in the right are produced from the same primitive geometry. The plan drawing presents the object’s configuration in horizontal, and the spherical drawing presents the object’s configuration in a panorama condition.

After the two drawings being produced, the “distance” between each point in the plan drawing and its corresponding point on the primitive geometry in the orthogonal system will be calculated and stored as a parameter. Then by reversing the projection, the spherical drawing on the right side will be projected back to 3D space to translate the geometric information from the spherical drawing into a new 3D geometry. And in the reversing projection process, the calculated distance parameters in the left system will determine how long the points of the spherical drawing will be projected back into 3D space. In another word, the distance parameter works as an interface to keep the same drawing-object distance in both projection systems. For example, in Fig. 8, the distance between the object’s vertex Ds and the drawing’s corresponding point Ds’ is the same with the distance DD’ in the orthogonal system presented by Fig. 6.

Finally, a new 3D object will be generated in the spherical perspective system. And because of the continuity of spherical projection, the new object shows certain degree of non-Euclidean formal effect.

3.4 Tool Development 01: Introducing Double Curvature

Any 3D object could be interpreted as existing in two different realities, a “rigid” one and a “soft” one. In the former, the object follows a rigorous geometrical hierarchy of point, line and surface. And it’s usually the vertexes and edges that define the object’s geometric information. In the latter, the object would be directly composed by countless pixels. Comparing to the first reality, the second one has more freedom to transform an object into double curvature condition. Therefore, the spherical perspective based generative tool is developed further to be able to work with a large amount of points (pixels) on the object. With the computational tool, the two drawings will be divided into thousands of points, and the distance parameters will be calculated one by one. Then each point on the spherical drawing will follow the reversing projection process under the control of the distance parameters to generate double curvature geometry. The projection process and the projective result are presented in Fig. 9 and Fig. 10.
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**Fig. 7.** Orthogonal projection system and spherical projection system producing different formal results based on the same drawing-object distance. Drawing by the author.

**Fig. 8.** Spherical projection system as “architectural working space” to generate fully three-dimensional geometry. Drawing by the author.
Fig. 9. Three-dimensional objects being generated in both orthogonal projection and spherical projection, with the “meshwork interpretation” of geometry. Drawing by the author.

Fig. 10. Three-dimensional geometry being interpreted as countless pixels in the process of spherical projection. Drawing by the author.
3.5 Tool Development 02: Altering Projection Type

As orthogonal projection system notates 3D object through plan, oblique projection system produces 3D object through axonometric (Fig. 11). The primary difference between the two systems is whether the projection lines are perpendicular to the drawing. The further development of the generative tool tests its working process with an “oblique version” of spherical drawing. As Fig. 11 and Fig. 12 show, corresponding to the oblique projection lines in the axonometric system in Fig.11, each projection line of the spherical perspective in Fig.12 is rotated in 45 degree with the same geometrical logic. In consequence, by reversing the projection process, a complete new object is generated from the oblique drawing of the tool. This experiment shows the great variability of the generative system to work with different types of drawing.

Fig. 11. Oblique projection being used in the imaginary cube to generate three-dimensional formal result. Drawing by the author

Fig. 12. Projection lines of spherical perspective being rotated into oblique for the geometrical construction. Drawing by the author.
4 Result and Discussion

4.1 Exploring a new Design Media

The generative design tool, which is based on spherical perspective, generated a series of non-Euclidean objects as the research results (Fig. 13). On one hand, these objects contain the features like double curvature and non-orthogonal posture, which could usually be found in the result of coding process or freeform modeling. On the other hand, these objects are highly geometrical constructed, so the location of each point on the objects could be precisely traced back to 2D drawing. The research provides us an opportunity to rethink the problem of the post-rationalization of the generated form in building construction. Instead of treating form generation and rationalization with different geometrical logic, we could now use one single drawing mechanism to define form in both design and fabrication process.

![Variable results from the developed generative design tool, photos of physical model](image)

Fig. 13. Variable results from the developed generative design tool, photos of physical model

However, to allow the further development of this generative tool and to make it available to architecture practice, there are still some difficulties that need to be overcome. One difficulty is that the projection mechanism of spherical perspective is very unusual in the traditional design process. Sometimes the spherical drawing of space would be too strange or complicated to be perceived in a normal way. Confronting this situation, contemporary virtual reality technology might bring new solutions to the problem. Because of the working mechanism of human eyeballs, the spherical perspective could be actually more closed to our real visual experience than planar perspective.

Other difficulties would be how to unfold a spherical drawing into 2D media and how to extract useful geometric information from the unfolded result. Although right now the problem cannot be resolved easily, we still conducted a series of experiments to test the potential of this tool in producing 2D geometrical drawings. Derived from map making techniques for the globe, we created an approximation of spherical projection by developing a mechanism to translate the geometric information from a sphere to another geometrical type, which could be unfolded into flat sheet, for example a cylinder or a polyhedral (Fig. 14, 15). The former, also named as Mercator projection, was a mapping technique developed by Gerardus Mercator in 1569 and
commonly used for navigation [9]. And the most famous example of the latter would be Buckminster Fuller’s “Dymaxion map”, in which the globe is projected into an icosahedron [10]. Regarding to the “Dymaxion map”, John Parr Snyder also indicated in his article “An Equal-Area Map Projection For Polyhedral Globes”,

‘Folded polyhedral globes are easier to assemble without special techniques than spherical globes and serve as instructional tools, but they are bulky and small-scale. Like globes. Unfolded and flattened polyhedral globes form world maps on projections, which can have less distortion than other interrupted projections...’ [11]

Then if we have a working mechanism to define the geometric information in these unfolded cylinder or polyhedral drawings, we would be able to reverse the process and to conceive 3D architecture space. In another word, architects would be able to design on a flat drawing, and then to fold it into a sphere to generate 3D building (Fig. 16).

![Fig. 14. Cylinder, as an approximation of the spherical drawing, being unfolded into two-dimensional surface. Drawing by the author.](image)

5 Conclusion

In these studies above, with the transformation of the drawing/projection mechanism from parallel to spherical, original rectangular geometries were also transformed into objects with non-orthogonal and double curvature features. The studies show that spherical perspective, as an alternative system for architectural projection, could have great potentials to conceive and represent non-Euclidean geometry. Meanwhile, the research also reveals that the internal working mechanism of spherical perspective could provide us a fresh view to explore new techniques in generative design. And then by using the new tool for geometrical construction, we would be able to reactivate the role of drawing in contemporary digital scenario. And this new spherical drawing, being more closely related to our visual experience, could relink human creativity to the reality of our spatial perception.
Fig. 15. Polyhedral, as an approximation of the spherical drawing, being unfolded into two-dimensional surface. Drawing by the author.

Fig. 16. Spherical projection as a way of representing architectural space. Drawing by the author.
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